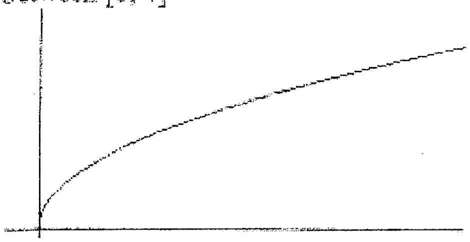
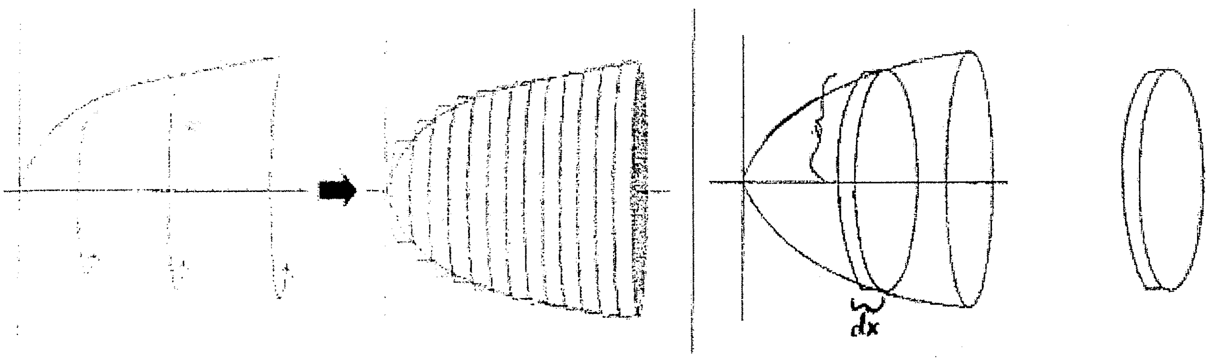
Recall, finding area under the curve $y = \sqrt{x}$ between $[0, 4]$





$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ graph\ curve$

$$\textbf{Disc Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

Example 1: Find the volume of the solid formed by rotating the graph of y = $\sqrt{x}$ around the x-axis between x = 0 and x = 4

$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ graph\ curve$

$$\textbf{Disc Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

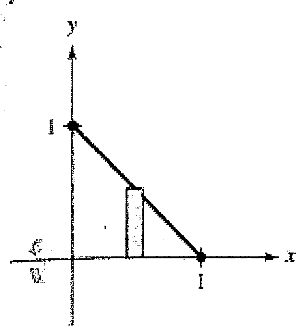
Example 2:

Find the volume of the solid created by $f(x) = 2 - x^2$ revolved around the line $y = 1$

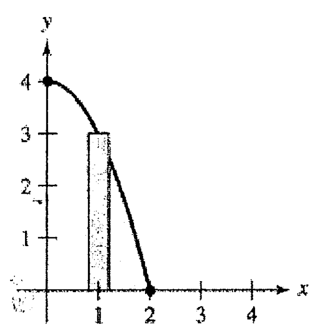
7.2a Disc Method Classwork Problems

Finding the Volume of a Solid In Exercises 1–6, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

1. $y = -x + 1$



2. $y = 4 - x^2$



Radius $[R(x)]$ = *distance from the AOR (Axis of Revolution) to the graph curve*

Disc Method: *Volume* $= \pi \int_{x_1}^{x_2} [R(x)]^2 dx$

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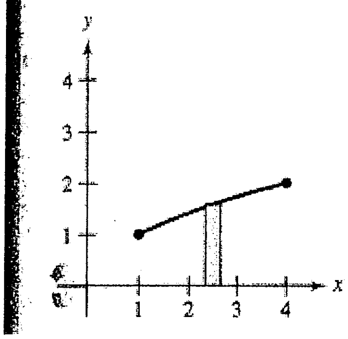
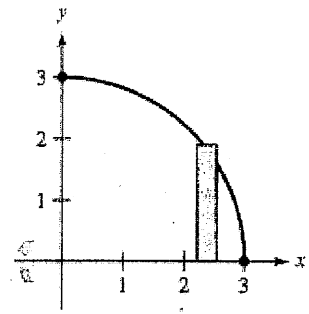
3. $y = \sqrt{x}$



4. $y = \sqrt{9 - x^2}$



Finding the Volume of a Solid **In Exercises 11–14, find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.**

