7.4 Euler's Method HW

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3) Approximate $y(0.4)$ if $\frac{dy}{dx} = x^2 - y$ and $y=1$ when $x=0$.
Use $h=0.2$ as the increment

$\Delta x = 0.2$

x	y	$y' = x^2 - y$	New y = old y + $y'\Delta x$
0	1	$y' = (0)^2 - 1 = -1$	$y = 1 + (-1)(0.2) = 0.8$
0.2	0.8	$y' = (0.2)^2 - 0.8 = -0.76$	$y = 0.8 + (-0.76)(0.2) = 0.648$
0.4	$\boxed{0.648}$		

$$\boxed{y(0.4) \approx 0.648}$$

5) Approximate $y(1.4)$ if $\frac{dy}{dx} = xy^2 - x$ and $y=2$ when $x=1$
Use $h=0.2$ as step size. $(\Delta x = 0.2)$

x	y	$y' = xy^2 - x$	New y = old y + $y'\Delta x$
1	2	$y' = 1(2)^2 - 1 = 3$	$y = 2 + 3(0.2) = 2.6$
1.2	2.6	$y' = (1.2)(2.6)^2 - 1.2 = 6.912$	$y = 2.6 + 6.912(0.2) = 3.9824$
1.4	$\boxed{3.9824}$		

$$\boxed{y(1.4) \approx 3.9824}$$

7.4 HW

7) Approximate $y(0.6)$ if $\frac{dy}{dx} = 1 - x - 2y$ and $y = 3$ when $x = 1$

Use $h = -0.2$ as step size

$\Delta x = -0.2$

x	y_0	$y' = 1 - x - 2y$	$y = y_0 + y'\Delta x$
1	3	$y' = 1 - (1) - 2(3) = -6$	$y = 3 + (-6)(-0.2) = 4.2$
0.8	4.2	$y' = 1 - (0.8) - 2(4.2) = -8.2$	$y = 4.2 + (-8.2)(-0.2) = 5.84$
0.6	$\boxed{5.84}$		

$\boxed{y(0.6) \approx 5.84}$

9) a) Approximate $y(1)$ when $\frac{dy}{dx} = \cos(\pi x)$ and $x = 0$ and $y = 1$

Use step size 0.2 $(\Delta x = 0.2)$

x	y_0	$y' = \cos(\pi x)$	$y = y_0 + y'\Delta x$
0	1	$y' = \cos(\pi(0)) = 1$	$y = 1 + 1(0.2) = 1.2$
0.2	1.2	$y' = \cos(\pi(0.2)) = 0.809$	$y = 1.2 + 0.809(0.2) = 1.3618$
0.4	1.3618	$y' = \cos(0.4\pi) = 0.309$	$y = 1.3618 + 0.309(0.2) = 1.4236$
0.6	1.4236	$y' = \cos(0.6\pi) = -0.309$	$y = 1.4236 + -0.309(0.2) = 1.3617$
0.8	1.3617	$y' = \cos(0.8\pi) = -0.809$	$y = 1.3617 + -0.809(0.2) = 1.199$
1	$\boxed{1.2}$		

$\boxed{y(1) \approx 1.2}$

b) Solve $\frac{dy}{dx} = \cos(\pi x)$ at $(0, 1)$

$\int dy = \int \cos(\pi x)\,dx$

$y = \frac{1}{\pi}\sin(\pi x) + C$

$1 = \frac{1}{\pi}\sin(0) + C$

$1 = C$

$y = \frac{1}{\pi}\sin(\pi x) + 1$

c) Evaluate $y(1)$

$y(1) = \frac{1}{\pi}\sin(\pi) + 1$

$= \frac{1}{\pi}(0) + 1$

$= \boxed{1}$