AP Calculus   Ch. 7.2b: Volume by Washer Method

Reviewing Disc Method

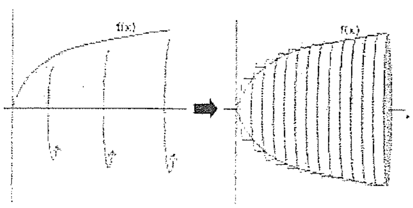
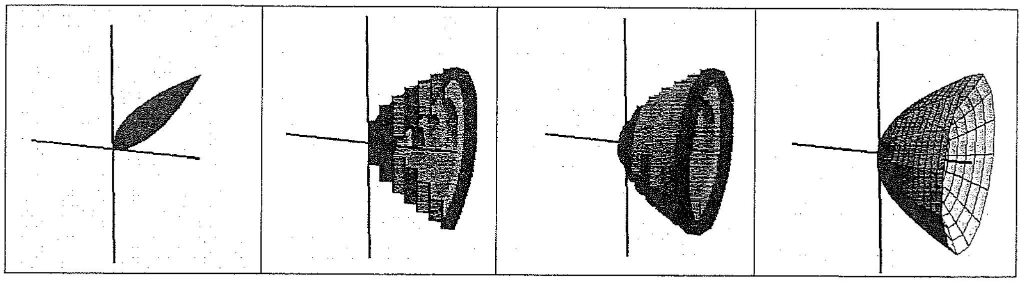
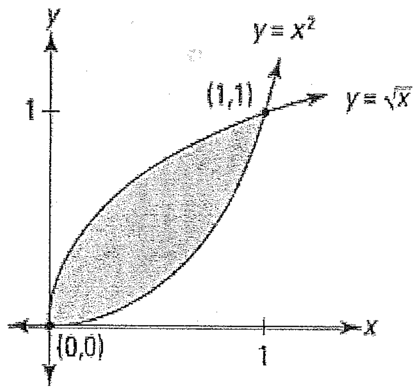


Illustration of Washer Method





Radius [R(x) or R(y)] - distance from the AOR (Axis of Revolution) to the **outer**(further)curve

radius[r(x) or r(y)] - distance from the AOR (Axis of Revolution) to the **inner**(closer) curve

Top/Bottom Vertical radius (rectangles):	Right/Left Horizontal radius (rectangles)
$V = \pi \int_{x_1}^{x_2} [R(x)^2 - r(x)^2] dx$ *Note: All values in integral are in terms of x * AOR is a **horizontal** line (example. y = 2)	$V = \pi \int_{y_1}^{y_2} [R(y)^2 - r(y)^2] dy$ *Note: All values in integral are in terms of y * AOR is a **vertical** line (example. x = -3)

Steps for Finding Volume using Washer Method:

1. Graph equations and shade the enclosed region
2. Draw the axis of revolution (AOR) using dashed lines
   a. If vertical radius (rectangles), then use top/bottom method [R(x) and r(x)]
   b. If horizontal radius (rectangles), then use right/left method [R(y) and r(y)]
3. ** If a **gap** exists between the AOR and the shaded region, then Washer method is required **
4. **STARTING** at the AOR, draw Radius(rectangle) connecting the AOR to the **outer** (further) curve/graph: This is the Radius [R(x) or R(y)]
5. **STARTING** at the AOR , draw radius(rectangle) connecting the AOR to the **inner** (closer) curve/graph : This is the radius [r(x) or r(y)]
6. (If necessary) Set equations equal to each other to find intersecting end points
7. Plug values into definite integral to find Volume

***Important Reminders**:

a. Remember that both your radius (R(x) and r(x)) need to connect to AOR
b. NEVER draw radius R(x) or r(x) between the 2 curves when finding Volume
c. R(x) is ALWAYS longer in length than r(x)

Top/Bottom Vertical radius (rectangles):

$$V = \pi \int_{x_1}^{x_2} [R(x)^2 - r(x)^2]dx$$

*Note: All values in integral are in terms of x
* AOR is a **horizontal** line (example. y = 2)

Radius [R(x)] - distance from the AOR (Axis of Revolution) to the further/outer curve/graph

radius [r(x)] - distance from the AOR (Axis of Revolution) to the closer/inner curve/graph

Right/Left Horizontal radius (rectangles)

$$V = \pi \int_{y_1}^{y_2} [R(y)^2 - r(y)^2]dy$$

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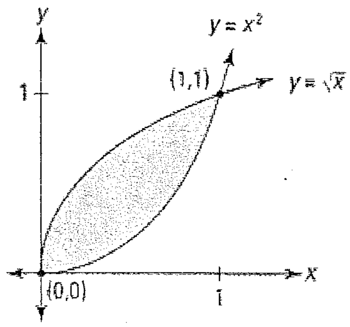
Example 2: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ revolved about the x-axis.

