7.2c Volume Cross Section Worksheet Practice

Find Area for cross section using below formulas:

Squares : Area = $(base)^2$

Equilateral Triangles: Area = $\frac{\sqrt{3}}{4}(base)^2$

Semicircles: Area = $\frac{\pi}{8}[base]^2$

$\boldsymbol{base = top - bottom}$ *or* $\boldsymbol{right - left}$

Isosceles Right Triangles (leg): Area = $\frac{1}{2}(base)^2$

Isosceles Right Triangles (hypotenuse): $Area = \frac{1}{4}(base)^2$

$$\boldsymbol{Volume} = \int_{x_1}^{x_2} (\boldsymbol{Area})\, \boldsymbol{dx} \text{ or } \int_{y_1}^{y_2} (\boldsymbol{Area})\, \boldsymbol{dy}$$

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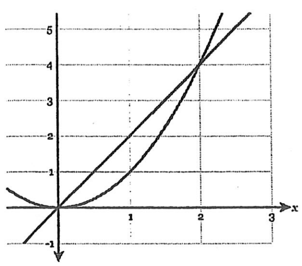
1)

Let the region R be the area enclosed by the function $f(x) = x^2$ and $g(x) = 2x$. If the region R is the base of a solid such that each cross section perpendicular to the x-axis is a square, find the volume of the solid. You may use a calculator and round to the nearest thousandth.



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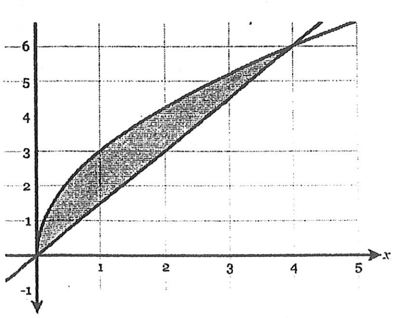
2)

Let the region R be the area enclosed by the function $f(x) = 3\sqrt{x}$ and $g(x) = \frac{3}{2}x$. If the region R is the base of a solid such that each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in the region R, find the volume of the solid. You may use a calculator and round to the nearest thousandth.



3. The base of a solid is bounded by $y = x^3$, $y = 0$, and $x = 1$. Find the volume of the solid for each of the following cross sections taken perpendicular to the y-axis.

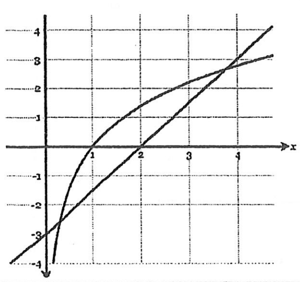
(a) Squares

(b) Semicircles

(c) Equilateral triangles

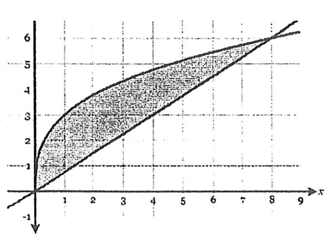
4)

Let the region R be the area enclosed by the function $f(x) = 2\ln(x)$ and $g(x) = \frac{3}{2}x - 3$. If the region R is the base of a solid such that each cross section perpendicular to the x-axis is a rectangle whose height is twice the length of its base in the region R, find the volume of the solid. You may use a calculator and round to the nearest thousandth.



5)

Let the region R be the area enclosed by the function $f(x) = 3x^{\frac{1}{3}}$ and $g(x) = \frac{3}{4}x$. If the region R is the base of a solid such that each cross section perpendicular to the x-axis is a semi-circle with diameters extending through the region R, find the volume of the solid. You may use a calculator and round to the nearest thousandth.



Key

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$Volume = \int_{x_1}^{x_2}(Area)\,dx$ or $\int_{y_1}^{y_2}(Area)\,dy$

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1)

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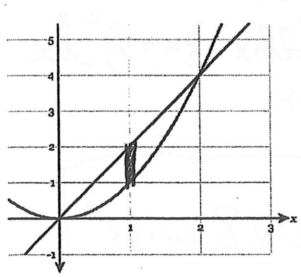

*intersections:

$x^2 = 2x$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, x = 2$

$base = 2x - x^2$ $\quad A = (base)^2$

$A = (2x - x^2)^2$

$V = \int_0^2 (2x - x^2)^2\,dx$

$\boxed{V = 1.067 \text{ units}^3}$

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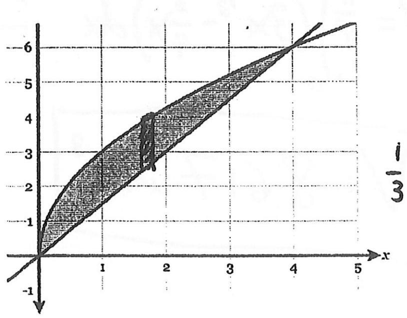
2)

Let the region R be the area enclosed by the function $f(x) = 3\sqrt{x}$ and $g(x) = \frac{3}{2}x$. If the region R is the base of a solid such that each cross section perpendicular to the x-axis is an isosceles right triangle with a leg in the region R, find the volume of the solid. You may use a calculator and round to the nearest thousandth.



