Reviewing Disc Method

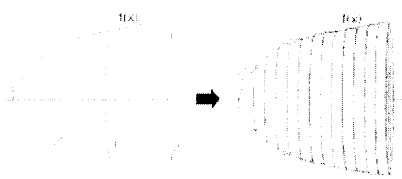
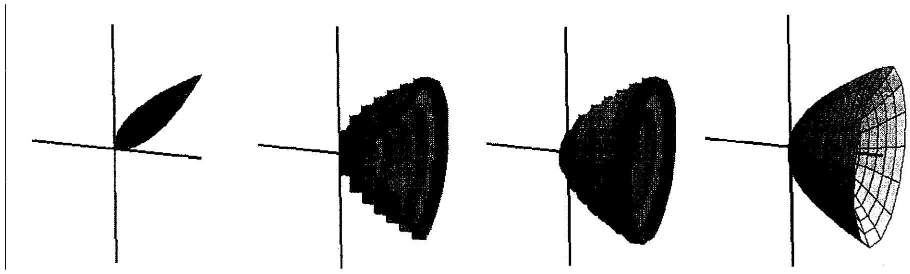


Illustration of Washer Method

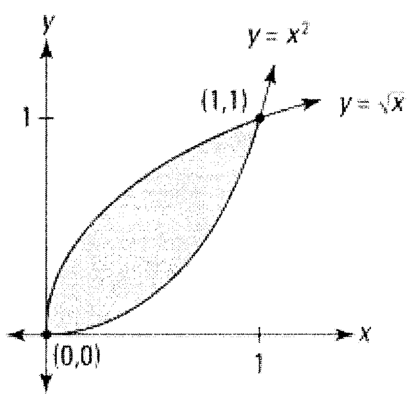


$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{further}\ graph\ curve$

$radius\ [r(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{closer}\ graph\ curve$

$$\textbf{Washer Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

Example 1: Find the volume of the solid created enclosed region of $y = x^2$ and $y = \sqrt{x}$ revolving about the x-axis



Example 2: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ revolved about the x-axis.

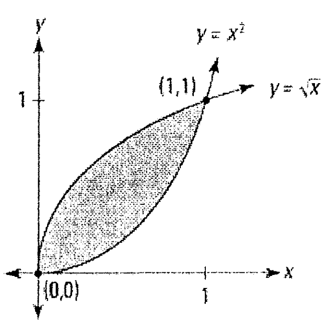
$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{further}\ graph\ curve$

$radius\ [r(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{closer}\ graph\ curve$

$$\textbf{Washer Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

Example 3: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ and the y-axis about the line $y = 4$

Example 4: Find the volume of the solid created enclosed region of $y = x^2$ and $y = \sqrt{x}$ revolving about the line $y = 1$



7.2b Washer Method Classwork Problems

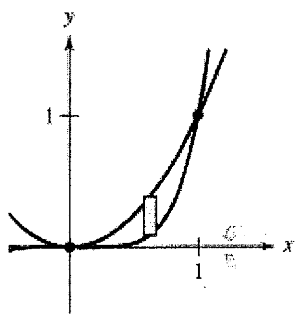
$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{further}\ graph\ curve$

$radius\ [r(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{closer}\ graph\ curve$

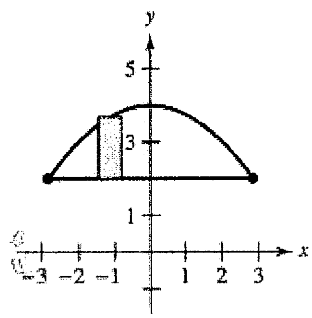
$$\textbf{Washer Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

Finding the Volume of a Solid In Exercises 1–6, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

5. $y = x^2, \quad y = x^5$



6. $y = 2, \quad y = 4 - \dfrac{x^2}{4}$



Finding the Volume of a Solid In Exercises 11–14, find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.

