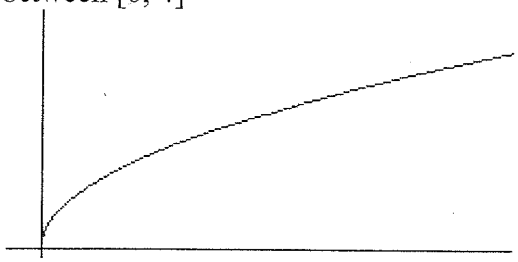
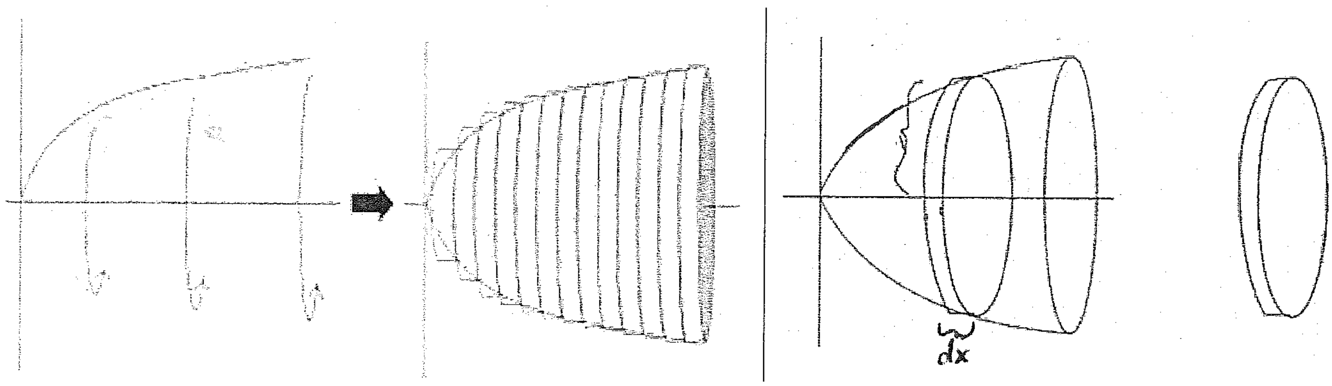
AP Calculus   Ch. 7.2a Notes: Volume by Disc Method

Recall, finding area under the curve $y = \sqrt{x}$
between [0, 4]





Radius [R(x) or R(y)] - distance from the AOR (Axis of Revolution) to the curve

Top/Bottom Vertical radius (rectangles):	Right/Left Horizontal radius (rectangles)
$V = \pi \int_{x_1}^{x_2} [R(x)]^2 \, dx$	$V = \pi \int_{y_1}^{y_2} [R(y)]^2 \, dy$
*Note: All values in integral are in terms of x	*Note: All values in integral are in terms of y
* AOR (Axis of Revolution) is a **horizontal** line	* AOR (Axis of Revolution) is a **vertical** line

Steps for Finding Volume using Disc Method:

1. Graph equations and shade the enclosed region
2. Draw the axis of revolution (AOR) using dashed lines
   a. If horizontal AOR, use Top/Bottom method
   b. If vertical AOR, use Right/Left method
3. **STARTING** at the AOR, draw the radius(rectangle) from AOR to curve: This is the Radius [R(x) or R(y)]
4. Determine whether Top/Bottom or Right/Left method
   a. If vertical radius (rectangles), then use top/bottom method [R(x)]
   b. If horizontal radius (rectangles), then use right/left method [R(y)]
5. (If necessary) Set equations equal to each other to find intersecting end points
6. Plug values into definite integral to find Volume (leave π in your solution)

Top/Bottom Vertical radius (rectangles):	Right/Left Horizontal radius (rectangles)	
$V = \pi \int_{x_1}^{x_2} [R(x)]^2\,dx$ *Note: All values in integral are in terms of x * AOR is a **horizontal** line Radius [R(x) or R(y)] - distance from the AOR (Axis of Revolution) to the curve/graph	$V = \pi \int_{y_1}^{y_2} [R(y)]^2\,dy$ *Note: All values in integral are in terms of y * AOR is a **vertical** line Radius [R(x) or R(y)] - distance from the AOR (Axis of Revolution) to the curve/graph	1. Graph equations and shade the enclosed region 2. Draw the axis of revolution (AOR) using dashed lines a. If horizontal AOR, use Top/Bottom method b. If vertical AOR, use Right/Left method 3. STARTING at the AOR, draw the radius(rectangle) from AOR to curve: This is the Radius [R(x) or R(y)] 4. Determine whether Top/Bottom or Right/Left method a. If vertical radius (rectangles), then use top/bottom method [R(x)] b. If horizontal radius (rectangles), then use right/left method [R(y)] 5. (If necessary) Set equations equal to each other to find intersecting end points 6. Plug values into definite integral to find Volume (leave π in your solution)

