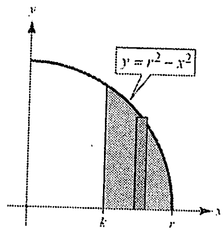
60. (a) $p(x) = x,\ h(x) = r^2 - x^2$

Shell method:

$$V = 2\pi \int_k^r x(r^2 - x^2)\,dx$$
$$= -\pi \int_k^r (r^2 - x^2)(-2x)\,dx$$
$$= -\pi \left[\frac{(r^2 - x^2)^2}{2}\right]_k^r$$
$$= -\pi\left[0 - \frac{(r^2 - k^2)^2}{2}\right] = \frac{\pi}{2}(r^2 - k^2)^2$$

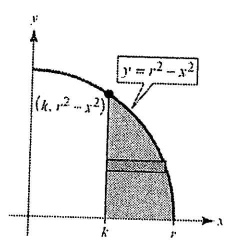


(b) $y = r^2 - x^2$

$x = \sqrt{r^2 - y}$

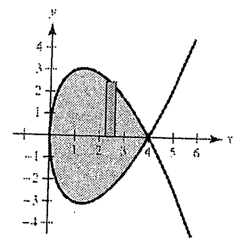
Disk method:

$$V = \pi \int_0^{r^2 - k^2} \left[\left(\sqrt{r^2 - y}\right)^2 - k^2\right] dy$$
$$= \pi \int_0^{r^2 - k^2} \left[r^2 - y - k^2\right] dy$$
$$= \pi\left[(r^2 - k^2)y - \frac{y^2}{2}\right]_0^{r^2 - k^2}$$
$$= \pi\left[(r^2 - k^2)^2 - \frac{(r^2 - k^2)^2}{2}\right] = \frac{\pi}{2}(r^2 - k^2)^2$$



61. $y^2 = x(4 - x)^2, \quad 0 \le x \le 4$

$$y_1 = \sqrt{x(4 - x)^2} = (4 - x)\sqrt{x}$$
$$y_2 = -\sqrt{x(4 - x)^2} = -(4 - x)\sqrt{x}$$



(a) $V = \pi \int_0^4 x(4 - x)^2\,dx$

$$= \pi \int_0^4 (x^3 - 8x^2 + 16x)\,dx$$
$$= \pi\left[\frac{x^4}{4} - \frac{8x^3}{3} + 8x^2\right]_0^4 = \frac{64\pi}{3}$$

(b) $V = 4\pi \int_0^4 x(4 - x)\sqrt{x}\,dx$

$$= 4\pi \int_0^4 (4x^{3/2} - x^{5/2})\,dx$$
$$= 4\pi\left[\frac{8}{5}x^{5/2} - \frac{2}{7}x^{7/2}\right]_0^4 = \frac{2048\pi}{35}$$

(c) $V = 4\pi \int_0^4 (4 - x)(4 - x)\sqrt{x}\,dx$

$$= 4\pi \int_0^4 \left(16\sqrt{x} - 8x^{3/2} + x^{5/2}\right) dx$$
$$= 4\pi\left[\frac{32}{3}x^{3/2} - \frac{16}{5}x^{5/2} + \frac{2}{7}x^{7/2}\right]_0^4 = \frac{8192\pi}{105}$$

Section 7.4 Arc Length and Surfaces of Revolution

1. $(0, 0), (8, 15)$

(a) $d = \sqrt{(8 - 0)^2 + (15 - 0)^2}$

$$= \sqrt{64 + 225}$$
$$= \sqrt{289} = 17$$

(b) $y = \frac{15}{8}x$

$y' = \frac{15}{8}$

$$s = \int_0^8 \sqrt{1 + \left(\frac{15}{8}\right)^2}\,dx = \int_0^8 \frac{17}{8}\,dx = \left[\frac{17}{8}x\right]_0^8 = 17$$

2. $(1, 2), (7, 10)$

(a) $d = \sqrt{(7-1)^2 + (10-2)^2} = 10$

(b) $y = \frac{4}{3}x + \frac{2}{3}$

$y' = \frac{4}{3}$

$s = \int_1^7 \sqrt{1 + \left(\frac{4}{3}\right)^2}\, dx = \left[\frac{5}{3}x\right]_1^7 = 10$

3. $y = \frac{2}{3}(x^2+1)^{3/2}$

$y' = (x^2+1)^{1/2}(2x), \quad 0 \le x \le 1$

$1 + (y')^2 = 1 + 4x^2(x^2+1)$

$= 4x^4 + 4x^2 + 1 = (2x^2+1)^2$

$s = \int_0^1 \sqrt{1 + (y')^2}\, dx$

$= \int_0^1 (2x^2+1)\, dx = \left[\frac{2x^3}{3} + x\right]_0^1 = \frac{5}{3}$

4. $y = \frac{x^3}{6} + \frac{1}{2x}$

$y' = \frac{x^2}{2} - \frac{1}{2x^2} = \frac{1}{2}\left(x^2 - \frac{1}{x^2}\right), \quad 1 \le x \le 2$

