

Reviewing Disc Method

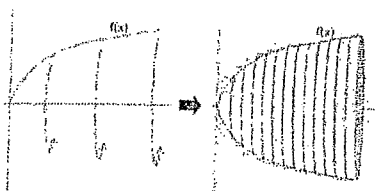
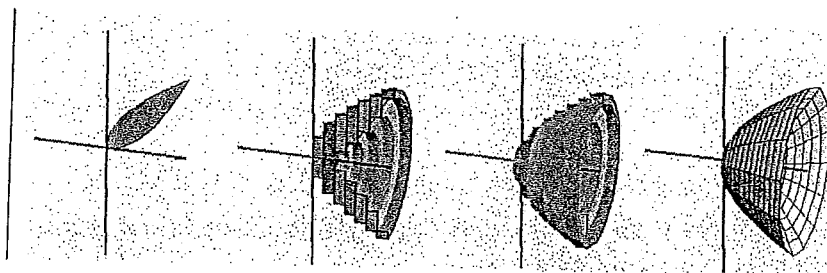


Illustration of Washer Method

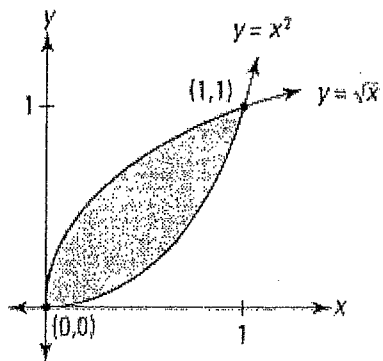


Radius $[R(x)] =$ distance from the AOR (Axis of Revolution) to the **further** graph curve

radius $[r(x)] =$ distance from the AOR (Axis of Revolution) to the **closer** graph curve

Washer Method: Volume =
$$\pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

Example 1: Find the volume of the solid created enclosed region of $y = x^2$ and $y = \sqrt{x}$ revolving about the x-axis



Example 2: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ revolved about the x-axis.

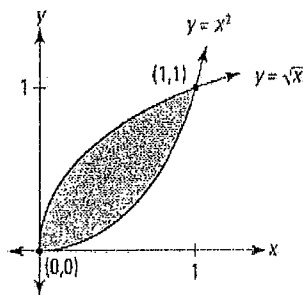
Radius $[R(x)]$ = distance from the AOR (Axis of Revolution) to the **further** graph curve

radius $[r(x)]$ = distance from the AOR (Axis of Revolution) to the **closer** graph curve

$$\text{Washer Method: Volume} = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

Example 3: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ and the y -axis about the line $y = 4$

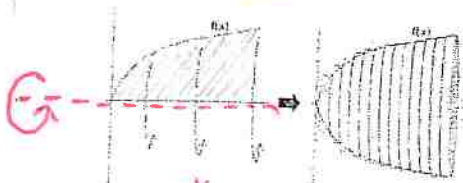
Example 4: Find the volume of the solid created enclosed region of $y = x^2$ and $y = \sqrt{x}$ revolving about the line $y = 1$



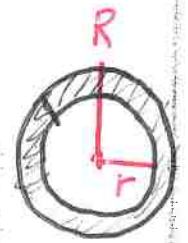
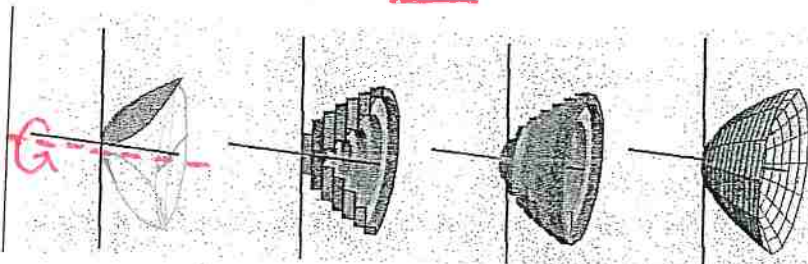
circular rings

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Illustration of Washer Method



$$V = \pi \int_{x_1}^{x_2} [R(x)]^2 dx$$



$$\pi R^2 - \pi r^2$$

$$\cdot \pi (R^2 - r^2)$$

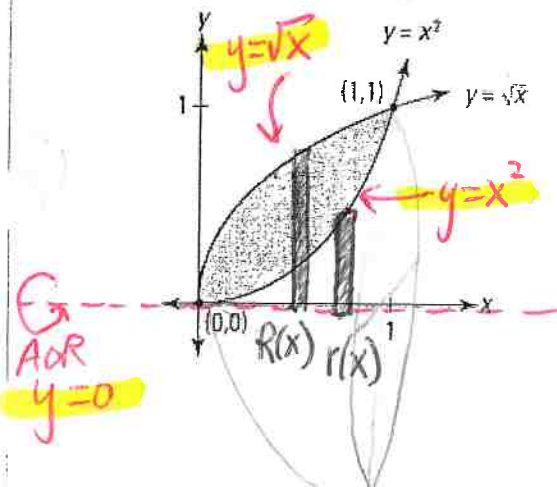
Radius $[R(x)]$ = distance from the AOR (Axis of Revolution) to the **further** graph curve

radius $[r(x)]$ = distance from the AOR (Axis of Revolution) to the **closer** graph curve