11. $y = \sqrt{x}, \quad y = 0, \quad x = 3$

AOR (axis of revolution) : $y = -3$

12. $y = 2x^2, \quad y = 0, \quad x = 2$

AOR (axis of revolution) : $y = -2$

$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{further}\ graph\ curve$

$radius\ [r(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{closer}\ graph\ curve$

$$\textbf{Washer Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

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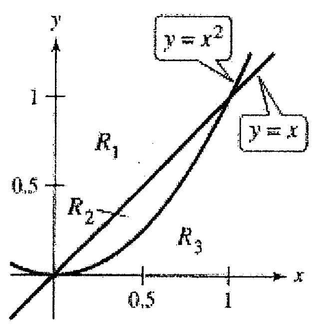
15. $y = x, \quad y = 3, \quad x = 0$

AOR: y = 4

16. $y = \frac{1}{2}x^3, \quad y = 4, \quad x = 0$

AOR: y = 4

Finding the Volume of a Solid **In Exercises 41–48, find the volume generated by rotating the given region about the specified line.**



41) $R_2\ about\ AOR\ y = 1$

42) $R_1\ about\ AOR\ y = -1$

Reviewing Disc Method

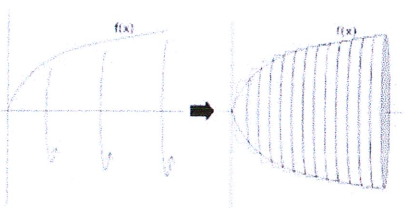
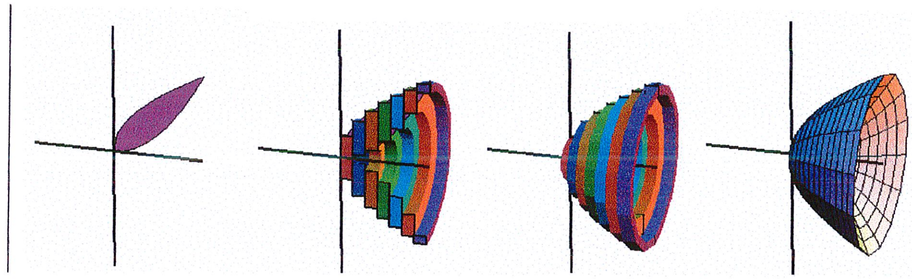


Illustration of Washer Method



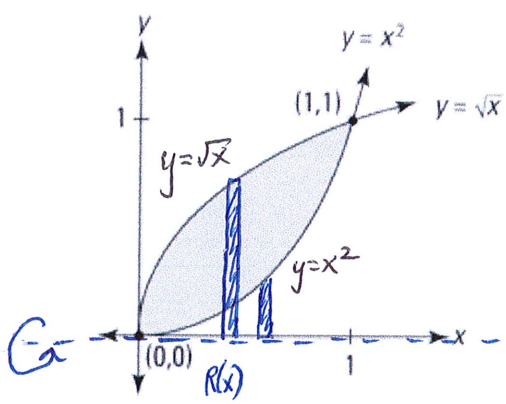
Radius $[R(x)]$ = *distance from the AOR (Axis of Revolution) to the* ***further*** *graph curve*

radius $[r(x)]$ = *distance from the AOR (Axis of Revolution) to the* ***closer*** *graph curve*

Washer Method: $Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$

$\pi R^2 - \pi r^2$

Example 1: Find the volume of the solid created enclosed region of $y = x^2$ and $y = \sqrt{x}$ revolving about the x-axis



$R(x) = \sqrt{x} - 0 = \sqrt{x}$

$r(x) = x^2 - 0 = x^2$

$V = \pi \int_0^1 [\sqrt{x}]^2 - [x^2]^2 dx$

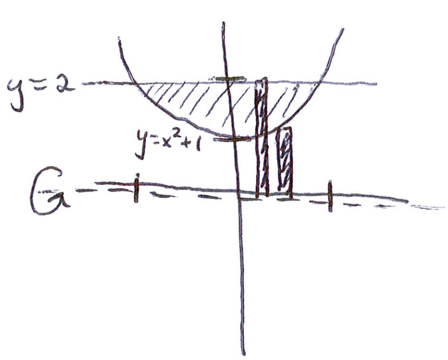
$V = \pi \int_0^1 x - x^4 dx$

$\left.\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 = \frac{1}{2} - \frac{1}{5} - (0-0)$

$= \frac{5}{10} - \frac{2}{10} = \frac{3}{10}\pi$

$\boxed{V = \frac{3}{10}\pi \text{ units}^3}$

Example 2: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ revolved about the x-axis.