$1 + (y')^2 = 1 + \frac{1}{4}\left(x^4 - 2 + \frac{1}{x^4}\right)$

$= \frac{1}{4}\left(x^4 + 2 + \frac{1}{x^4}\right)$

$= \left[\frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)\right]^2$

$s = \int_1^2 \sqrt{1 + (y')^2}\, dx = \int_1^2 \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right) dx$

$= \frac{1}{2}\left[\frac{x^3}{3} - \frac{1}{x}\right]_1^2$

$= \frac{1}{2}\left[\left(\frac{8}{3} - \frac{1}{2}\right) - \left(\frac{1}{3} - 1\right)\right]$

$= \frac{17}{12}$

5. $y = \frac{2}{3}x^{3/2} + 1$

$y' = x^{1/2}, \quad 0 \le x \le 1$

$s = \int_0^1 \sqrt{1+x}\, dx$

$= \left[\frac{2}{3}(1+x)^{3/2}\right]_0^1 = \frac{2}{3}\left(\sqrt{8} - 1\right) \approx 1.219$

6. $y = 2x^{3/2} + 3$

$y' = 3x^{1/2}, \quad 0 \le x \le 9$

$s = \int_0^9 \sqrt{1 + 9x}\, dx$

$= \left[\frac{2}{27}(1+9x)^{3/2}\right]_0^9 = \frac{2}{27}\left(82^{3/2} - 1\right) \approx 54.929$

7. $y = \frac{3}{2}x^{2/3}$

$y' = \frac{1}{x^{1/3}}, \quad 1 \le x \le 8$

$s = \int_1^8 \sqrt{1 + \left(\frac{1}{x^{1/3}}\right)^2}\, dx$

$= \int_1^8 \sqrt{\frac{x^{2/3}+1}{x^{2/3}}}\, dx$

$= \frac{3}{2}\int_1^8 \sqrt{x^{2/3}+1}\left(\frac{2}{3x^{1/3}}\right) dx$

$= \frac{3}{2}\left[\frac{2}{3}\left(x^{2/3}+1\right)^{3/2}\right]_1^8$

$= 5\sqrt{5} - 2\sqrt{2} \approx 8.352$

8. $y = \frac{x^4}{8} + \frac{1}{4x^2}$

$y' = \frac{1}{2}x^3 - \frac{1}{2x^3}, \quad 1 \le x \le 3$

$1 + (y')^2 = \left(\frac{1}{2}x^3 + \frac{1}{2x^3}\right)^2, \quad [1, 3]$

$s = \int_a^b \sqrt{1 + (y')^2}\, dx$

$= \int_1^3 \left(\frac{1}{2}x^3 + \frac{1}{2x^3}\right) dx$

$= \left[\frac{1}{8}x^4 - \frac{1}{4x^2}\right]_1^3$

$= \frac{92}{9} \approx 10.222$

9. $$y = \frac{x^5}{10} + \frac{1}{6x^3}, \quad 2 \le x \le 5$$

$$y' = \frac{x^4}{2} - \frac{1}{2x^4} = \frac{1}{2}\left(x^4 - \frac{1}{x^4}\right)$$

$$1 + (y')^2 = 1 + \frac{1}{4}\left(x^4 - \frac{1}{x^4}\right)^2 = 1 + \frac{1}{4}\left(x^8 - 2 + \frac{1}{x^8}\right)$$

$$= \frac{1}{4}\left(x^8 + 2 + \frac{1}{x^8}\right) = \frac{1}{4}\left(x^4 + \frac{1}{x^4}\right)^2$$

$$s = \int_2^5 \sqrt{1 + (y')^2}\,dx = \int_2^5 \frac{1}{2}\left(x^4 + \frac{1}{x^4}\right)dx$$

$$= \frac{1}{2}\left[\frac{x^5}{5} - \frac{1}{3x^3}\right]_2^5 = \frac{1}{2}\left[\left(625 - \frac{1}{375}\right) - \left(\frac{32}{5} - \frac{1}{24}\right)\right]$$

$$= \frac{618639}{2000} \approx 309.320$$

10. $$y = \frac{3}{2}x^{2/3} + 4$$

$$y' = x^{-1/3}, \quad 1 \le x \le 27$$

$$s = \int_1^{27} \sqrt{1 + \left(\frac{1}{x^{1/3}}\right)^2}\,dx$$

$$= \int_1^{27} \sqrt{\frac{x^{2/3} + 1}{x^{2/3}}}\,dx$$

$$= \frac{3}{2}\int_1^{27} \sqrt{x^{2/3} + 1}\left(\frac{2}{3x^{1/3}}\right)dx$$

$$= \left[\frac{3}{2} \cdot \frac{2}{3}\left(x^{2/3} + 1\right)^{3/2}\right]_1^{27}$$

