7.2b Volume - Washer Method Practice Problems Worksheet

Key

Washer Method: (Top – Bottom) – Vertical Radius	Washer Method: (Right – Left) – Horizontal Radius
$V = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 \, dx$	$V = \pi \int_{y_1}^{y_2} [R(y)]^2 - [r(y)]^2 dy$
(expression(s) used above has form: " y = ___ ")	(expression(s) used above has form: " x = ___ ")

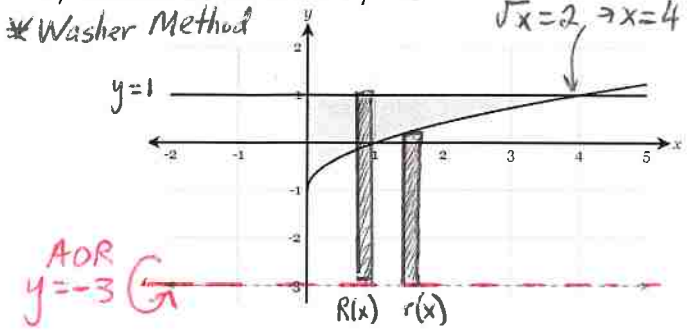
1) Let the region R be the area enclosed the the function $f(x) = \sqrt{x} - 1$, the horizontal line y=1, and the y-axis. Find the volume of the solid generated when the region is:

a) revolved about the line y = -3

*Washer Method

$\sqrt{x} - 1 = 1$

$\sqrt{x} = 2 \Rightarrow x = 4$



$R(x) = 1 - (-3) = 4$

$r(x) = \sqrt{x} - 1 - (-3) = \sqrt{x} + 2$

$V = \pi \int_0^4 [4]^2 - [\sqrt{x}+2]^2 \, dx$

$V = 18.667\pi \text{ units}^3$

b) revolved about the line x = - 1

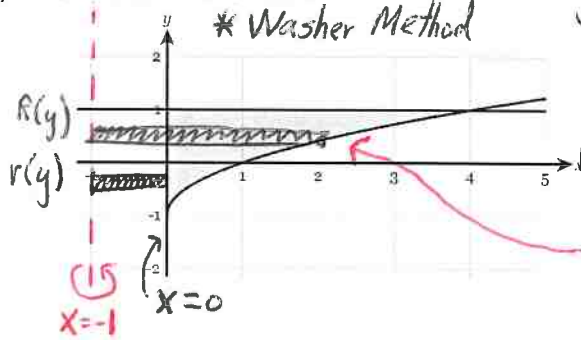
*Washer Method

$y = \sqrt{x} - 1$

$y + 1 = \sqrt{x}$

$(y+1)^2 = (\sqrt{x})^2$

$x = (y+1)^2$



$R(y) = (y+1)^2 - (-1) = (y+1)^2 + 1$

$r(y) = 0 - (-1) = 1$

$V = \pi \int_{-1}^{1} [(y+1)^2 + 1]^2 - [1]^2 \, dy$

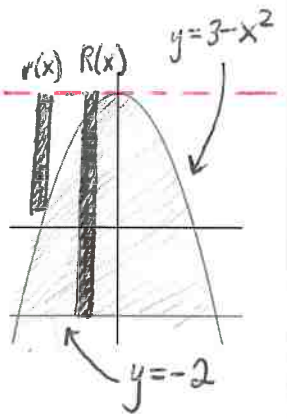
$V = \frac{176}{15}\pi \text{ units}^3$

2) Let the region R be the area enclosed the the function $f(x) = 3 - x^2$ the line y = -2. Find the volume of the solid generated when the region is:

a) revolved about the line y = 3

*Washer Method

AOR $y = 3$



$R(x) = 3 - (-2) = 5$

$r(x) = 3 - (3 - x^2) = x^2$

*Find intersection (bounds)

$3 - x^2 = -2$

$5 = x^2$

$\sqrt{5} = \sqrt{x^2}$

$\pm\sqrt{5} = x$

$V = \pi \int_{-\sqrt{5}}^{\sqrt{5}} (5)^2 - (x^2)^2 \, dx$

$V = 89.443\pi \text{ units}^3$

b) revolved about the line y = - 2

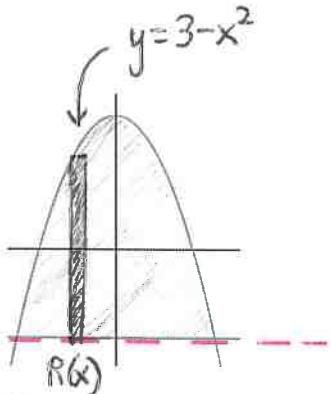
*Disc Method

$R(x) = 3 - x^2 - (-2)$

$R(x) = 5 - x^2$

$V = \pi \int_{-\sqrt{5}}^{\sqrt{5}} [5 - x^2]^2 \, dx$

AOR $y = -2$



*intersections:

$3 - x^2 = -2$

$x = \pm\sqrt{5}$

$V = 59.628\pi \text{ units}^3$

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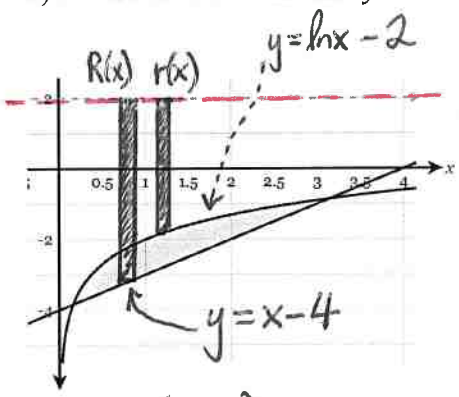
$V = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 \, dx$	$V = \pi \int_{y_1}^{y_2} [R(y)]^2 - [r(y)]^2 dy$
(expression(s) used above has form: " y = ___ ")	(expression(s) used above has form: " x = ___ ")

3) Let the region R be the area enclosed the function $f(x) = \ln x - 2$ and $g(x) = x - 4$. Find the volume of the solid generated when the region is:

a) revolved about the line $y = 2$ *washer Method



$R(x) = 2-(x-4) = 2-x+4 = 6-x$

$r(x) = 2-(\ln x-2) = 2-\ln x+2 = 4-\ln x$

*find bounds:

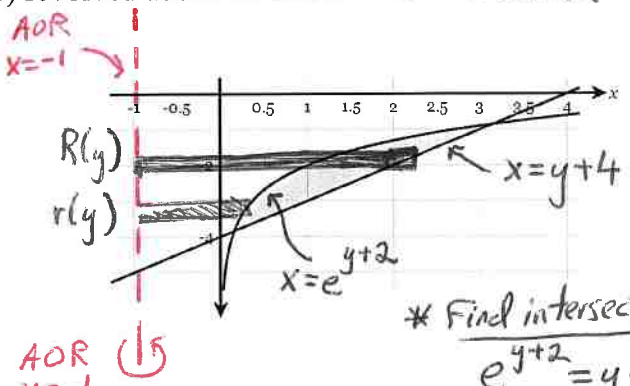
set $\ln x - 2 = x - 4$

$x = 0.1586, x = 3.146$

$V = \pi \int_{0.159}^{3.146} (6-x)^2 - (4-\ln x)^2 \, dx$

$V = 16.402\pi$ units^3

b) revolved about the line x = –1 *washer Method



*Find intersections:

$e^{y+2} = y+4$

$y = -0.853, y = -3.841$

*Rewrite equations:

$y = \ln x - 2$

$y + 2 = \ln x$

$e^{y+2} = e^{\ln x}$

$e^{y+2} = x$

$x = e^{y+2}$

$y = x - 4$

$y + 4 = x$

$R(y) = y+4-(-1) = y+5$

$r(y) = e^{y+2}-(-1) = e^{y+2}+1$

$V = \pi \int_{-3.841}^{-0.853} (y+5)^2 - (e^{y+2}+1)^2 \, dy$

$V = 9.341\pi$ units^3

4) Let the region R be the area enclosed by the function $f(x) = x^2 + 2$, the horizontal line y=2, & the vertical lines x=0 & x=4. Find volume of the solid generated when region is:

a) revolved about the line $x = 5$

*Washer Method

*rewrite equation:

$y = x^2 + 2$

$y - 2 = x^2$

$\pm\sqrt{y-2} = \sqrt{x^2}$

$x = \sqrt{y-2}$

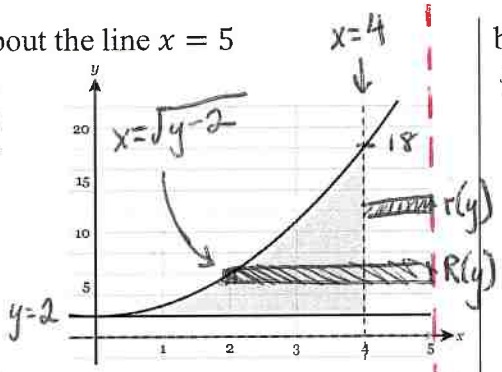
*find intersection:

$\sqrt{y-2} = 4$

$(\sqrt{y-2})^2 = (4)^2$

$y - 2 = 16$

$y = 18$



$R(y) = 5-\sqrt{y-2}$

$r(y) = 5-(4) = 1$

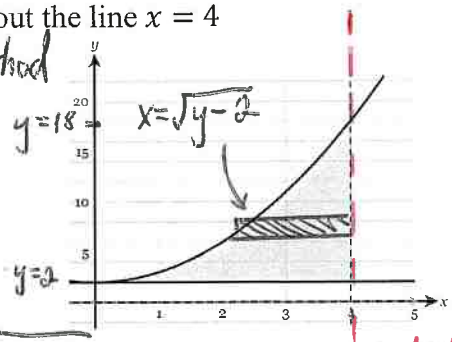
AOR x=5

$V = \pi \int_{2}^{18} [5-\sqrt{y-2}]^2 - [1]^2 \, dy$

$V = 85.333\pi$ units^3

b) revolved about the line $x = 4$

*Disc Method



$R(y) = 4-\sqrt{y-2}$

AOR x=4

$V = \pi \int_{2}^{18} [4-\sqrt{y-2}]^2 \, dy$

$V = 42.667\pi$ units^3