Example 3: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ and the y-axis about the line $y = 4$

Example 4: Find the volume of the solid created enclosed region of $y = x^2$ and $y = \sqrt{x}$ revolving about the line $x = 2$



AP Calculus Ch. 7.2b: Volume by Washer Method (circular rings)

$\pi R^2 - \pi r^2$

Reviewing Disc Method

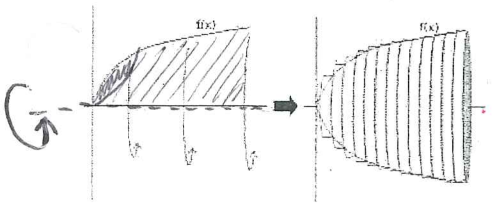
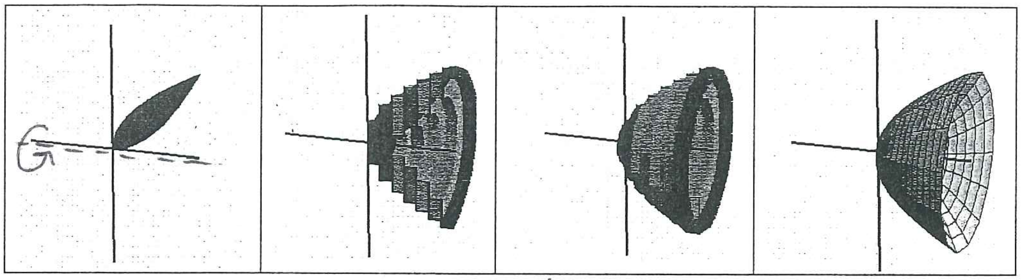
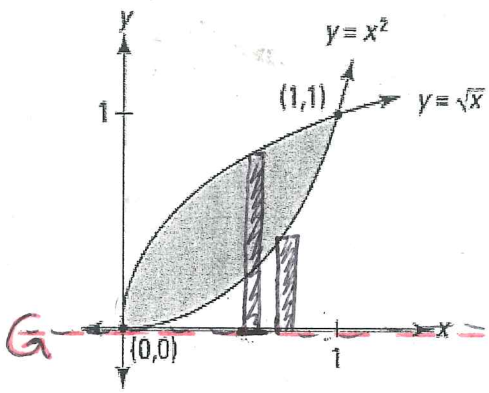


Illustration of Washer Method





washer method required:

$R(x) = \sqrt{x} - 0 = \sqrt{x}$

$r(x) = x^2 - 0 = x^2$

$V = \pi \int_a^b R(x)^2 - r(x)^2 \, dx$

$V = \pi \int_0^1 [\sqrt{x}]^2 - [x^2]^2 \, dx$

$= \pi \int_0^1 x - x^4 \, dx \quad \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$

$= \pi\left(\frac{1}{2} - \frac{1}{5}\right) = \frac{5}{10} - \frac{2}{10} = \frac{3}{10}$

$= \boxed{\frac{3}{10}\pi \text{ units}^3}$

Radius [R(x) or R(y)] - distance from the AOR (Axis of Revolution) to the **outer**(further)curve

radius[r(x) or r(y)] - distance from the AOR (Axis of Revolution) to the **inner**(closer) curve

Top/Bottom Vertical radius (rectangles):	Right/Left Horizontal radius (rectangles)
$V = \pi \int_{x_1}^{x_2} [R(x)^2 - r(x)^2] dx$	$V = \pi \int_{y_1}^{y_2} [R(y)^2 - r(y)^2] dy$
*Note: All values in integral are in terms of x * AOR is a **horizontal** line (example. y = 2)	*Note: All values in integral are in terms of y * AOR is a **vertical** line (example. x = -3)

Steps for Finding Volume using Washer Method:

1. Graph equations and shade the enclosed region
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****Important Reminders**:

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Top/Bottom Vertical radius (rectangles):

$$V = \pi \int_{x_1}^{x_2} [R(x)^2 - r(x)^2]dx$$

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Radius [R(x)] - distance from the AOR (Axis of Revolution) to the further/outer curve/graph

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Right/Left Horizontal radius (rectangles)

$$V = \pi \int_{y_1}^{y_2} [R(y)^2 - r(y)^2]dy$$

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Radius [R(y)] - distance from the AOR (Axis of Revolution) to the further/outer curve/graph

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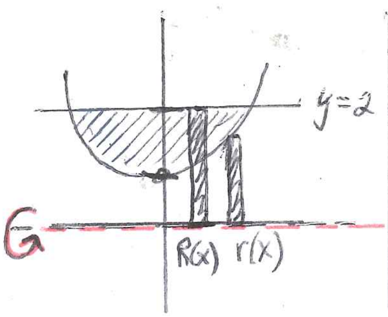
Example 2: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ revolved about the x-axis.



