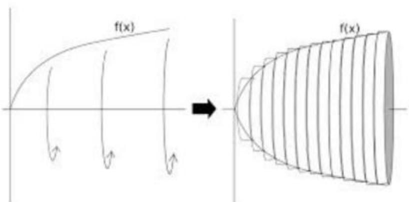
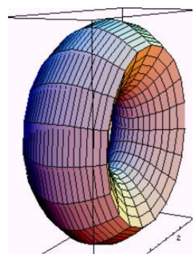
AP Calculus Ch. 7.2c Volumes with Known Cross Section

Cross-section is the shape that results if we cut the object in half and look at the resulting shape sideways:

Disc Method



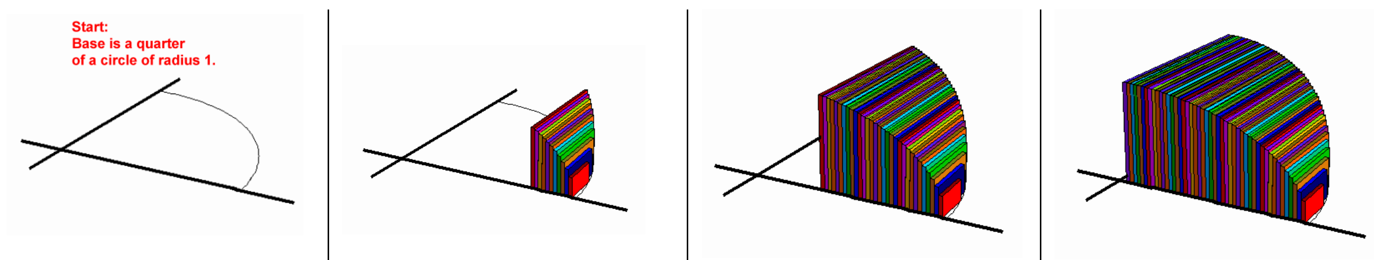
Washer Method



$V = \pi \int [Area \text{ of cross section}]\, dx$

The volume problems we have covered so far(Disc, Washer) have involved taking the shaded region and rotate it about a center, (AOR), resulting in circular Area, either $Area = \pi[R(x)]^2$ or $Area = \pi[R(x)]^2 - \pi[r(x)]^2$

Now, we still have a shaded region, but now it will act as the base of the 3D object. We will now build the cross-section area on top of this base. (no longer rotating around an axis)



Top-Bottom Vertical base	**Right-Left Horizontal base**
$V = \int_{x_1}^{x_2} [\boldsymbol{Area} \text{ of cross section}]dx$ *Note: All values in integral are in terms of x (in the form of " y = ____")	$V = \int_{y_1}^{y_2} [\boldsymbol{Area} \text{ of cross section}]dy$ *Note: All values in integral are in terms of y (in the forms of " x = _______")

Area formulas for Cross sections:

1. Square: $A = (base)^2$	2. Isosceles Right Triangle (**leg** on base): $A = \frac{1}{2}(base)^2$	3. Isosceles Right Triangle (**hypotenuse** on base): $A = \frac{1}{4}(base)^2$
4. Rectangle: A = (base)(height)	5. Equilateral Triangle: $A = \frac{\sqrt{3}}{4}(base)^2$	6. Semicircle: $A = \frac{\pi}{8}(base)^2$

Example 1: Find the volume of the solid if the base is bounded by curve y = $\sqrt{1-x^2}$, y = 0, x = 0 (in the first quadrant) and the cross sections are squares parallel to the y-axis.

Top-Bottom Vertical base	Right-Left Horizontal base
$V = \int_{x_1}^{x_2} [\textbf{Area of cross section}]dx$ *Note: All values in integral are in terms of x (equations in the form of " y = ____")	$V = \int_{y_1}^{y_2} [\textbf{Area of cross section}]dy$ *Note: All values in integral are in terms of y (equations in the form of " x = ____")

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1. Square: $A = (base)^2$	2. Isosceles Right Triangle (**leg** on base): $A = \frac{1}{2}(base)^2$	3. Isosceles Right Triangle (**hypotenuse** on base): $A = \frac{1}{4}(base)^2$
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Example 2: Find the volume of the solid if the base is bounded by the curve $y = x^2$ and the line y = 4 and the cross sections are isosceles right triangles whose hypotenuse lie on the base and are parallel to the x-axis.

Example 3: Find the volume of the solid if the base is bounded by the curve $y = x^2$ and the line y = 4 and the cross sections are semicircles whose base are perpendicular to the y-axis.

Example 4: Let the region R be the area enclosed by the function $f(x) = \ln x$ and $g(x) = \frac{1}{2}x - 1$. If the region R is the base of a solid such that each cross section perpendicular to the *x*-axis is an isosceles right triangle with a leg in the region R, find the volume of the solid.