$R(x) = 2 - 0$

$r(x) = x^2 + 1 - 0$

*intersections:

$x^2 + 1 = 2$

$x^2 = 1$, $\boxed{x = \pm 1}$

$V = \pi \int_{-1}^1 [2]^2 - [x^2+1]^2 dx$

$V = \int_{-1}^1 4 - (x^4 - 2x^2 + 1) dx$

$\int_{-1}^1 3 - x^4 + 2x^2 dx = \left. 3x - \frac{x^5}{5} + \frac{2x^3}{3} \right]_{-1}^1$

$= 3 - \frac{1}{5} + \frac{2}{3} - \left(-3 + \frac{1}{5} - \frac{2}{3}\right)$

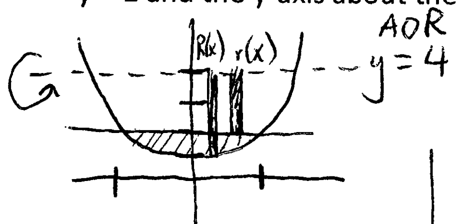
$= \boxed{\frac{64}{15}\pi \text{ units}^3}$

$$Radius\ [R(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{further}\ graph\ curve$$

$$radius\ [r(x)] = distance\ from\ the\ AOR\ (Axis\ of\ Revolution)\ to\ the\ \textbf{closer}\ graph\ curve$$

$$\textbf{Washer Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

Example 3: Find the volume of the solid created by revolving the function y = x^2 + 1 bounded by the line y = 2 and the y-axis about the line y = 4



$R(x) = 4 - (x^2+1)$

$R(x) = 4 - x^2 - 1$

$R(x) = 3 - x^2$

$r(x) = 4 - 2 = 2$

* intersections:

$x^2 + 1 = 2$

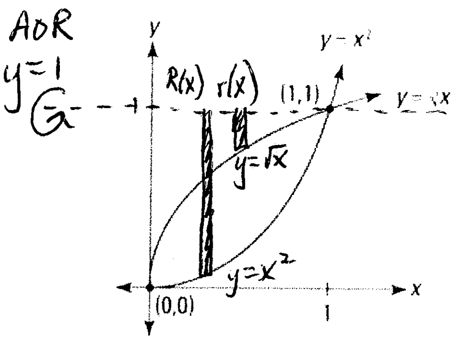
$x^2 = 1$

$x = \pm 1$

$$V = \pi \int_{-1}^{1} [3-x^2]^2 - [2]^2 dx$$

$$\boxed{V = 6.4\pi \text{ or } \frac{32}{5}\pi \text{ units}^3}$$

Example 4: Find the volume of the solid created enclosed region of y = x^2 and $y = \sqrt{x}$ revolving about the line y = 1



$R(x) = 1 - x^2$

$r(x) = 1 - \sqrt{x}$

* intersections:

$x^2 = \sqrt{x}$

$(x^2)^2 = (\sqrt{x})^2$

$x^4 = x$

$x^4 - x = 0$

$x(x^3 - 1) = 0$

$x = 0, 1$

$$V = \pi \int_{0}^{1} [1-x^2]^2 - [1-\sqrt{x}]^2 dx$$

$$\boxed{V = 0.3667\pi \text{ units}^3}$$

7.2a Disc Method Classwork Problems

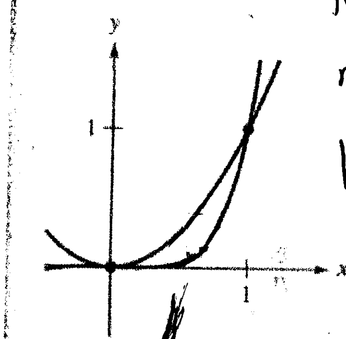
Radius $[R(x)] =$ *distance from the AOR (Axis of Revolution) to the* **further** *graph curve*

radius $[r(x)] =$ *distance from the AOR (Axis of Revolution) to the* **closer** *graph curve*

$$\textbf{Washer Method}: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

Finding the Volume of a Solid **In Exercises 1–6, set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.**

