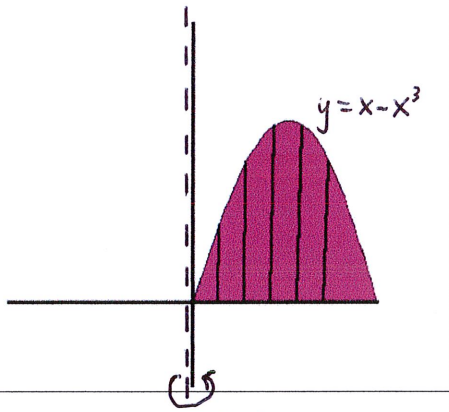
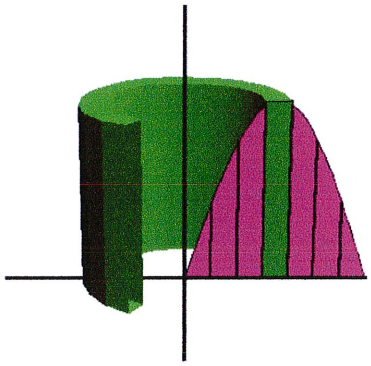
7.3 Shell Method Notes 2

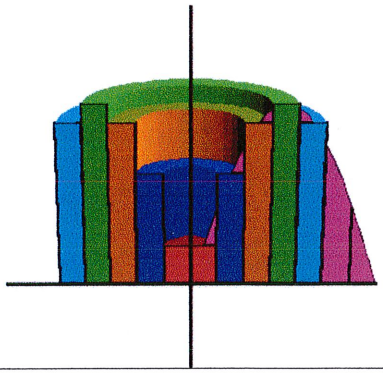
Generating a Partition



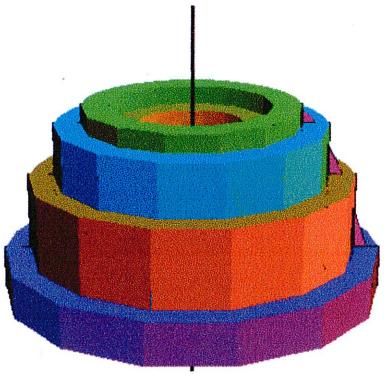
Generation of Typical Shell

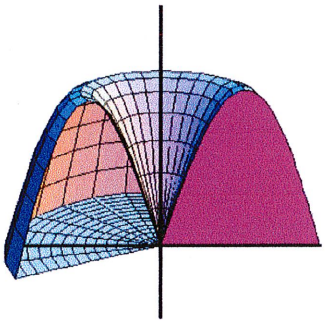


Drawing Half Shell 5

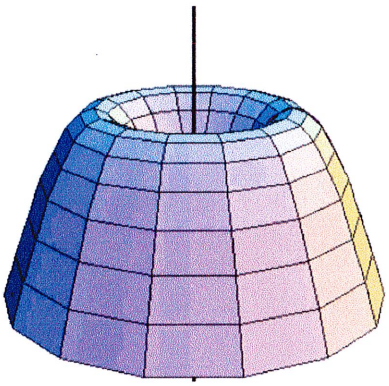


Drawing Shell 7

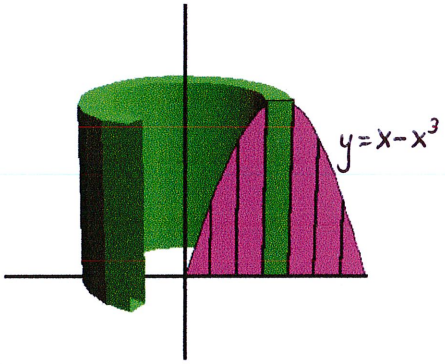


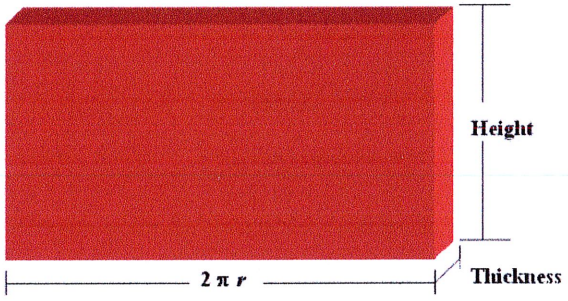


Generation of Surface



Generation of Typical Shell