*intersections:

$3\sqrt{x} = \frac{3}{2}x$

$\frac{1}{3}(3\sqrt{x} = \frac{3}{2}x)$

$\sqrt{x} = \frac{1}{2}x$

$(\sqrt{x})^2 = (\frac{1}{2}x)^2$

$x = \frac{x^2}{4}$

$4x = x^2$

$4x - x^2 = 0$

$x(4-x) = 0$

$x = 0, 4$

$base = 3\sqrt{x} - \frac{3}{2}x$

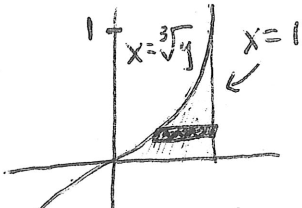
$A = \frac{1}{2}(base)^2$

$A = \frac{1}{2}(3\sqrt{x} - \frac{3}{2}x)^2$

$V = \frac{1}{2}\int_0^4 (3\sqrt{x} - \frac{3}{2}x)^2\,dx$

$\boxed{V = 2.4 \text{ units}^3}$

3. The base of a solid is bounded by $y = x^3$, $y = 0$, and $x = 1$. Find the volume of the solid for each of the following cross sections taken perpendicular to the y-axis.
(a) Squares
(b) Semicircles
(c) Equilateral triangles



intersection:
$\sqrt[3]{y} = 1$
$y = 1$

*Right-Left
base $= 1 - \sqrt[3]{y}$

a) squares: $A = (\text{base})^2$
$A = [1 - \sqrt[3]{y}]^2$
$V = \int_0^1 [1 - \sqrt[3]{y}]^2 \, dy$
$\boxed{V = 0.1 \text{ units}^3}$

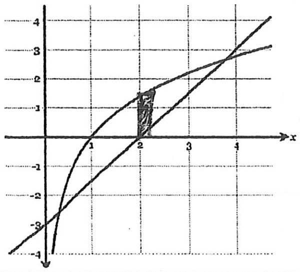
b) semicircles
$A = \frac{\pi}{8}(\text{base})^2$
$A = \frac{\pi}{8}[1 - \sqrt[3]{y}]^2$
$V = \frac{\pi}{8}\int_0^1 [1 - \sqrt[3]{y}]^2 \, dy$
$\boxed{V = 0.039 \text{ or } \pi/80 \text{ units}^3}$

c) Equilateral triangles
$A = \frac{\sqrt{3}}{4}(\text{base})^2$
$A = \frac{\sqrt{3}}{4}[1 - \sqrt[3]{y}]^2$
$V = \frac{\sqrt{3}}{4}\int_0^1 [1 - \sqrt[3]{y}]^2 \, dy$
$\boxed{V = 0.043 \text{ or } \frac{\sqrt{3}}{40} \text{ units}^3}$

4)

Let the region R be the area enclosed by the function $f(x) = 2\ln(x)$ and $g(x) = \frac{3}{2}x - 3$. If the region R is the base of a solid such that each cross section perpendicular to the x-axis is a rectangle whose height is twice the length of its base in the region R, find the volume of the solid. You may use a calculator and round to the nearest thousandth.

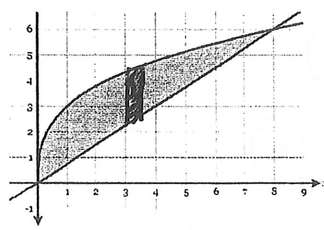
intersections: $x = 0.274,\ 3.769$
base $= 2\ln x - (\frac{3}{2}x - 3) = 2\ln x - \frac{3}{2}x + 3$
Area = base × height
height $= 2(2\ln x - \frac{3}{2}x + 3)$



$\text{Area} = 2(2\ln x - \frac{3}{2}x + 3)^2$
$V = \int_{0.274}^{3.769} 2(2\ln x - \frac{3}{2}x + 3)^2 \, dx$
$\boxed{V = 9.028 \text{ units}^3}$

5)

Let the region R be the area enclosed by the function $f(x) = 3x^{\frac{1}{3}}$ and $g(x) = \frac{3}{4}x$. If the region R is the base of a solid such that each cross section perpendicular to the x-axis is a semi-circle with diameters extending through the region R, find the volume of the solid. You may use a calculator and round to the nearest thousandth.



*intersections:
$\frac{1}{3}(3x^{1/3} = \frac{3}{4}x)$
$(x^{1/3})^3 = (\frac{x}{4})^3$
$x = \frac{x^3}{64}$
$64x = x^3$
$64x - x^3 = 0$
$x(64 - x^2) = 0$

$x = 0,\ x = 8$
base $= 3x^{1/3} - \frac{3}{4}x$
$\text{Area} = \frac{\pi}{8}(\text{base})^2$
$\text{Area} = \frac{\pi}{8}(3x^{1/3} - \frac{3}{4}x)^2$

$V = \frac{\pi}{8}\int_0^8 (3x^{1/3} - \frac{3}{4}x)^2 \, dx$
$\boxed{V = 8.617 \text{ units}^3}$