11. $y = \sqrt{x}, \quad y = 0, \quad x = 3$

AOR (axis of revolution) : x-axis

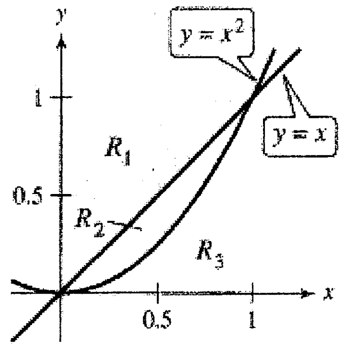
12. $y = 2x^2, \quad y = 0, \quad x = 2$

AOR (axis of revolution) : x-axis

$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ graph\ curve$

$$\textbf{Disc Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

Finding the Volume of a Solid **In Exercises 41–48, find the volume generated by rotating the given region about the specified line.**

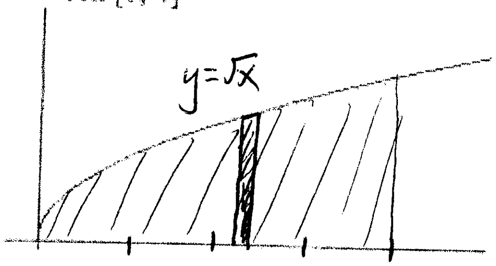


41) R_1 about $AOR\ y = 1$

42) R_3 about $AOR\ x - axis$

Key

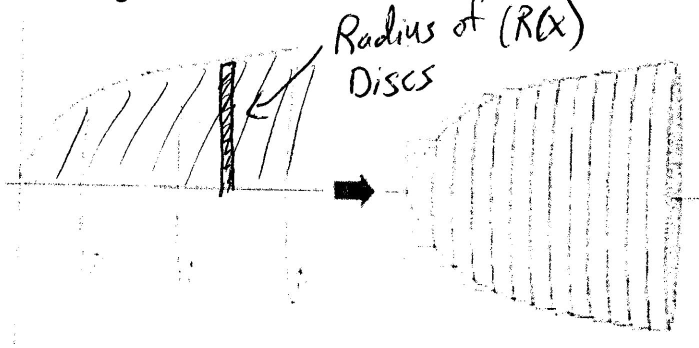
Recall, finding area under the curve $y = \sqrt{x}$ between [0, 4]

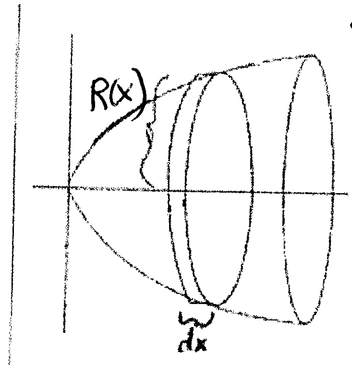


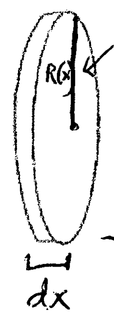
$$Area = \int_0^4 \sqrt{x} - 0\,dx = \int_0^4 x^{1/2}dx$$

$$\rightarrow \left.\frac{x^{3/2}}{3/2}\right]_0^4 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}(0)^{3/2}$$

$$= \frac{2}{3}(2)^3 = \boxed{\frac{16}{3}}$$





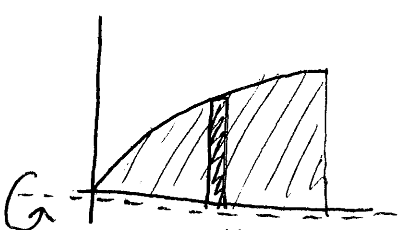


$Area = \pi r^2 = \pi[R(x)]^2$

Radius $[R(x)]$ = *distance from the AOR (Axis of Revolution) to the graph curve*

Disc Method: *Volume* $= \underbrace{\pi \int_{x_1}^{x_2} [R(x)]^2}_{Area} \cdot \underbrace{dx}_{width}$

__Example 1:__ Find the volume of the solid formed by rotating the graph of $y = \sqrt{x}$ around the x-axis between x = 0 and x = 4



$R(x) = \sqrt{x} - 0 = \sqrt{x}$

$$V = \pi\int_{x_1}^{x_2}[R(x)]^2dx \rightarrow V = \pi\int_0^4[\sqrt{x}]^2dx = \pi\int_0^4 x\,dx$$

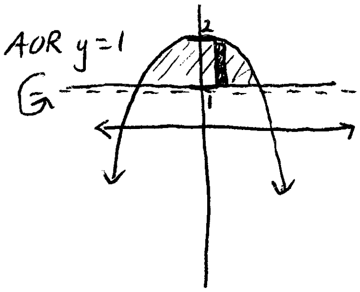
$$= \pi \cdot \left.\frac{x^2}{2}\right]_0^4 = \frac{16}{2}\pi - 0\pi = \boxed{8\pi \text{ units}^3}$$