$$= 10^{3/2} - 2^{3/2} \approx 28.794$$

11. $$y = \ln(\sin x), \quad \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$$

$$y' = \frac{1}{\sin x}\cos x = \cot x$$

$$1 + (y')^2 = 1 + \cot^2 x = \csc^2 x$$

$$s = \int_{\pi/4}^{3\pi/4} \csc x\,dx$$

$$= \Big[\ln|\csc x - \cot x|\Big]_{\pi/4}^{3\pi/4}$$

$$= \ln\left(\sqrt{2} + 1\right) - \ln\left(\sqrt{2} - 1\right) \approx 1.763$$

12. $$y = \ln(\cos x), \quad 0 \le x \le \frac{\pi}{3}$$

$$y' = \frac{-\sin x}{\cos x} = -\tan x$$

$$1 + (y')^2 = 1 + \tan^2 x = \sec^2 x$$

$$s = \int_0^{\pi/3} \sqrt{\sec^2 x}\,dx$$

$$= \int_0^{\pi/3} \sec x\,dx$$

$$= \ln|\sec x + \tan x|\Big]_0^{\pi/3}$$

$$= \ln\left(2 + \sqrt{3}\right) \approx 1.3170$$

13. $$y = \frac{1}{2}\left(e^x + e^{-x}\right)$$

$$y' = \frac{1}{2}\left(e^x - e^{-x}\right), \quad [0, 2]$$

$$1 + (y')^2 = \left[\frac{1}{2}\left(e^x + e^{-x}\right)\right]^2, \quad [0, 2]$$

$$s = \int_0^2 \sqrt{\left[\frac{1}{2}\left(e^x + e^{-x}\right)\right]^2}\,dx$$

$$= \frac{1}{2}\int_0^2 \left(e^x + e^{-x}\right)dx$$

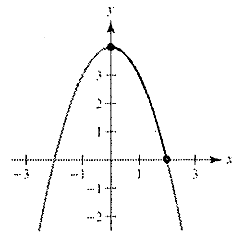
$$= \frac{1}{2}\left[e^x - e^{-x}\right]_0^2 = \frac{1}{2}\left(e^2 - \frac{1}{e^2}\right) \approx 3.627$$

14.
$$y = \ln\left(\frac{e^x + 1}{e^x - 1}\right) = \ln(e^x + 1) - \ln(e^x - 1)$$
$$\frac{dy}{dx} = \frac{e^x}{e^x + 1} - \frac{e^x}{e^x - 1} = \frac{-2e^x}{e^{2x} - 1} = \frac{2e^x}{1 - e^{2x}}$$
$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{4e^{2x}}{1 - 2e^{2x} + e^{4x}}$$
$$= \frac{1 + 2e^{2x} + e^{4x}}{(1 - e^{2x})^2} = \left(\frac{1 + e^{2x}}{1 - e^{2x}}\right)^2$$
$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_{\ln 2}^{\ln 3} \frac{1 + e^{2x}}{e^{2x} - 1}\,dx$$
$$= \int_{\ln 2}^{\ln 3} \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx = \int_{\ln 2}^{\ln 3} \coth x\,dx$$
$$= \ln(\sinh(x))\Big]_{\ln 2}^{\ln 3} = \ln\left(\frac{4}{3}\right) - \ln\left(\frac{3}{4}\right)$$
$$= \ln\left(\frac{4/3}{3/4}\right) = \ln\frac{16}{9} - 2\ln\left(\frac{4}{3}\right) \approx 0.57536$$

15. $x = \frac{1}{3}(y^2 + 2)^{3/2}, \quad 0 \le y \le 4$
$$\frac{dx}{dy} = y(y^2 + 2)^{1/2}$$
$$s = \int_0^4 \sqrt{1 + y^2(y^2 + 2)}\,dy$$
$$= \int_0^4 \sqrt{y^4 + 2y^2 + 1}\,dy$$
$$= \int_0^4 (y^2 + 1)\,dy$$
$$= \left[\frac{y^3}{3} + y\right]_0^4 = \frac{64}{3} + 4 = \frac{76}{3}$$