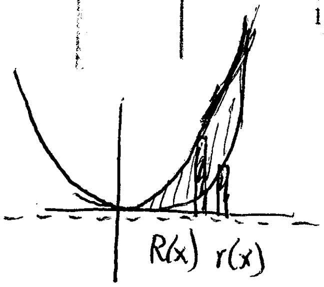
5. $y = x^2, \; y = x^5$



$R(x) = x^2 - 0 = x^2$

$r(x) = x^5 - 0 = x^5$

$V = \pi \int_0^1 [x^2]^2 - [x^5]^2 dx$

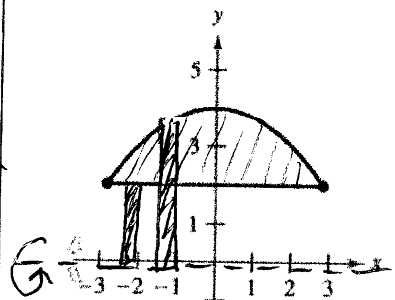


$V = \pi \int_0^1 x^4 - x^{10} dx$

$\frac{x^5}{5} - \frac{x^{11}}{11} \Big]_0^1 = \frac{1}{5} - \frac{1}{11}$

$\boxed{V = \frac{6\pi}{55} \text{ units}^3}$

6. $y = 2, \; y = 4 - \frac{x^2}{4}$



$R(x) = 4 - \frac{1}{4}x^2 - 0$

$r(x) = 2 - 0$

Intersections:

$2 = 4 - \frac{x^2}{4}$

$\frac{x^2}{4} = 2$

$\sqrt{x^2} = \sqrt{8}$

$x = \pm\sqrt{8}$

$V = \pi \int_{-\sqrt{8}}^{\sqrt{8}} [4 - \frac{1}{4}x^2]^2 - [2]^2 dx$

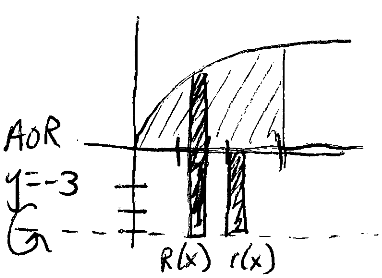
$V = \frac{448\sqrt{2}}{15}\pi$

$\boxed{V \approx 42.238\pi \text{ units}^3}$

Finding the Volume of a Solid **In Exercises 11–14, find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.**

11. $y = \sqrt{x}, \; y = 0, \; x = 3$

AOR (axis of revolution) : $y = -3$



$V = \pi \int_0^3 [\sqrt{x} + 3]^2 - [3]^2 dx$

$\boxed{V = 25.285\pi \text{ units}^3}$

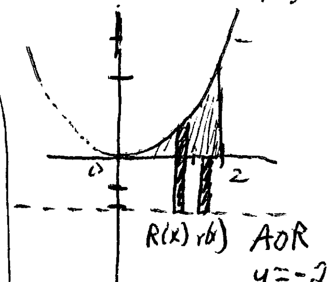
Washer:

$R(x) = \sqrt{x} - (-3) = \sqrt{x} + 3$

$r(x) = 0 - (-3) = 3$

12. $y = 2x^2, \; y = 0, \; x = 2$

AOR (axis of revolution) : $y = -2$



$V = \pi \int_0^2 [2x^2 + 2]^2 - [2]^2 dx$

$\boxed{V = 46.933\pi \text{ or } \frac{704}{15}\pi \text{ units}^3}$

Washer:

$R(x) = 2x^2 - (-2) = 2x^2 + 2$

$r(x) = 0 - (-2) = 2$

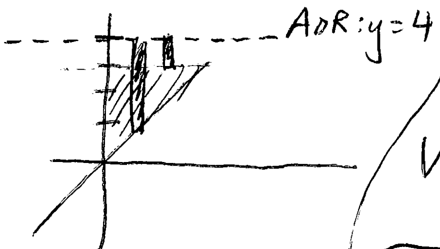
Radius $[R(x)]$ = *distance from the AOR (Axis of Revolution) to the* ***further*** *graph curve*

radius $[r(x)]$ = *distance from the AOR (Axis of Revolution) to the* ***closer*** *graph curve*

Washer Method: *Volume* $= \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$

Disc: $V = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$

15) $y = x,\ y = 3,\ x = 0 \quad$ AOR: $y = 4$



$R(x) = 4 - x$

$r(x) = 4 - 3 = 1$

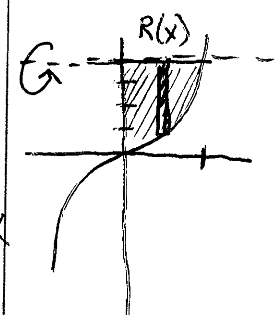
Intersections:

$x = 3,\ 0$

$$V = \pi \int_0^3 [4-x]^2 - [1]^2 dx$$

$$\boxed{V = 18\pi \text{ units}^3}$$

16) $y = \frac{1}{2}x^3,\ y = 4,\ x = 0 \quad$ AOR: $y = 4$



Disc: $V = \pi \int [R(x)]^2 dx$

$R(x) = 4 - \frac{1}{2}x^3$

intersections:

$\frac{1}{2}x^3 = 4$

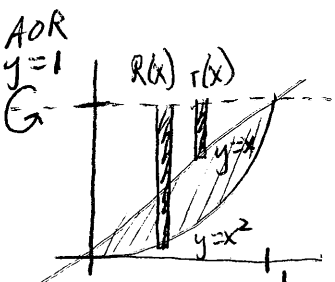
$x^3 = 8$

$x = 2,\ 0$

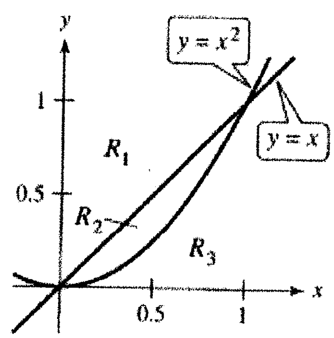
$$V = \pi \int_0^2 \left[4 - \tfrac{1}{2}x^3\right]^2 dx$$

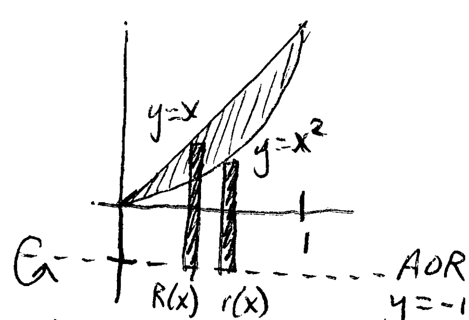
$$\boxed{V = \frac{144}{7}\pi \text{ units}^3}$$

Finding the Volume of a Solid **In Exercises 41–48, find the volume generated by rotating the given region about the specified line.**



*washer method





41) R_2 *about AOR* $y = 1$

$R(x) = 1 - x^2$

$r(x) = 1 - x$

$$V = \pi \int_0^1 [1-x^2]^2 - [1-x]^2 dx$$

$$\boxed{V = \frac{\pi}{5} \text{ units}^3}$$

42) R_1 *about AOR* $y = -1$

*washer method

$R(x) = x - (-1) = x + 1$

$r(x) = x^2 - (-1) = x^2 + 1$

$$V = \pi \int_0^1 [x+1]^2 - [x^2+1]^2 dx$$

$$\boxed{V = \frac{7}{15}\pi \text{ units}^3}$$