Washer Method: Volume = $\pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$ (Top-Bottom)

$$V = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

Example 1: Find the volume of the solid created enclosed region of $y = x^2$ and $y = \sqrt{x}$ revolving about the x-axis (AOR $y=0$)



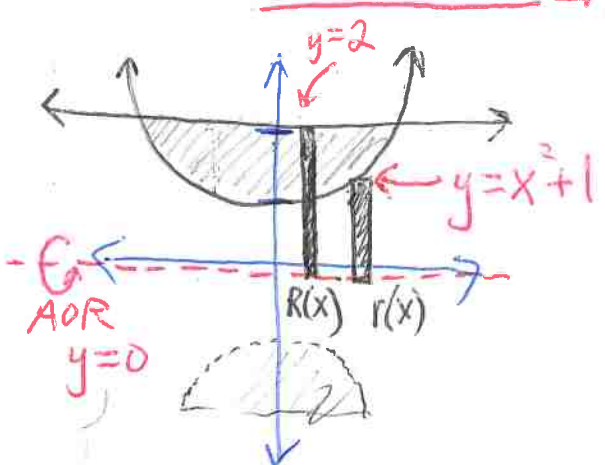
$$R(x) = \sqrt{x} - (0) \rightarrow \sqrt{x}$$

$$r(x) = x^2 - (0) \rightarrow x^2$$

$$V = \pi \int_0^1 [\sqrt{x}]^2 - [x^2]^2 dx$$

$$V = 0.3\pi \text{ or } \frac{3}{10}\pi \text{ units}^3$$

Example 2: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ revolved about the x-axis. \rightarrow AOR $y=0$



$$R(x) = 2 - (0) \rightarrow 2$$

$$r(x) = x^2 + 1 - (0) \rightarrow x^2 + 1$$

* find intersection (bounds)

$$x^2 + 1 = 2$$

$$\sqrt{x^2} = \pm 1$$

$$x = 1, x = -1$$

$$V = \pi \int_{-1}^1 [2]^2 - [x^2 + 1]^2 dx$$

$$V = \frac{64}{15}\pi \text{ units}^3$$

Radius $[R(x)] =$ distance from the AOR (Axis of Revolution) to the **further** graph curve

radius $[r(x)] =$ distance from the AOR (Axis of Revolution) to the **closer** graph curve

Washer Method: $Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$

Example 3: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ and the y-axis about the line $y = 4$

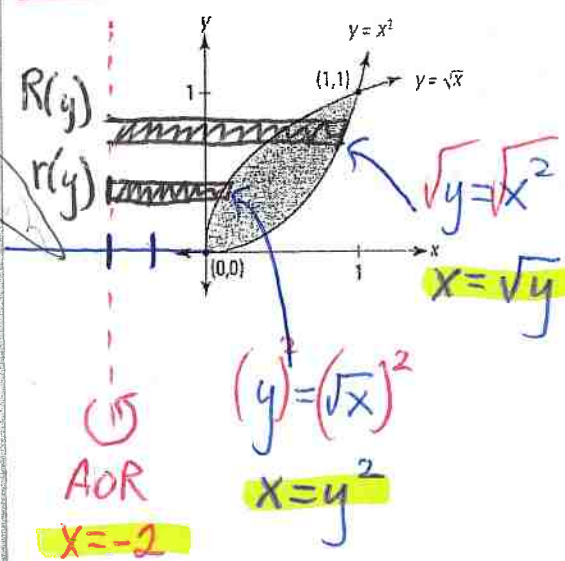
Washer Method
(Right-Left)

$$V = \pi \int_{y_1}^{y_2} [R(y)]^2 - [r(y)]^2 dy$$

needs the form
"x = _"

Change
Problem

Example 4: Find the volume of the solid enclosed region of $y = x^2$ and $y = \sqrt{x}$ revolving about the line ~~$y = 1$~~ Revolve about $x = -2$



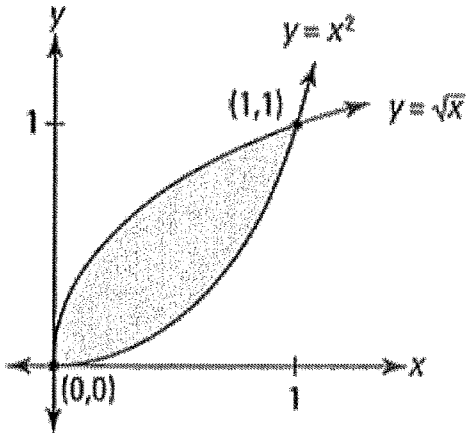
$$R(y) = \sqrt{y} - (-2) \rightarrow \sqrt{y} + 2$$

$$r(y) = y^2 - (-2) \rightarrow y^2 + 2$$

$$V = \pi \int_0^1 [\sqrt{y} + 2]^2 - [y^2 + 2]^2 dy$$

$$V = 1.633\pi \text{ units}^3$$

or 5.131 units³



now revolve this shaded area about the x-axis