16.
$$x = \frac{1}{3}\sqrt{y}(y - 3), \quad 1 \le y \le 4$$
$$x = \frac{1}{3}(y^{3/2} - 3y^{1/2})$$
$$\frac{dx}{dy} = \frac{1}{2}y^{1/2} - \frac{1}{2}y^{-1/2}$$
$$1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{1}{4}y + \frac{1}{4}y^{-1} - \frac{1}{2}$$
$$= \frac{1}{4}(y + 2 + y^{-1}) = \frac{1}{4}\left(\sqrt{y} + \frac{1}{\sqrt{y}}\right)^2$$
$$s = \int_1^4 \frac{1}{2}\left(\sqrt{y} + \frac{1}{\sqrt{y}}\right)dy$$
$$= \left[\frac{1}{2}\left(\frac{2}{3}y^{3/2} + 2y^{1/2}\right)\right]_1^4$$
$$= \frac{1}{2}\left(\frac{16}{3} + 4\right) - \frac{1}{2}\left(\frac{2}{3} + 2\right) = \frac{10}{3}$$

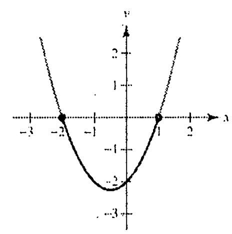
17. (a) $y = 4 - x^2, \quad 0 \le x \le 2$



(b)
$$y' = -2x$$
$$1 + (y')^2 = 1 + 4x^2$$
$$L = \int_0^2 \sqrt{1 + 4x^2}\,dx$$

(c) $L \approx 4.647$

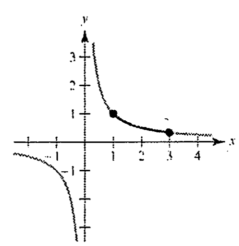
18. (a) $y = x^2 + x - 2, \quad -2 \le x \le 1$



(b)
$$y' = 2x + 1$$
$$1 + (y')^2 = 1 + 4x^2 + 4x + 1$$
$$L = \int_{-2}^1 \sqrt{2 + 4x + 4x^2}\,dx$$

(c) $L \approx 5.653$

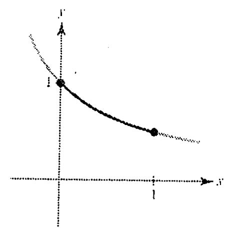
19. (a) $y = \frac{1}{x}, \quad 1 \le x \le 3$



(b)
$$y' = -\frac{1}{x^2}$$
$$1 + (y')^2 = 1 + \frac{1}{x^4}$$
$$L = \int_1^3 \sqrt{1 + \frac{1}{x^4}}\,dx$$

(c) $L \approx 2.147$

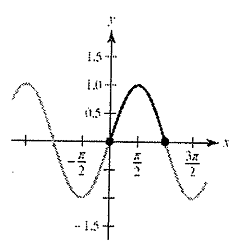
20. (a) $y = \dfrac{1}{1+x}, \quad 0 \le x \le 1$



(b) $$y' = -\frac{1}{(1+x)^2}$$
$$1 + (y')^2 = 1 + \frac{1}{(1+x)^4}$$
$$L = \int_0^1 \sqrt{1 + \frac{1}{(1+x)^4}}\, dx$$

(c) $L \approx 1.132$

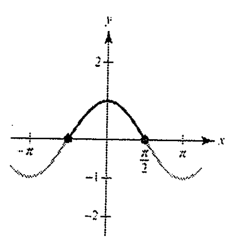
21. (a) $y = \sin x, \quad 0 \le x \le \pi$



(b) $$y' = \cos x$$
$$1 + (y')^2 = 1 + \cos^2 x$$
$$L = \int_0^{\pi} \sqrt{1 + \cos^2 x}\, dx$$

(c) $L \approx 3.820$

22. (a) $y = \cos x, \quad -\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$



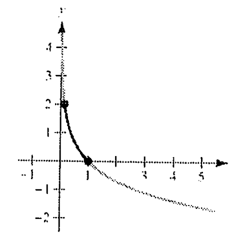
(b) $$y' = -\sin x$$
$$1 + (y')^2 = 1 + \sin^2 x$$
$$L = \int_{-\pi/2}^{\pi/2} \sqrt{1 + \sin^2 x}\, dx$$

(c) 3.820

23. (a) $x = e^{-y}, \quad 0 \le y \le 2$

$y = -\ln x$

$1 \ge x \ge e^{-2} \approx 0.135$



(b) $$y' = -\frac{1}{x}$$
$$1 + (y')^2 = 1 + \frac{1}{x^2}$$
$$L = \int_{e^{-2}}^{1} \sqrt{1 + \frac{1}{x^2}}\, dx$$

(c) $L \approx 2.221$

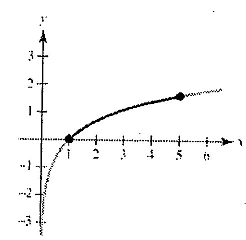
Alternatively, you can do all the computations with respect to y.