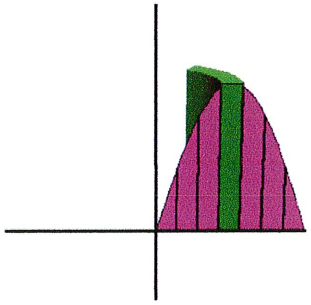




Volume of shell $\approx 2\pi r \times$ *Thickness* $\times$ Height

$$V = 2\pi \int [\text{shell radius}][\text{shell height}]\, dx$$

Generation of Typical Shell



Example: Shell Method vs Disc/Washer Method

Find the volume of the solid formed by revolving the region bounded by the graphs of

$$y = x^2 + 1, \quad y = 0, \quad x = 0, \quad \text{and} \quad x = 1$$

about the y-axis, as shown in Figure 7.21.

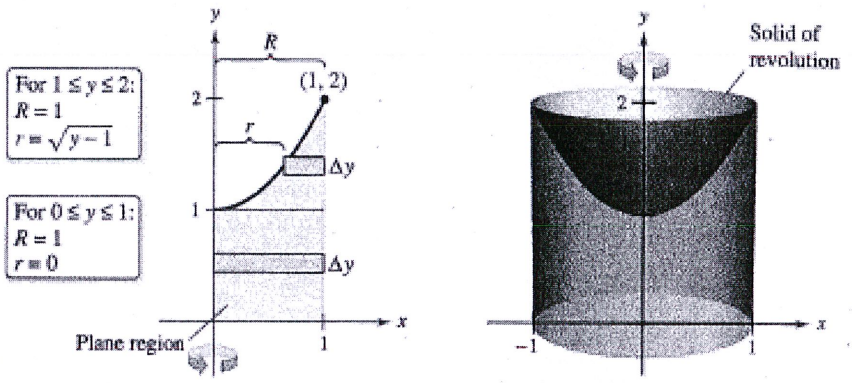
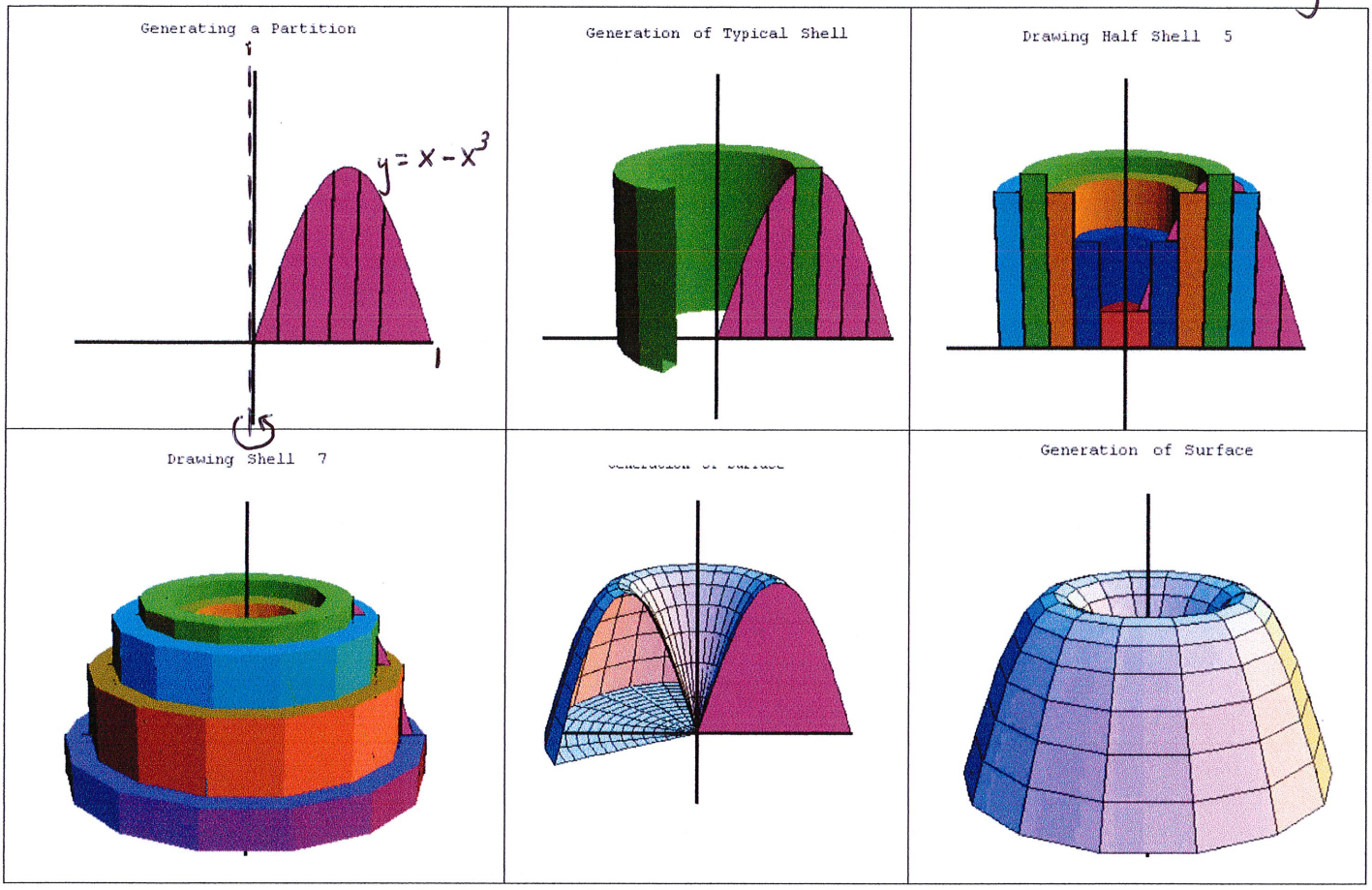


