7.2a Disc Method Practice Problems Worksheet

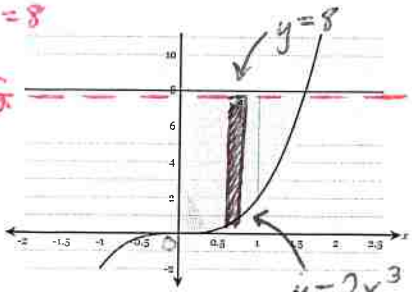
Key

Disc Method: (Top – Bottom)	Disc Method: (Right – Left)
$V = \pi \int_{x_1}^{x_2} [R(x)]^2 \, dx$	$V = \pi \int_{y_1}^{y_2} [R(y)]^2 \, dy$
(expression(s) used above has form: " y = ___ ")	(expression(s) used above has form: " x = ___ ")

1. Let the region R be the area enclosed the function $f(x) = 2x^3$ the horizontal line y=8, and the y-axis. Find the volume of the solid generated when the shaded region is:

a) rotated about the line y = 8

AOR $y=8$



$R(x) = 8 - 2x^3$

* intersection:

$2x^3 = 8$

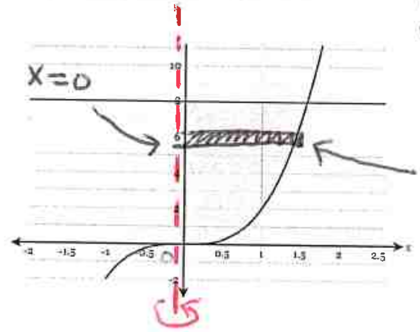
$x^3 = 4$

$x = \sqrt[3]{4} \approx 1.587$

$V = \pi \int_0^{1.587} [8 - 2x^3]^2 \, dx$

$\boxed{V = 65.310\pi \text{ units}^3}$

b) rotated about the y-axis



$y = 2x^3$

$\frac{y}{2} = x^3$

$\sqrt[3]{\frac{y}{2}} = x$

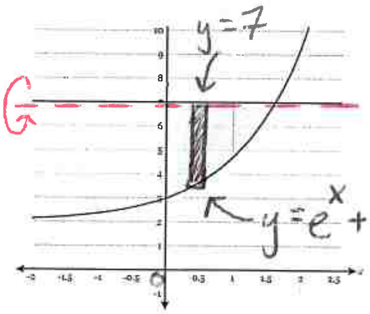
$R(y) = \sqrt[3]{\frac{y}{2}} - 0 = \sqrt[3]{\frac{y}{2}}$

$V = \pi \int_0^8 \left[\sqrt[3]{\frac{y}{2}}\right]^2 dy = \boxed{12.095\pi \text{ units}^3}$

2) Let the region R be the area enclosed the function $f(x) = e^x + 2$, the horizontal line y=7, and the y-axis. Find the volume of the solid generated when the shaded region is:

a) rotated about the line y = 7

$y=7$



$V = \pi \int_0^{\ln 5} [5 - e^x]^2 \, dx$

$\boxed{V = 12.236\pi \text{ units}^3}$

$R(x) = 7 - (e^x + 2) = 5 - e^x$

* intersection:

$e^x + 2 = 7$

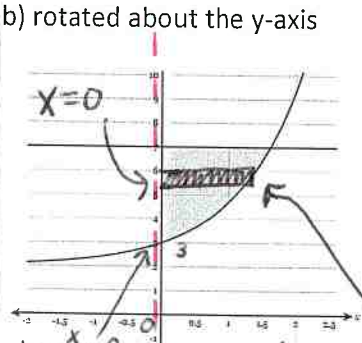
$e^x = 5$

$\ln e^x = \ln 5$

$x \cancel{\ln e} = \ln 5$

$x = \ln 5$

b) rotated about the y-axis



$y = e^x + 2$

$y = e^0 + 2$

$y = 1 + 2 = 3$ (LS)

