A.P. Calculus AB   Worksheet 7.2 a,b,c (Disc Method / Washer Method / Volume of Cross Section)

1. Determine the volume of the solid generated by revolving the region bounded by $y = \frac{x^3}{2}$, y = 4, and the y-axis (in the first quadrant)

a) x= 0

b) y = 4

c) y = -2

d) x = 5

1. Find the volume of the solid if the base is bounded by the circle $x^2 + y^2 = 4$

a. rectangles of height 4 (cross section perpendicular to x-axis)

b. equilateral triangles (cross section parallel to x-axis)

c. semicircles (cross section perpendicular to y-axis)

d. isosceles right triangles whose hypotenuses lie on the base of the solid. (cross section parallel to y-axis)

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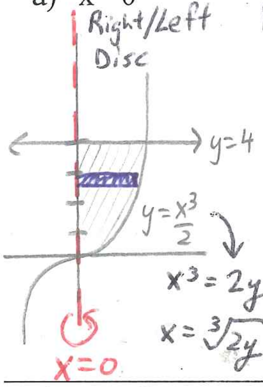
1. Determine the volume of the solid generated by revolving the region bounded by $y = \frac{x^3}{2}$, y = 4, and the y-axis (in the first quadrant)

a) x= 0

Right/Left Disc



$y = \frac{x^3}{2}$, $x^3 = 2y$, $x = \sqrt[3]{2y}$

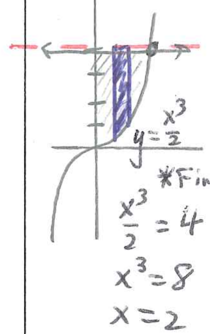
$R(y) = \sqrt[3]{2y} - 0$

$V = \pi\int_0^4 \left[\sqrt[3]{2y}\right]^2 dy$

$V = 9.6\pi$ units^3

b) y = 4

Top/bottom Disc



$R(x) = 4 - \frac{x^3}{2}$

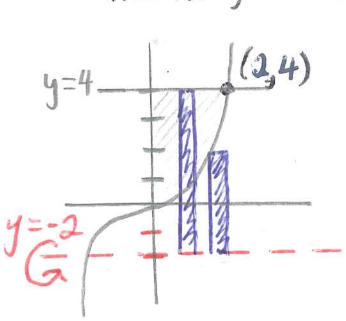
*Find bounds: $\frac{x^3}{2} = 4$, $x^3 = 8$, $x = 2$

$V = \pi\int_0^2 \left[4 - \frac{x^3}{2}\right]^2 dx$

$V = \frac{144}{7}\pi$ units^3

c) y = -2

Top/bottom, Washer



$R(x) = 4 - (-2) = 6$

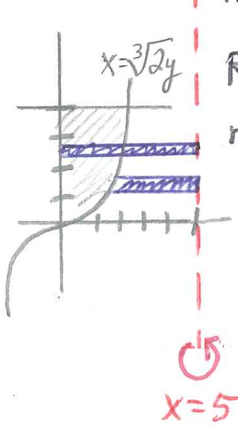
$r(x) = \frac{x^3}{2} - (-2) = \frac{x^3}{2} + 2$

$V = \pi\int_0^2 6^2 - \left[\frac{x^3}{2} + 2\right]^2 dx$

$= \frac{360}{7}\pi$ units^3

d) x = 5

Right/Left, washer



$R(y) = 5 - 0$

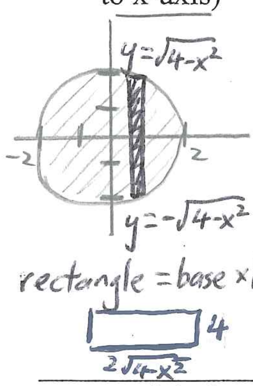
$r(y) = 5 - \sqrt[3]{2y}$

$V = \pi\int_0^4 5^2 - \left[5 - \sqrt[3]{2y}\right]^2 dy$

$V = 50.4\pi$ units^3

1. Find the volume of the solid if the base is bounded by the circle $x^2 + y^2 = 4$

a. rectangles of height 4 (cross section perpendicular to x-axis)



$y = \pm\sqrt{4-x^2}$

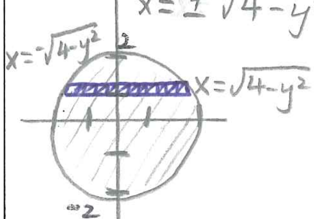
base $= \sqrt{4-x^2} - (-\sqrt{4-x^2})$
$= 2\sqrt{4-x^2}$

rectangle = base × height ($2\sqrt{4-x^2}$ by 4)

$V = \int_{-2}^{2} 2\sqrt{4-x^2} \cdot 4\, dx$

$= \int_{-2}^{2} 8\sqrt{4-x^2}\, dx = 50.266$ units^3

b. equilateral triangles (cross section parallel to x-axis)



$x = \pm\sqrt{4-y^2}$

base $= \sqrt{4-y^2} - (-\sqrt{4-y^2})$
$= 2\sqrt{4-y^2}$

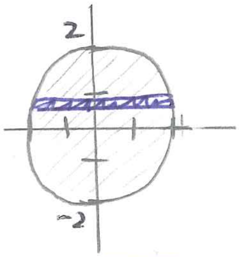
$A = \frac{\sqrt{3}}{4}(\text{base})^2$

$V = \frac{\sqrt{3}}{4}\int_{-2}^{2} \left[2\sqrt{4-y^2}\right]^2 dy$

$= \frac{\sqrt{3}}{4}\left(\frac{128}{3}\right)$

$= \frac{32\sqrt{3}}{3}$ units^3

c. semicircles (cross section perpendicular to y-axis)



base $= \sqrt{4-y^2} - (-\sqrt{4-y^2})$
$= 2\sqrt{4-y^2}$

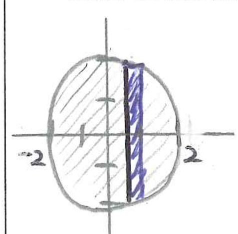
$A = \frac{\pi}{8}(\text{diameter})^2$

$= \frac{\pi}{8}\left(2\sqrt{4-y^2}\right)^2$

$V = \frac{\pi}{8}\int_{-2}^{2} 4(4-y^2)\, dy$

$= \frac{\pi}{8}\left(\frac{128}{3}\right) = \frac{16}{3}\pi$ units^3

d. isosceles right triangles whose hypotenuses lie on the base of the solid. (cross section parallel to y-axis)



base $= \sqrt{4-x^2} - (-\sqrt{4-x^2})$
$= 2\sqrt{4-x^2}$

$A = \frac{1}{4}(\text{hypotenuse})^2$

$V = \frac{1}{4}\int_{-2}^{2} \left[2\sqrt{4-x^2}\right]^2 dx = \frac{1}{4}\left(\frac{128}{3}\right)$

$V = \frac{32}{3}$ units^3