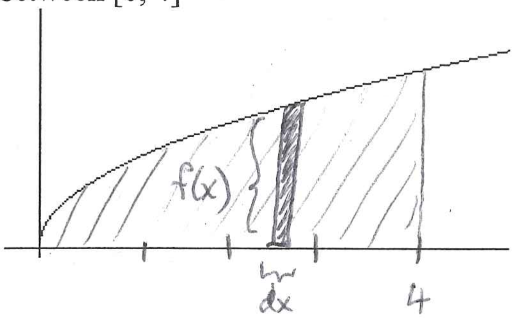
Example 1: Find the volume of the solid formed by rotating the graph of y = $\sqrt{x}$ around the x-axis between x = 0 and x = 4

Example 2: Find the volume of revolution of y = $\frac{1}{2}x$ revolved about the y-axis between y = 0 and y = 4.

Example 3: Find the volume of the solid created by f(x) = $2 - x^2$ revolved around the line y = 1

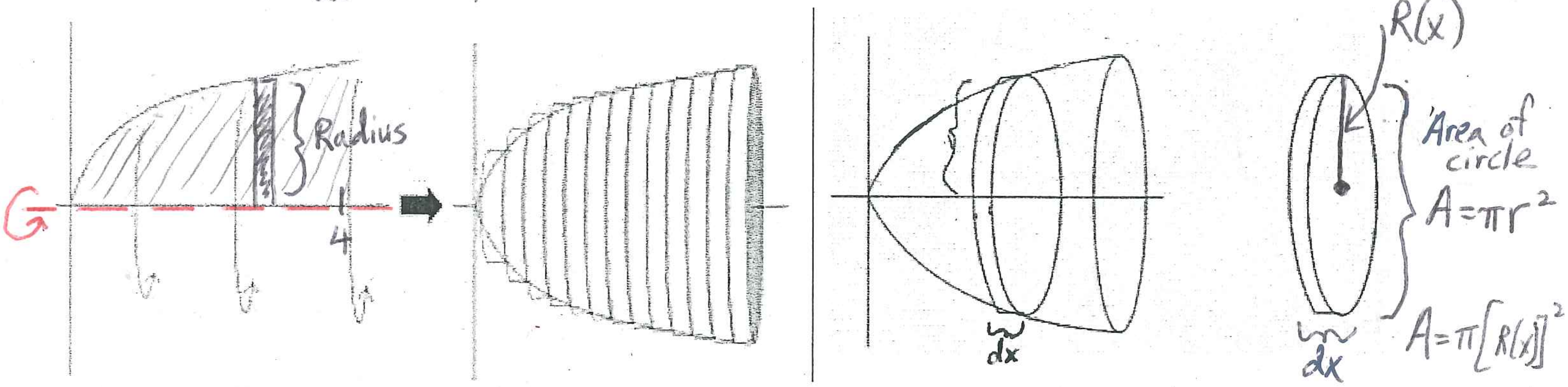
AP Calculus Ch. 7.2a Notes: Volume by Disc Method

Recall, finding area under the curve $y = \sqrt{x}$ between [0, 4]



$$A = \int_a^b \underbrace{f(x)}_{\text{height}} \underbrace{dx}_{\text{width}}$$

$$A = \int_0^4 \sqrt{x}\, dx$$



$$V = \int \underbrace{\pi [R(x)]^2}_{\text{Area}} \underbrace{dx}_{\text{width}}$$

Radius [R(x) or R(y)] - distance from the AOR (Axis of Revolution) to the curve

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Steps for Finding Volume using Disc Method:

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Top/Bottom Vertical radius (rectangles):

$$V = \pi \int_{x_1}^{x_2} [R(x)]^2 \, dx$$

*Note: All values in integral are in terms of x
* AOR is a **horizontal** line

Radius [R(x) or R(y)] - distance from the <u>AOR (Axis of Revolution)</u> to the curve/graph

Right/Left Horizontal radius (rectangles)

$$V = \pi \int_{y_1}^{y_2} [R(y)]^2 \, dy$$

*Note: All values in integral are in terms of y
* AOR is a **vertical** line

Radius [R(x) or R(y)] - distance from the <u>AOR (Axis of Revolution)</u> to the curve/graph

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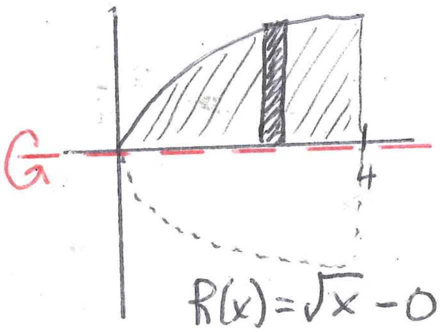
<u>Example 1</u>: Find the volume of the solid formed by rotating the graph of y = $\sqrt{x}$ around the x-axis between x = 0 and x = 4



$$V = \pi \int_a^b [R(x)]^2 dx$$

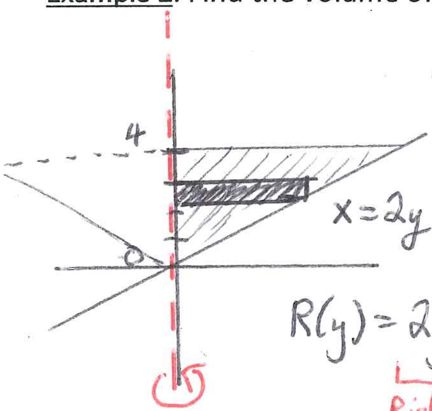
$$= \pi \int_0^4 [\sqrt{x}]^2 dx = \pi \int_0^4 x \, dx = \frac{x^2}{2}\Big]_0^4 = \frac{16}{2} - 0$$

$$= \boxed{8\pi \text{ units}^3}$$

$$R(x) = \underbrace{\sqrt{x}}_{\text{top}} - \underbrace{0}_{\text{bottom}}$$

$$R(x) = \sqrt{x}$$

<u>Example 2</u>: Find the volume of revolution of y = $\frac{1}{2}x$ revolved about the y-axis between y = 0 and y = 4.



x=0

$$V = \pi \int_a^b [R(y)]^2 dy$$

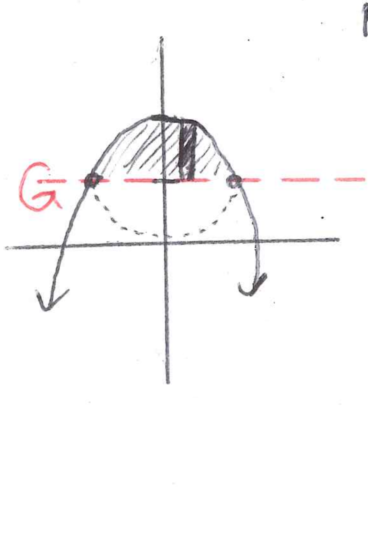
$$V = \pi \int_0^4 (2y)^2 dy$$

$$R(y) = \underbrace{2y}_{\text{Right}} - \underbrace{0}_{\text{Left}} = 2y$$

$$= \pi \int_0^4 4y^2 dy = \frac{4y^3}{3}\Big]_0^4$$

$$\frac{256}{3} - 0 = \boxed{\frac{256}{3}\pi \text{ units}^3}$$

<u>Example 3</u>: Find the volume of the solid created by f(x) = 2 – x^2 revolved around the line y = 1



$$R(x) = \underbrace{2 - x^2}_{\text{top}} - \underbrace{1}_{\text{bottom}}$$

$$R(x) = 1 - x^2$$

*Find intersection (bounds)