$R(x) = 2 - 0 = 2$

$r(x) = x^2 + 1 - 0 = x^2 + 1$

*Find intersection:

$x^2 + 1 = 2$

$x^2 = 1$

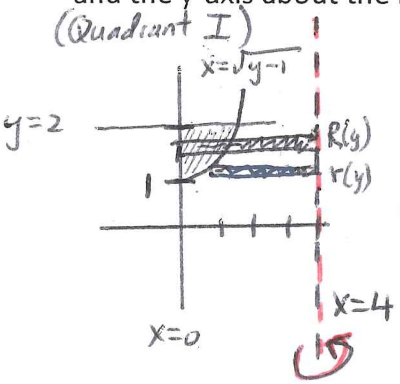
$x = \pm 1$

$$V = \pi \int_{-1}^{1} [2]^2 - [x^2+1]^2 dx = \pi \int_{-1}^{1} 4 - x^4 - 2x^2 - 1\, dx$$

$$\boxed{V = \frac{64}{15}\pi \text{ units}^3}$$

Example 3: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ and the y-axis about the line $x = 4$

(Quadrant I)



$x^2 = y - 1$

$x = \sqrt{y-1}$

$R(y) = 4 - 0 = 4$

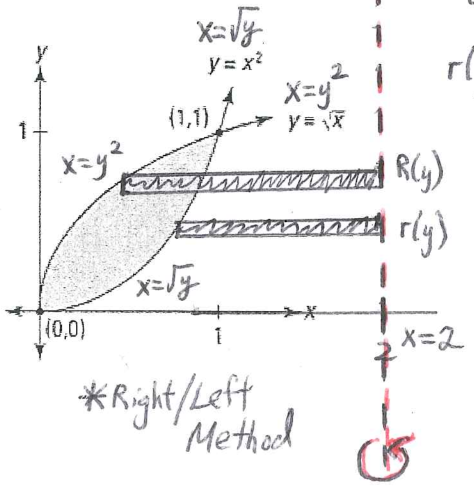
$r(y) = 4 - \sqrt{y-1}$

*Right/Left Method

$$V = \pi \int_{1}^{2} 4^2 - [4 - \sqrt{y-1}]^2 dy$$

$$\boxed{V = 4.833\pi \text{ units}^3}$$

Example 4: Find the volume of the solid created enclosed region of $y = x^2$ and $y = \sqrt{x}$ revolving about the line $x = 2$



$R(y) = 2 - y^2$

$r(y) = 2 - \sqrt{y}$

$$V = \pi \int_{0}^{1} (2-y^2)^2 - (2-\sqrt{y})^2 dy$$

$$\boxed{V = 1.033\pi \text{ units}^3}$$

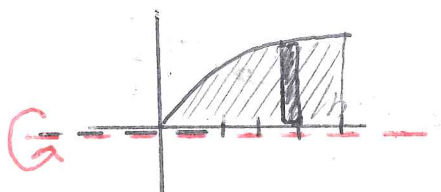
*Right/Left Method

7.2b Homework p.463-464 #5, 11-17 odd, 21, 46

Volume by Washer Method

11) $y=\sqrt{x}$, $y=0$, $x=4$

a) Revolve about x-axis



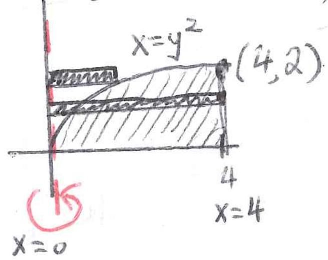
$R(x)=\sqrt{x}-0=\sqrt{x}$

$$V=\pi\int_0^4 (\sqrt{x})^2 dx$$

Disc Method

$$= \left.\frac{x^2}{2}\right]_0^4 = \frac{16}{2} = \boxed{8\pi \text{ units}^3}$$

b) Revolve about y-axis (washer method)