$x=0$

$y = e^x + 2$

$y - 2 = e^x$

$\ln(y-2) = \ln e^x$

$\ln(y-2) = x \cdot \cancel{\ln e}$

$x = \ln(y-2)$

$V = \pi \int_3^7 [\ln(y-2)]^2 \, dy$

$R(y) = \ln(y-2) - 0$

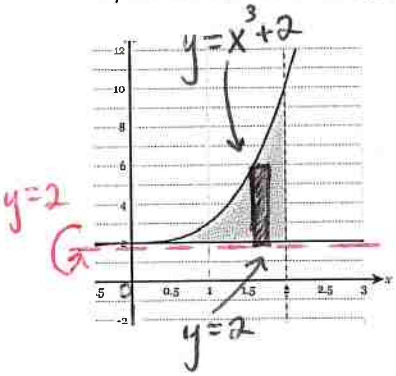
$R(y) = \ln(y-2)$

$\boxed{V = 4.857\pi \text{ units}^3}$

Disc Method: (Top – Bottom)	**Disc Method: (Right – Left)**
$V = \pi \int_{x_1}^{x_2} [R(x)]^2\, dx$	$V = \pi \int_{y_1}^{y_2} [R(y)]^2\, dy$
(expression(s) used above has form: " y = ___ ")	(expression(s) used above has form: " x = ___ ")

3) Let the region R be the area enclosed by the function $f(x) = x^3 + 2$, the horizontal line y=2, and the vertical lines x=0 and x=2. Find the volume of the solid generated when shaded region is:

a) rotated about the line y = 2



$R(x) = x^3+2-(2) = x^3$

$V = \pi \int_0^2 [x^3]^2\, dx$

$\boxed{V = \frac{128}{7}\pi \text{ units}^3}$

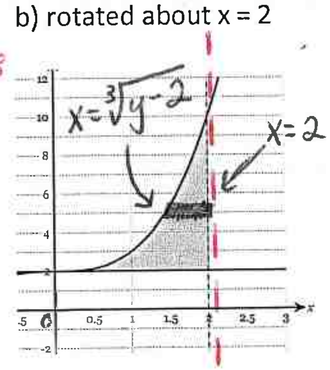
*intersection:

$(\sqrt[3]{y-2})^3 = (2)^3$

$y-2=8$

$y=10$

b) rotated about x = 2



$f(x) = x^3+2$

$y = x^3+2$

$y-2 = x^3$

$\sqrt[3]{y-2} = \sqrt[3]{x^3}$

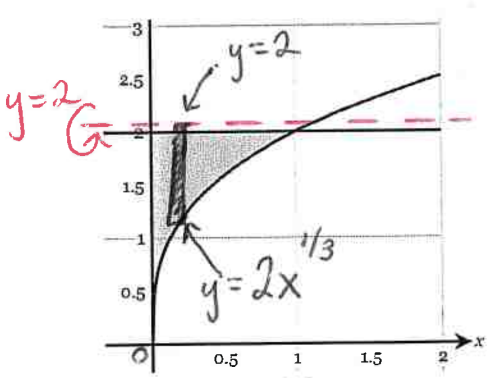
$\sqrt[3]{y-2} = x$

$R(y) = 2-\sqrt[3]{y-2}$

$V = \pi \int_2^{10} [2-\sqrt[3]{y-2}]^2\, dy = \boxed{3.199\pi \text{ units}^3}$

4. Let the region R be the area enclosed the function $f(x) = 2x^{\frac{1}{3}}$, the horizontal line y=2, and the y-axis. Find the volume of the solid generated when shaded region is

a) rotated about the line y = 2



$R(x) = 2-2x^{1/3}$

$V = \pi \int_0^1 [2-2x^{1/3}]^2\, dx$

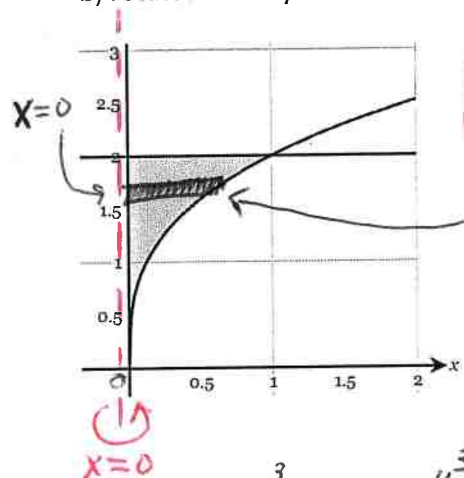
$\boxed{V = 0.399\pi \text{ units}^3}$

b) rotated about y-axis

$y = 2x^{1/3}$

$\left(\frac{y}{2}\right)^3 = \left(x^{1/3}\right)^3$

$\frac{y^3}{8} = x$



$R(y) = \frac{y^3}{8} - 0 = \frac{y^3}{8}$

$V = \pi \int_0^2 \left(\frac{y^3}{8}\right)^2 dy = \boxed{\frac{2}{7}\pi \text{ units}^3}$