set $2 - x^2 = 1$

$x^2 = 1$

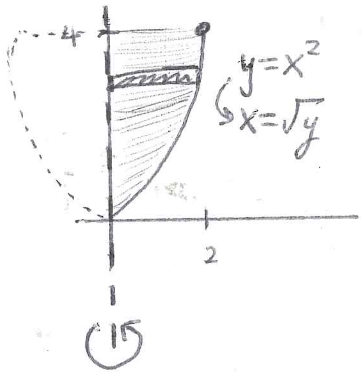
$x = \pm 1$

$$V = \pi \int_{-1}^{1} [1 - x^2]^2 dx$$

$$= \boxed{\frac{16}{15}\pi \text{ units}^3}$$

7.2a Homework p.463-464 #1, 3, 7, 9, 23-27 odds
*Disc Method (Volume) 31, 45, 53

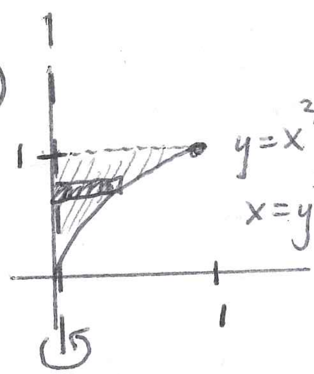
7) Revolve region about y-axis



$R(y)=\sqrt{y}-0=\sqrt{y}$

$$V=\pi\int_0^4\left[\sqrt{y}\right]^2dy=\pi\int_0^4 y\,dy=\frac{y^2}{2}\Big]_0^4=\frac{16}{2}=\boxed{8\pi\text{ units}^3}$$

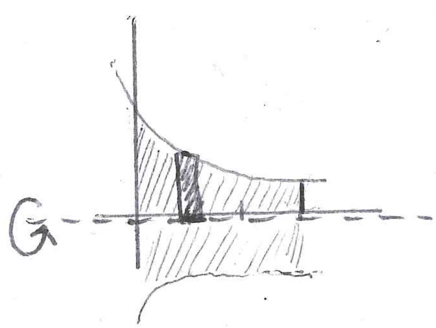
9)



$$V=\pi\int_0^1\left[y^{3/2}\right]^2dy=\int y^3\,dy=\frac{y^4}{4}\Big]_0^1=\frac{1}{4}\pi=\boxed{\frac{\pi}{4}\text{ units}^3}$$

23) Find Volume of solid revolved about x-axis

$y=\frac{1}{\sqrt{x+1}}$, $y=0$, $x=0$, $x=3$] All these lines and graphs contribute as boundaries for enclosed region.



$R(x)=\frac{1}{\sqrt{x+1}}-0=\frac{1}{\sqrt{x+1}}$

$$V=\pi\int_0^3\left[\frac{1}{\sqrt{x+1}}\right]^2dx=\pi\int_0^3\frac{1}{x+1}dx$$

$u=x+1$

$\frac{du}{dx}=1$

$dx=du$

$$\int\frac{1}{u}\,du=\ln|u|\Big]_1^4$$

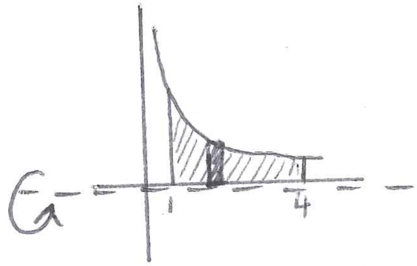
if $x=0$, $u=1$
if $x=3$, $u=4$

$$\ln|u|\Big]_1^4$$

$\ln|4|-\ln|1|$

$=\pi\ln|4|\approx\boxed{4.355}$

25) $y=\frac{1}{x}, y=0, x=1, x=4$ Revolved about x-axis.



$V=\pi\int_1^4 \left[\frac{1}{x}\right]^2 dx$

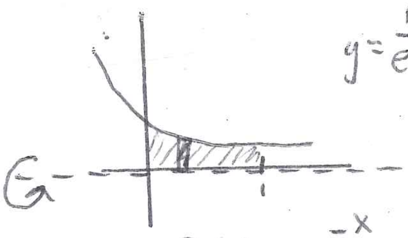
$R(x)=\frac{1}{x}-0=\frac{1}{x}$

$=\pi\int \frac{1}{x^2}dx = \int x^{-2}dx = \frac{x^{-1}}{-1} = \left.\frac{-1}{x}\right]_1^4 = \frac{-1}{4}-\left(\frac{-1}{1}\right)$