$R(y)=4-0$

$r(y)=y^2-0$

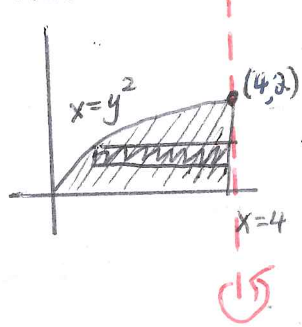
$$V=\pi\int_0^2 4^2-(y^2)^2 dy$$

$$= \left.16y-\frac{y^5}{5}\right]_0^2 = 32-\frac{32}{5}-0$$

$$= \boxed{\frac{128}{5}\pi \text{ units}^3}$$

c) Revolve about $x=4$

Disc Method

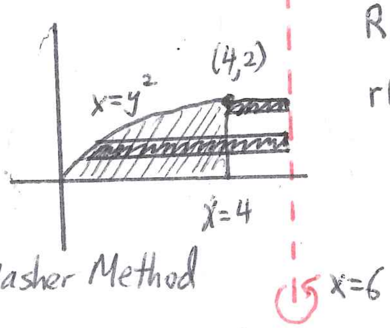


$R(y)=4-y^2$

$$V=\pi\int_0^2 (4-y^2)^2 dy = \pi\int_0^2 16-8y^2+y^4 dy$$

$$\left.16y-\frac{8y^3}{3}+\frac{y^5}{5}\right]_0^2 = 32-\frac{64}{3}+\frac{32}{5} = \boxed{\frac{256}{15}\pi \text{ units}^3}$$

d) Revolve about line $x=6$.



$R(y)=6-y^2$

$r(y)=6-4=2$

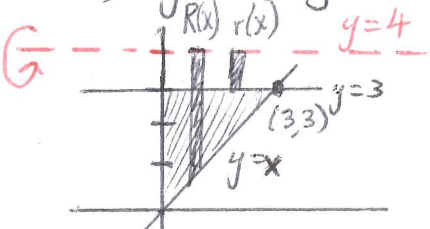
*Washer Method

$$V=\pi\int_0^2 (6-y^2)^2-2^2 dy$$

$$V=\boxed{\frac{192}{5}\pi \text{ units}^3}$$

7.2b HW (continued)

15) $y=x,\ y=3,\ x=0$ Revolved about $y=4$



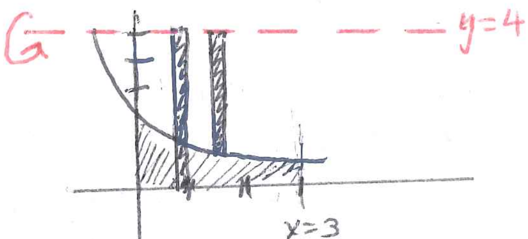
$R(x)=4-x$

$r(x)=4-3=1$

$$V=\pi\int_0^3 [4-x]^2-1^2\,dx = \boxed{18\pi \text{ units}^3}$$

*Washer Method Top/bottom

17) $y=\frac{1}{1+x},\ y=0,\ x=0,\ x=3$ Revolve about $y=4$



$R(x)=4-0$

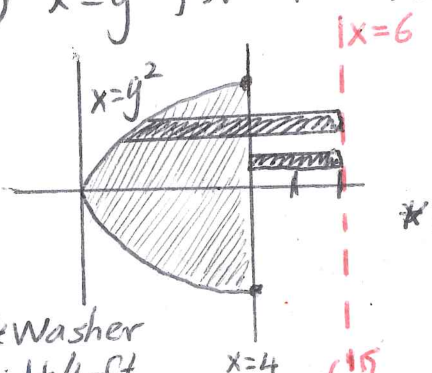
$r(x)=4-\frac{1}{1+x}$

$$V=\pi\int_0^3 4^2-\left[4-\frac{1}{1+x}\right]^2 dx$$

$$\boxed{V=10.340\pi \text{ units}^3}$$

*Washer Top/bottom

21) $x=y^2,\ x=4$ revolved about $x=6$



$R(y)=6-y^2$

$r(y)=6-4=2$

*Find intersection/bounds.

$y^2=4$

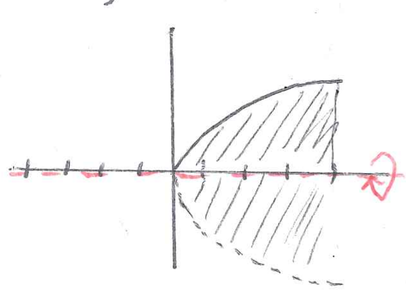
$y=\pm 2$

$$V=\pi\int_{-2}^{2}[6-y^2]^2-2^2\,dy=\boxed{\frac{384}{5}\pi}$$

*Washer Right/Left

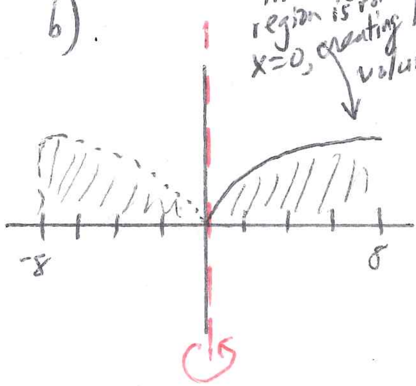
least		greatest
	$a<c<b$	

46) a) x-axis



b)

this large area region is rotated about x=0, creating largest volume.



c) $x=8$