$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ graph\ curve$

$$\textbf{Disc Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

Example 2:

Find the volume of the solid created by $f(x) = 2 - x^2$ revolved around the line $y = 1$ and $y=1$



$R(x) = \underbrace{2-x^2}_{Top} - \underbrace{1}_{bottom}$

$R(x) = 1-x^2$

*find intersections:

$1 = 2-x^2$

$x^2 = 1$

$x = \pm 1$

$V = \pi \int_{-1}^{1} [1-x^2]^2 dx$

$\boxed{V = \frac{16}{15}\pi \text{ units}^3}$

$V = \pi \int 1 - 2x^2 + x^4 dx = x - \frac{2x^3}{3} + \frac{x^5}{5} \Big]_{-1}^{1}$

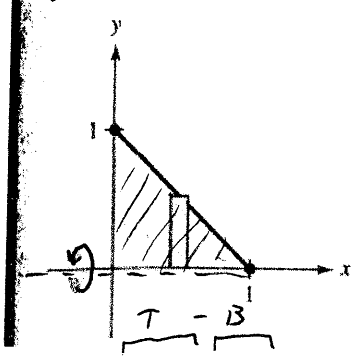
$= 1 - \frac{2}{3} + \frac{1}{5} - \left(-1 + \frac{2}{3} - \frac{1}{5}\right)$

$= \boxed{\frac{16}{15}\pi}$

7.2a Disc Method Classwork Problems

Finding the Volume of a Solid In Exercises 1–6, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

1. $y = -x + 1$



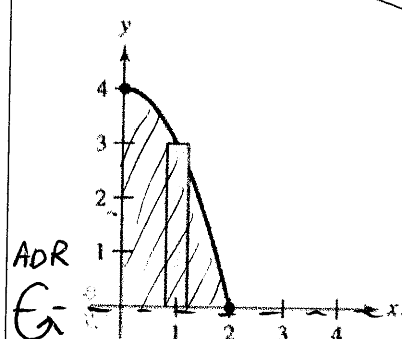
$R(x) = -x+1 - 0$

$R(x) = -x+1$

$V = \pi \int_0^1 (-x+1)^2 dx = \int_0^1 x^2 - 2x + 1\, dx$

$= \pi \cdot \frac{x^3}{3} - \frac{2x^2}{2} + x \Big]_0^1 = \frac{1}{3} - 1 + 1 = \boxed{\frac{1}{3}\pi} \text{ units}^3$

2. $y = 4 - x^2$



$R(x) = 4 - x^2 - 0$

$V = \pi \int_0^2 [4-x^2]^2 dx$

$= \pi \int_0^2 16 - 8x^2 + x^4 dx$

$= \pi \cdot 16x - \frac{8x^3}{3} + \frac{x^5}{5}$

$16x - \frac{8}{3}x^3 + \frac{x^5}{5} \Big]_0^2$

$16(2) - \frac{8}{3}(2)^3 + \frac{2^5}{5} - (0-0-0)$

$= \boxed{\frac{256}{15}\pi}$

Radius $[R(x)]$ = *distance from the AOR (Axis of Revolution) to the graph curve*

Disc Method: *Volume* $= \pi \int_{x_1}^{x_2} [R(x)]^2 dx$

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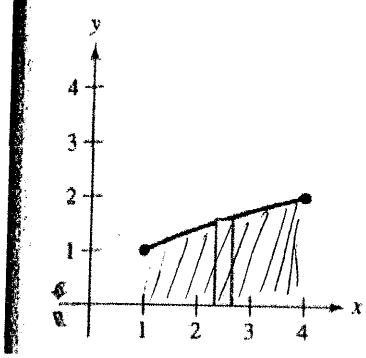
3. $y = \sqrt{x}$