(a) $x = e^{-y}, \quad 0 \le y \le 2$

(b) $$\frac{dx}{dy} = -e^{-y}$$
$$1 + \left(\frac{dx}{dy}\right)^2 = 1 + e^{-2y}$$
$$L = \int_0^2 \sqrt{1 + e^{-2y}}\, dy$$

(c) $L \approx 2.221$

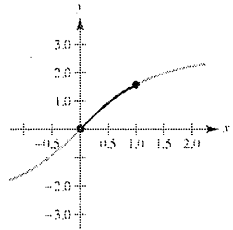
24. (a) $y = \ln x, \quad 1 \le x \le 5$



(b) $$y' = \frac{1}{x}$$
$$1 + (y')^2 = 1 + \frac{1}{x^2}$$
$$L = \int_1^5 \sqrt{1 + \frac{1}{x^2}}\, dx$$

(c) $L \approx 4.367$

25. (a) $y = 2 \arctan x, \quad 0 \le x \le 1$



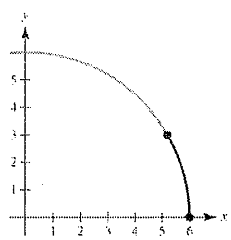
(b) $y' = \dfrac{2}{1 + x^2}$

$$L = \int_0^1 \sqrt{1 + \frac{4}{\left(1 + x^2\right)^2}}\, dx$$

(c) $L \approx 1.871$

26. (a) $x = \sqrt{36 - y^2}, \quad 0 \le y \le 3$

$y = \sqrt{36 - x^2}, \quad 3\sqrt{3} \le x \le 6$



(b) $\dfrac{dx}{dy} = \dfrac{1}{2}\left(36 - y^2\right)^{-1/2}(-2y) = \dfrac{-y}{\sqrt{36 - y^2}}$

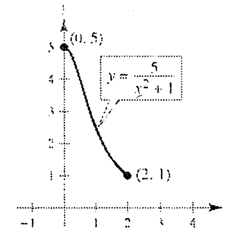
$$L = \int_0^3 \sqrt{1 + \frac{y^2}{36 - y^2}}\, dy = \int_0^3 \frac{6}{\sqrt{36 - y^2}}\, dy$$

(c) $L \approx 3.142 \quad (\pi)$

27. $\displaystyle\int_0^2 \sqrt{1 + \left[\frac{d}{dx}\left(\frac{5}{x^2 + 1}\right)\right]^2}\, dx$

$s \approx 5$

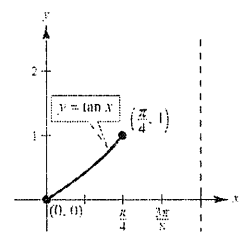
Matches (b)



28. $\displaystyle\int_0^{\pi/4} \sqrt{1 + \left[\frac{d}{dx}(\tan x)\right]^2}\, dx$

$s \approx 1$

Matches (e)



29. $y = x^3, \quad [0, 4]$

(a) $d = \sqrt{(4 - 0)^2 + (64 - 0)^2} \approx 64.125$

(b) $d = \sqrt{(1 - 0)^2 + (1 - 0)^2} + \sqrt{(2 - 1)^2 + (8 - 1)^2} + \sqrt{(3 - 2)^2 + (27 - 8)^2} + \sqrt{(4 - 3)^2 + (64 - 27)^2} \approx 64.525$

(c) $s = \int_0^4 \sqrt{1 + \left(3x^2\right)^2}\, dx = \int_0^4 \sqrt{1 + 9x^4}\, dx \approx 64.666$ (Simpson's Rule, $n = 10$)

(d) 64.672

30. $f(x) = \left(x^2 - 4\right)^2, \quad [0, 4]$

(a) $d = \sqrt{(4 - 0)^2 + (144 - 16)^2} \approx 128.062$

(b) $d = \sqrt{(1 - 0)^2 + (9 - 16)^2} + \sqrt{(2 - 1)^2 + (0 - 9)^2} + \sqrt{(3 - 2)^2 + (25 - 0)^2} + \sqrt{(4 - 3)^2 + (144 - 25)^2} \approx 160.151$

(c) $s = \int_0^4 \sqrt{1 + \left[4x\left(x^2 - 4\right)\right]^2}\, dx \approx 159.087$

(d) 160.287

31. $y = 20\cosh\dfrac{x}{20}, \quad -20 \le x \le 20$

$y' = \sinh\dfrac{x}{20}$

$1 + (y')^2 = 1 + \sinh^2\dfrac{x}{20} = \cosh^2\dfrac{x}{20}$

$$L = \int_{-20}^{20}\cosh\frac{x}{20}\,dx = 2\int_0^{20}\cosh\frac{x}{20}\,dx = \left[2(20)\sinh\frac{x}{20}\right]_0^{20} = 40\sinh(1) \approx 47.008 \text{ m}$$