$=\frac{-1}{4}+1=\frac{3}{4}$

$=\boxed{\frac{3}{4}\pi \text{ units}^3}$

27) $y=e^{-x}, y=0, x=0, x=1$



$R(x)=e^{-x}-0=e^{-x}$

$V=\pi\int_0^1 \left[e^{-x}\right]^2 dx$

$=\pi\int e^{-2x}dx$

$\pi\int e^u \cdot \frac{du}{-2} = -\frac{\pi}{2}\int e^u du$

$u=-2x$

$\frac{du}{dx}=-2$

$dx=\frac{du}{-2}$

if $x=0$, $u=0$

if $x=1$, $u=-2$

$\left.e^u\right]_0^{-2}$

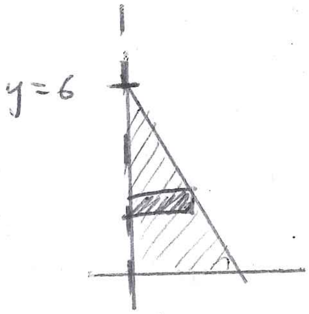
$=e^{-2}-e^0$

$=\frac{1}{e^2}-1$

$=-\frac{\pi}{2}\left(\frac{1}{e^2}-1\right)\approx\boxed{1.358}$

31) $y=3(2-x), y=0, x=0$

Revolve about y-axis



$y=6-3x$

$3x=6-y$

$x=-\frac{1}{3}y+2$

(15) $R(y)=2-\frac{1}{3}y-0$

$=2-\frac{1}{3}y$

*find intersection

$2-\frac{1}{3}y=0$

$2=\frac{1}{3}y$

$6=y$

$V=\pi\int_0^6\left[2-\frac{1}{3}y\right]^2 dy$

$\pi\int 4-\frac{4}{3}y+\frac{y^2}{9}\,dy$

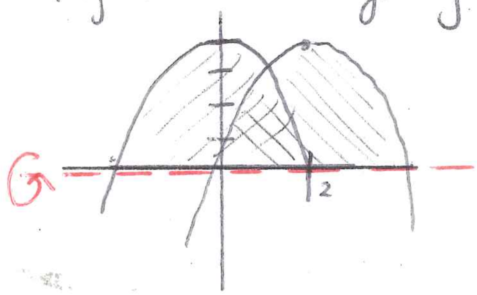
$\left.4y-\frac{4}{3}\left(\frac{y^2}{2}\right)+\frac{1}{9}\left(\frac{y^3}{3}\right)\right]_0^6$

$24-24+8$

$=\boxed{8\pi \text{ units}^3}$

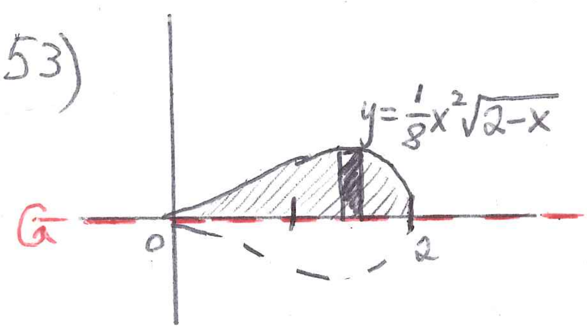
7.2a HW continued

45) Region bounded by $y = 4x - x^2$, x-axis Revolved about x-axis
Region bounded by $y = 4 - x^2$, x-axis Revolved about x-axis



Volumes of the 2 solids are the same because shaded regions are simply translated horizontally.

53)



Top/bottom formula:

$$R(x) = \underbrace{\tfrac{1}{8}x^2\sqrt{2-x}}_{\text{top}} - \underbrace{0}_{\text{bottom}}$$

$$R(x) = \tfrac{1}{8}x^2\sqrt{2-x}$$

$$V = \pi\int_0^2 \left[\tfrac{1}{8}x^2\sqrt{2-x}\right]^2 dx$$

$$= \pi\int \frac{x^4}{64}(2-x)\,dx$$

$$= \frac{\pi}{64}\int 2x^4 - x^5\,dx$$

$$\left.\frac{2x^5}{5} - \frac{x^6}{6}\right]_0^2 = \frac{64}{5} - \frac{64}{6} = \frac{32}{15}$$

$$V = \frac{\pi}{64}\left(\frac{32}{15}\right) = \boxed{\frac{\pi}{30}\text{ units}^3}$$