$R(x) = \sqrt{x} - 0$

$V = \pi \int_1^4 [\sqrt{x}]^2 dx$

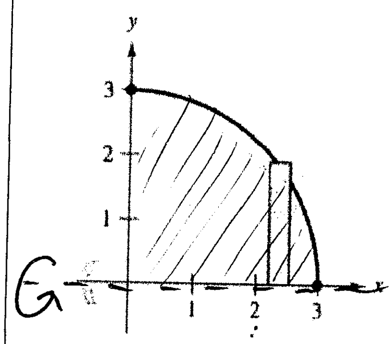
$= \pi \int_1^4 x\, dx$

$\left.\frac{x^2}{2}\right]_1^4 = \frac{4^2}{2} - \frac{1^2}{2} =$

$\frac{16}{2} - \frac{1}{2} = \frac{15}{2}\pi$

$\boxed{V = \frac{15}{2}\pi \text{ units}^3}$

4. $y = \sqrt{9 - x^2}$



$R(x) = \sqrt{9-x^2} - 0$

$V = \pi \int_0^3 [\sqrt{9-x^2}]^2 dx$

$V = \pi \int_0^3 9 - x^2 dx$

$\left. 9x - \frac{x^3}{3} \right]_0^3$

$9(3) - \frac{3^3}{3}$

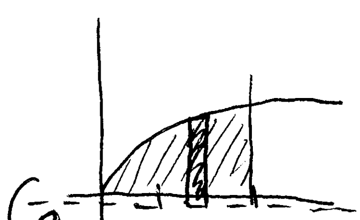
$27 - 9 = 18\pi$

$\boxed{V = 18\pi \text{ units}^3}$

Finding the Volume of a Solid **In Exercises 11–14, find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.**

11. $y = \sqrt{x}$, $y = 0$, $x = 3$

AOR (axis of revolution) : x-axis



$R(x) = \sqrt{x} - 0$

$V = \pi \int_0^3 [\sqrt{x}]^2 dx$

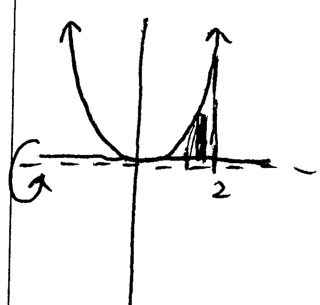
$V = \pi \int_0^3 x\, dx$

$\left.\frac{x^2}{2}\right]_0^3 = \frac{3^2}{2} - 0$

$= \boxed{\frac{9}{2}\pi \text{ units}^3}$

12. $y = 2x^2$, $y = 0$, $x = 2$

AOR (axis of revolution) : x-axis



$R(x) = 2x^2 - 0$

$V = \pi \int_0^2 [2x^2]^2 dx$

$V = \pi \int_0^2 4x^4 dx$

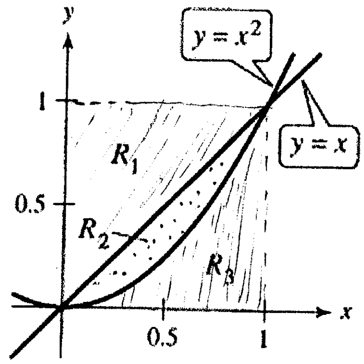
$\left.\frac{4x^5}{5}\right]_0^2 = \frac{4(2)^5}{5} - 0$

$\boxed{\text{Volume} = \frac{128}{5}\pi \text{ units}^3}$

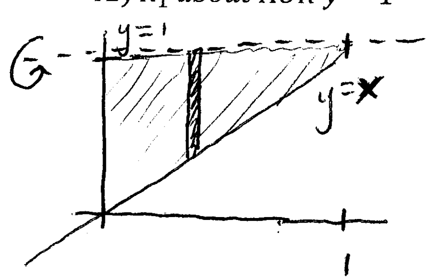
$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ graph\ curve$

$$\textbf{Disc Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$

Finding the Volume of a Solid **In Exercises 41–48, find the volume generated by rotating the given region about the specified line.**



41) R_1 about AOR $y = 1$



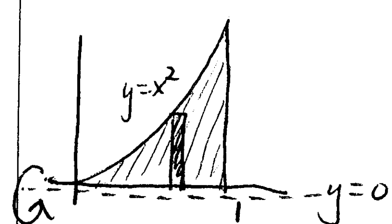
$R(x) = 1 - x$

$$V = \pi \int_0^1 [1-x]^2 dx = \int_0^1 1 - 2x + x^2 dx$$

$$x - \frac{\cancel{2}x^2}{\cancel{2}} + \frac{x^3}{3}\Bigg]_0^1 = 1 - 1 + \frac{1}{3} - (0 - 0 + 0)$$

$$= \boxed{\frac{1}{3}\pi \text{ units}^3}$$

42) R_3 about AOR x – axis



$R(x) = x^2 - 0$

$$V = \pi \int_0^1 [x^2]^2 dx = \pi \int_0^1 x^4 dx$$

$$= \frac{x^5}{5}\Bigg]_0^1 = \frac{1}{5} - 0 = \boxed{\frac{1}{5}\pi \text{ units}^3}$$