32. $y = 31 - 10\left(e^{x/20} + e^{-x/20}\right)$

$y' = -\dfrac{1}{2}\left(e^{x/20} - e^{-x/20}\right)$

$1 + (y')^2 = 1 + \dfrac{1}{4}\left(e^{x/10} - 2 + e^{-x/10}\right) = \left[\dfrac{1}{2}\left(e^{x/20} + e^{-x/20}\right)\right]^2$

$$s = \int_{-20}^{20}\sqrt{\left[\frac{1}{2}\left(e^{x/20} + e^{-x/20}\right)\right]^2}\,dx = \frac{1}{2}\int_{-20}^{20}\left(e^{x/20} + e^{-x/20}\right)dx = \left[10\left(e^{x/20} - e^{-x/20}\right)\right]_{-20}^{20} = 20\left(e - \frac{1}{e}\right) \approx 47 \text{ ft}$$

So, there are $100(47) = 4700$ square feet of roofing on the barn.

33. $y = 693.8597 - 68.7672\cosh 0.0100333x$

$y' = -0.6899619478\sinh 0.0100333x$

$$s = \int_{-299.2239}^{299.2239}\sqrt{1 + (-0.6899619478\sinh 0.0100333x)^2}\,dx \approx 1480$$

(Use Simpson's Rule with $n = 100$ or a graphing utility.)

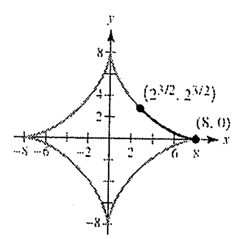
34. $x^{2/3} + y^{2/3} = 4$

$y^{2/3} = 4 - x^{2/3}$

$y = \left(4 - x^{2/3}\right)^{3/2}$

$y' = \dfrac{3}{2}\left(4 - x^{2/3}\right)^{1/2}\left(-\dfrac{2}{3}x^{-1/3}\right) = \dfrac{-\left(4 - x^{2/3}\right)^{1/2}}{x^{1/3}}$

In order to avoid division by 0, compute the arc length for $2^{3/2} \le x \le 8$, and multiply the answer by 8, as indicated in the figure.



$1 + (y')^2 = 1 + \dfrac{4 - x^{2/3}}{x^{2/3}}, \quad 2^{3/2} \le x \le 8$

$= \dfrac{4}{x^{2/3}}$

$s = 8\displaystyle\int_{2^{3/2}}^{8}\sqrt{\frac{4}{x^{2/3}}}\,dx$

$= 16\displaystyle\int_{2^{3/2}}^{8} x^{-1/3}\,dx$

$= 16\left[\dfrac{3}{2}x^{2/3}\right]_{2^{3/2}}^{8}$

$= 24(4 - 2) = 48$

35. $y = \sqrt{9 - x^2}$

$y' = \dfrac{-x}{\sqrt{9 - x^2}}$

$1 + (y')^2 = \dfrac{9}{9 - x^2}$

$s = \int_0^2 \sqrt{\dfrac{9}{9 - x^2}}\,dx = \int_0^2 \dfrac{3}{\sqrt{9 - x^2}}\,dx$

$= \left[3 \arcsin \dfrac{x}{3}\right]_0^2 = 3\left(\arcsin \dfrac{2}{3} - \arcsin 0\right)$

$= 3 \arcsin \dfrac{2}{3} \approx 2.1892$

36. $y = \sqrt{25 - x^2}$

$y' = \dfrac{-x}{\sqrt{25 - x^2}}$

$1 + (y')^2 = \dfrac{25}{25 - x^2}$

$s = \int_{-3}^4 \sqrt{\dfrac{25}{25 - x^2}}\,dx = \int_{-3}^4 \dfrac{5}{\sqrt{25 - x^2}}\,dx$

$= \left[5 \arcsin \dfrac{x}{5}\right]_{-3}^4 = 5\left[\arcsin \dfrac{4}{5} - \arcsin\left(-\dfrac{3}{5}\right)\right]$

≈ 7.8540

$\dfrac{1}{4}\left[2\pi(5)\right] \approx 7.8540 = s$

37. $y = \dfrac{x^3}{3}$

$y' = x^2, \quad [0, 3]$

$S = 2\pi \int_0^3 \dfrac{x^3}{3}\sqrt{1 + x^4}\,dx$

$= \dfrac{\pi}{6}\int_0^3 \left(1 + x^4\right)^{1/2}\left(4x^3\right) dx$

$= \left[\dfrac{\pi}{9}\left(1 + x^4\right)^{3/2}\right]_0^3$

$= \dfrac{\pi}{9}\left(82\sqrt{82} - 1\right) \approx 258.85$

38. $y = 2\sqrt{x}$

$y' = \dfrac{1}{\sqrt{x}}, \quad [4, 9]$

$S = 2\pi \int_4^9 2\sqrt{x}\sqrt{1 + \dfrac{1}{x}}\,dx$

$= 4\pi \int_4^9 \sqrt{x + 1}\,dx$

$= \left[\dfrac{8}{3}\pi(x + 1)^{3/2}\right]_4^9$

$= \dfrac{8\pi}{3}\left(10^{3/2} - 5^{3/2}\right) \approx 171.258$

39. $y = \dfrac{x^3}{6} + \dfrac{1}{2x}$

$y' = \dfrac{x^2}{2} - \dfrac{1}{2x^2}$

$1 + (y')^2 = \left(\dfrac{x^2}{2} + \dfrac{1}{2x^2}\right)^2, \quad [1, 2]$

$S = 2\pi \int_1^2 \left(\dfrac{x^3}{6} + \dfrac{1}{2x}\right)\left(\dfrac{x^2}{2} + \dfrac{1}{2x^2}\right) dx$