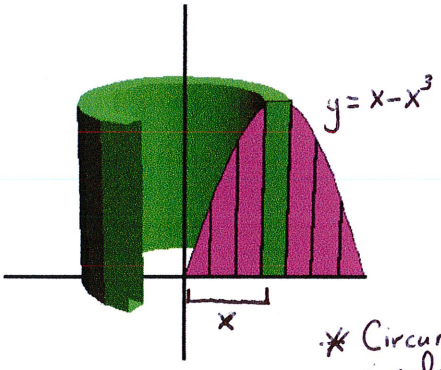
Figure 7.21

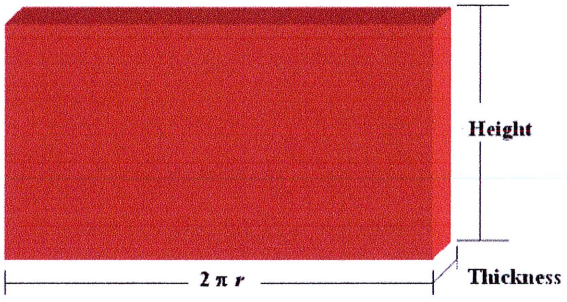
7.3 Shell Method Notes 2

Key



Generation of Typical Shell

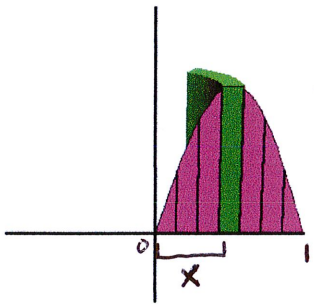




* Circumference of circular base ($C = 2\pi r$) also represents base length of rectangle

