7.2abc Volume: Disc/Washer/Cross Section Practice Worksheet

Disc Method:	Washer Method:
$Volume = \pi \int_{x_1}^{x_2} (R(x))^2 dx$ Top/Bottom AOR: y= __ | y= __	$Volume = \pi \int_{x_1}^{x_2} ((R(x)^2 - r(x)^2)\, dx$ Top/Bottom AOR: y= __ | y= __ y= __
or	or
$Volume = \pi \int_{y_1}^{y_2} (R(y))^2\, dy$ Right/Left AOR: x= __ | x= __	$Volume = \pi \int_{y_1}^{y_2} (R(y)^2 - r(y)^2)\, dy$ Right/Left: AOR: x= __ | x= __ x= __

1)

Let the region R be the area enclosed the function $f(x) = 2e^x$, the horizontal line $y = 5$, and the y-axis. Find the volume of the solid generated when the region R is

a) Revolved about y = 5

b) Revolved about y = -3

c) Revolved about the y-axis

d) Revolved about x = 2

Cross-Section Area formulas:

Squares : Area = $(base)^2$

Equilateral Triangles: Area = $\frac{\sqrt{3}}{4}(base)^2$

Semicircles: Area = $\frac{\pi}{8}[base]^2$

$base = top - bottom$ or $right - left$

Isosceles Right Triangles (leg): Area = $\frac{1}{2}(base)^2$

Isosceles Right Triangles (hypotenuse): $Area = \frac{1}{4}(base)^2$

Rectangle: Area = base × height

$$Volume = \int_{x_1}^{x_2}(Area)\,dx \text{ or } \int_{y_1}^{y_2}(Area)\,dy$$

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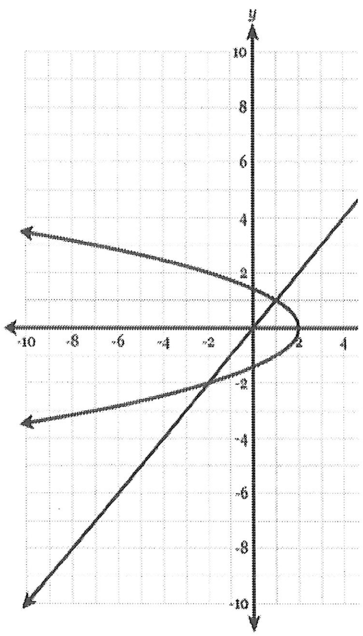
2)

Let the region R be the area enclosed by the the equations
$x = 2 - y^2$ and $x = y$
If the region R is the base of a solid such that each cross section perpendicular to the y-axis, find the volume of the solid for:

a) Isosceles right triangle with a leg on base in region R



b) Semicircles with base on region R

c) Rectangle with height 3 times that of base

Cross-Section Area formulas:

Squares : Area = $(base)^2$

Equilateral Triangles: Area = $\frac{\sqrt{3}}{4}(base)^2$

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$\boldsymbol{base = top - bottom}$ or $\boldsymbol{right - left}$

Isosceles Right Triangles (leg): Area = $\frac{1}{2}(base)^2$

Isosceles Right Triangles (hypotenuse): $Area = \frac{1}{4}(base)^2$

Rectangle: Area = base × height

$\boldsymbol{Volume} = \int_{x_1}^{x_2} (\boldsymbol{Area})\, \boldsymbol{dx}$ or $\int_{y_1}^{y_2} (\boldsymbol{Area})\, \boldsymbol{dy}$

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2)

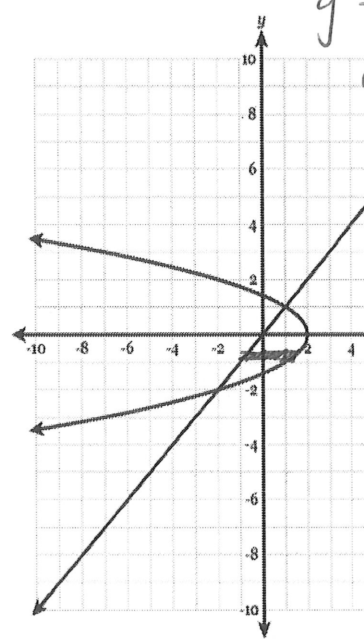
Let the region R be the area enclosed by the the equations
$x = 2 - y^2$ and $x = y$
If the region R is the base of a solid such that each cross section perpendicular to the y-axis, find the volume of the solid for:

*intersection: $y = 2 - y^2$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2,\ y = 1$



a) Isosceles right triangle with a leg on base in region R

base = right − left

$base = 2 - y^2 - y$

$Area = \frac{1}{2}(base)^2$

$Area = \frac{1}{2}(2-y^2-y)^2$

bounds: $y=-2, y=1$

$V = \int_{-2}^{1} \frac{1}{2}(2-y^2-y)^2\, dy$

$V = 4.05 \text{ units}^3$

b) Semicircles with base on region R

$base = 2 - y^2 - y$

$Area = \frac{\pi}{8}[base]^2$

$Area = \frac{\pi}{8}(2-y^2-y)^2$

$V = \frac{\pi}{8}\int_{-2}^{1}(2-y^2-y)^2\, dy$

$V = \frac{\pi}{8}\left(\frac{81}{10}\right)$

$V = \frac{81}{80}\pi$ or

$V = 1.0125\pi \text{ units}^3$

c) Rectangle with height 3 times that of base

$base = 2 - y^2 - y$

$height = 3(2-y^2-y)$

Area = base × height

$Area = (2-y^2-y) \times 3(2-y^2-y)$

$Area = 3(2-y^2-y)^2$

$V = 3\int_{-2}^{1}(2-y^2-y)^2\, dy$

$V = 24.3 \text{ units}^3$

Key

7.2abc Volume: Disc/Washer/Cross Section Practice Worksheet

Disc Method:

$$Volume = \pi \int_{x_1}^{x_2} (R(x))^2 dx$$

AOR: y = ___ Top/Bottom
y = ___

or

$$Volume = \pi \int_{y_1}^{y_2} (R(y))^2 dy$$

AOR: x = ___ Right/Left
x = ___

Washer Method:

$$Volume = \pi \int_{x_1}^{x_2} ((R(x)^2 - r(x)^2)\, dx$$

Top/Bottom
AOR: y = ___
y = ___ y = ___

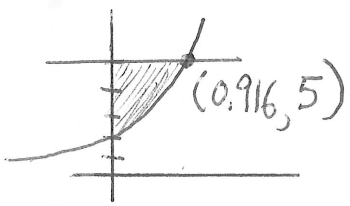
or

$$Volume = \pi \int_{y_1}^{y_2} (R(y)^2 - r(y)^2)\, dy$$

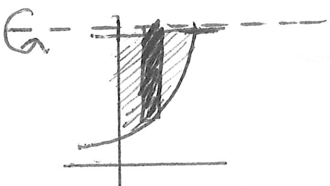
Right/Left
AOR: x = ___
x = ___ x = ___

1)

Let the region R be the area enclosed the function $f(x) = 2e^x$, the horizontal line $y = 5$, and the y-axis. Find the volume of the solid generated when the region R is



a) Revolved about y = 5



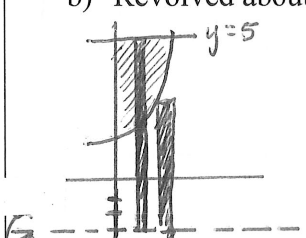
Disc, Top/Bottom

$R(x) = 5 - 2e^x$

$$V = \pi \int_0^{0.916} [5 - 2e^x]^2 dx$$

$V = 3.407\pi$ units^3

b) Revolved about y = -3



washer, Top/Bottom

$R(x) = 5 - (-3) = 8$

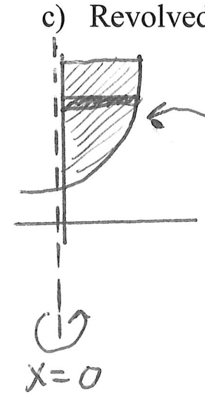
$r(x) = 2e^x - (-3) = 2e^x + 3$

$$V = \pi \int_0^{0.916} 8^2 - (2e^x + 3)^2 dx$$

$V = 21.896\pi$ units^3

c) Revolved about the y-axis

Disc, Right/Left



*convert

$y = 2e^x$

$\frac{y}{2} = e^x$

$\ln\left(\frac{y}{2}\right) = \ln e^x$

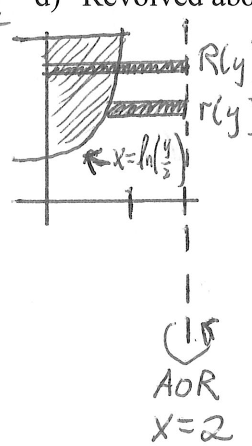
$\ln\left(\frac{y}{2}\right) = x$

$R(y) = \ln\left(\frac{y}{2}\right) - 0 = \ln\left(\frac{y}{2}\right)$

$$V = \pi \int_2^5 \left[\ln\left(\frac{y}{2}\right)\right]^2 dy = 1.035\pi \text{ units}^3$$

d) Revolved about x = 2

* washer, Right/Left



$R(y) = 2 - (0) = 2$

$r(y) = 2 - \ln\left(\frac{y}{2}\right)$

$$V = \pi \int_2^5 2^2 - \left(2 - \ln\left(\frac{y}{2}\right)\right)^2 dy$$

$V = 5.291\pi$ units^3