$= 2\pi \int_1^2 \left(\dfrac{x^5}{12} + \dfrac{x}{3} + \dfrac{1}{4x^3}\right) dx$

$= 2\pi\left[\dfrac{x^6}{72} + \dfrac{x^2}{6} - \dfrac{1}{8x^2}\right]_1^2 = \dfrac{47\pi}{16}$

40. $y = 3x$

$y' = 3$

$1 + (y')^2 = 10, \quad [0, 3]$

$S = 2\pi \int_0^3 3x\sqrt{10}\,dx$

$= 6\pi\sqrt{10}\left[\dfrac{x^2}{2}\right]_0^3$

$= 27\sqrt{10}\pi$

41. $y = \sqrt{4 - x^2}$

$y' = \dfrac{1}{2}\left(4 - x^2\right)^{-1/2}(-2x) = \dfrac{-x}{\sqrt{4 - x^2}}, \quad -1 \le x \le 1$

$1 + (y')^2 = 1 + \dfrac{x^2}{4 - x^2} = \dfrac{4}{4 - x^2}$

$S = 2\pi \int_{-1}^1 \sqrt{4 - x^2} \cdot \sqrt{\dfrac{4}{4 - x^2}}\,dx$

$= 4\pi \int_{-1}^1 dx = 4\pi[x]_{-1}^1 = 8\pi$

42. $y = \sqrt{9 - x^2}, \quad -2 \le x \le 2$

$y' = \dfrac{1}{2}\left(9 - x^2\right)^{-1/2}(-2x) = \dfrac{-x}{\sqrt{9 - x^2}}$

$1 + (y')^2 + 1 + \dfrac{x^2}{\left(9 - x^2\right)} = \dfrac{9}{9 - x^2}$

$S = 2\pi \int_{-2}^2 \sqrt{9 - x^2}\,\dfrac{3}{\sqrt{9 - x^2}}\,dx = 2\pi \int_{-2}^2 3\,dx$

$= 2\pi[3x]_{-2}^2 = 24\pi$

43. $y = \sqrt[3]{x} + 2$

$y' = \dfrac{1}{3x^{2/3}}, \quad [1, 8]$

$$S = 2\pi \int_1^8 x\sqrt{1 + \frac{1}{9x^{4/3}}}\,dx$$
$$= \frac{2\pi}{3}\int_1^8 x^{1/3}\sqrt{9x^{4/3} + 1}\,dx$$
$$= \frac{\pi}{18}\int_1^8 \left(9x^{4/3} + 1\right)^{1/2}\left(12x^{1/3}\right)dx$$
$$= \left[\frac{\pi}{27}\left(9x^{4/3} + 1\right)^{3/2}\right]_1^8$$
$$= \frac{\pi}{27}\left(145\sqrt{145} - 10\sqrt{10}\right) \approx 199.48$$

44. $y = 9 - x^2, \quad [0, 3]$

$y' = -2x$

$$S = 2\pi\int_0^3 x\sqrt{1 + 4x^2}\,dx$$
$$= \frac{\pi}{4}\int_0^3 \left(1 + 4x^2\right)^{1/2}(8x)\,dx$$
$$= \left[\frac{\pi}{6}\left(1 + 4x^2\right)^{3/2}\right]_0^3 = \frac{\pi}{6}\left(37^{3/2} - 1\right) \approx 117.319$$

45. $y = 1 - \dfrac{x^2}{4}$

$y' = -\dfrac{x}{2}, \quad 0 \le x \le 2$

$1 + (y')^2 = 1 + \dfrac{x^2}{4} = \dfrac{4 + x^2}{4}$

$$S = 2\pi\int_0^2 x\sqrt{\frac{4 + x^2}{4}}\,dx$$
$$= \pi\int_0^2 x\sqrt{4 + x^2}\,dx$$
$$= \frac{1}{2}\pi\int_0^2 \left(4 + x^2\right)^{1/2}(2x)\,dx$$
$$= \frac{1}{2}\pi\left[\frac{2}{3}\left(4 + x^2\right)^{3/2}\right]_0^2$$
$$= \frac{\pi}{3}\left(8^{3/2} - 4^{3/2}\right)$$
$$= \frac{\pi}{3}\left(16\sqrt{2} - 8\right) \approx 15.318$$