Volume of shell ≈ $2\pi r \times$ *Thickness* $\times$ *Height*

Generation of Typical Shell



height : $y = x - x^3$

$$V = 2\pi \int [\text{shell radius}][\text{shell height}]\,dx$$

$$V = 2\pi \int_0^1 (x)(x - x^3)\,dx$$

$$= 2\pi \int_0^1 x^2 - x^4\,dx = 2\pi \cdot \left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1$$

$$= 2\pi\left(\tfrac{1}{3} - \tfrac{1}{5}\right)$$

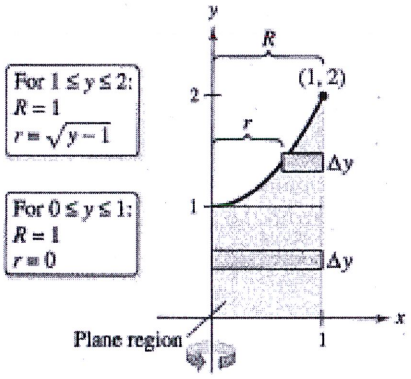
$$= 2\pi\left(\tfrac{2}{15}\right) = \boxed{\frac{4\pi}{15}\ \text{units}^3}$$

Example: Shell Method vs Disc/Washer Method

Find the volume of the solid formed by revolving the region bounded by the graphs of

$y = x^2 + 1, \quad y = 0, \quad x = 0, \quad \text{and} \quad x = 1$

about the y-axis, as shown in Figure 7.21.



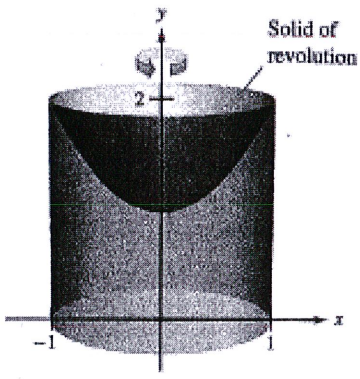
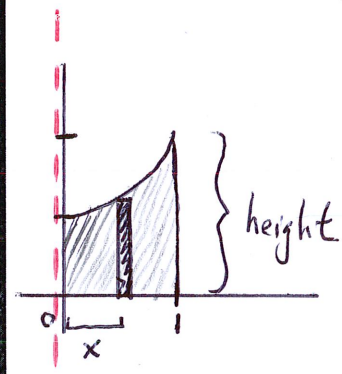


Figure 7.21

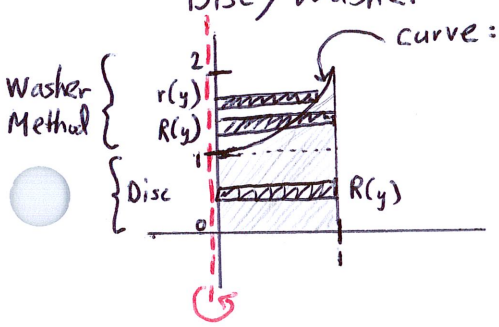


radius = x

height = $x^2 + 1$

Disc/Washer

curve: $x = \sqrt{y-1}$



Disc	Washer
$R(y) = 1 - 0 = 1$	$R(y) = 1 - 0 = 1$
$V = \pi \int_0^1 [1]^2 \, dy$	$r(y) = \sqrt{y-1} - 0$
	$V = \pi \int_1^2 [1]^2 - [\sqrt{y-1}]^2 \, dy$

$$V_{Total} = \pi + \pi(4 - 2 - 2 + 1/2)$$

$$= \boxed{\frac{3}{2}\pi \text{ units}^3}$$

Shell Method

* Vertical Axis (In terms of x)

$$V = 2\pi \int (\text{radius})(\text{height})\, dx$$

$$V = 2\pi \int_0^1 x(x^2+1)\, dx$$

$$= 2\pi \int_0^1 x^3 + x \, dx$$

$$= 2\pi \cdot \left[\frac{x^4}{4} + \frac{x^2}{2} \right]_0^1$$

$$= 2\pi \cdot \frac{1}{4} + 1/2 - 0$$

$$= 2\pi \left(\frac{1}{4} + 2/4 \right)$$

$$= 2\pi \left(\frac{3}{4} \right) = \boxed{\frac{3\pi}{2} \text{ units}^3}$$