46. $y = \dfrac{x}{2} + 3$

$y' = \dfrac{1}{2}$

$1 + (y')^2 = \dfrac{5}{4}, 1 \le x \le 5$

$$S = 2\pi\int_1^5 x\sqrt{\frac{5}{4}}\,dx$$
$$= \sqrt{5}\pi\left[\frac{x^2}{2}\right]_1^5$$
$$= \sqrt{5}\pi\left(\frac{25}{2} - \frac{1}{2}\right) = 12\sqrt{5}\pi$$

47. $y = \sin x$

$y' = \cos x, \quad [0, \pi]$

$$S = 2\pi\int_0^\pi \sin x\sqrt{1 + \cos^2 x}\,dx \approx 14.4236$$

48. $y = \ln x$

$y' = \dfrac{1}{x}$

$1 + (y')^2 = \dfrac{x^2 + 1}{x^2}, \quad [1, e]$

$$S = 2\pi\int_1^e x\sqrt{\frac{x^2 + 1}{x^2}}\,dx = 2\pi\int_1^e \sqrt{x^2 + 1}\,dx$$
$$\approx 22.943$$

49. A rectifiable curve is one that has a finite arc length.

50. The precalculus formula is the distance formula between two points. The representative element is

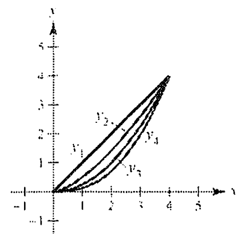
$$\sqrt{(\Delta x_i)^2 + (\Delta y_i)^2} = \sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\,\Delta xi.$$

51. The precalculus formula is the surface area formula for the lateral surface of the frustum of a right circular cone. The formula is $S = 2\pi rL$, where $r = \frac{1}{2}(r_1 + r_2)$, which is the average radius of the frustum, and L is the length of a line segment on the frustum. The representative element is

$$2\pi f(d_i)\sqrt{\Delta x_i^2 + \Delta y_i^2} = 2\pi f(d_i)\sqrt{1 + \left(\frac{\Delta y_i}{\Delta x_i}\right)^2}\,\Delta x_i.$$

52. The surface of revolution given by f_1 will be larger. $r(x)$ is larger for f_1.

53. (a)



(b) y_1, y_2, y_3, y_4

(c) $y_1' = 1, \quad s_1 = \int_0^4 \sqrt{2}\,dx \approx 5.657$

$$y_2' = \frac{3}{4}x^{1/2}, \quad s_2 = \int_0^4 \sqrt{1 + \frac{9x}{16}}\,dx \approx 5.759$$

$$y_3' = \frac{1}{2}x, \quad s_3 = \int_0^4 \sqrt{1 + \frac{x^2}{4}}\,dx \approx 5.916$$

$$y_4' = \frac{5}{16}x^{3/2}, \quad s_4 = \int_0^4 \sqrt{1 + \frac{25}{256}x^3}\,dx \approx 6.063$$

54. (a) Area of circle with radius L: $A = \pi L^2$

Area of sector with central angle θ (in radians):

$$S = \frac{\theta}{2\pi}A = \frac{\theta}{2\pi}\left(\pi L^2\right) = \frac{1}{2}L^2\theta$$

(b) Let s be the arc length of the sector, which is the circumference of the base of the cone. Here, $s = L\theta = 2\pi r$, and you have

$$S = \frac{1}{2}L^2\theta = \frac{1}{2}L^2\left(\frac{s}{L}\right) = \frac{1}{2}Ls = \frac{1}{2}L(2\pi r) = \pi rL.$$

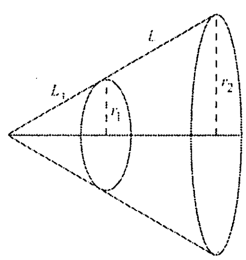
(c) The lateral surface area of the frustum is the difference of the large cone and the small one.

$$S = \pi r_2(L + L_1) - \pi r_1 L_1$$
$$= \pi r_2 L + \pi L_1(r_2 - r_1)$$

By similar triangles,

$$\frac{L + L_1}{r_2} = \frac{L_1}{r_1} \Rightarrow Lr_1 = L_1(r_2 - r_1). \text{ So,}$$

$$S = \pi r_2 L + \pi L_1(r_2 - r_1) = \pi r_2 L + \pi L r_1$$
$$= \pi L(r_1 + r_2).$$



55.
$$y = \frac{3x}{4}, \quad y' = \frac{3}{4}$$
$$1 + (y')^2 = 1 + \frac{9}{16} = 25/16$$
$$S = 2\pi\int_0^4 x\sqrt{\frac{25}{16}}\,dx = \frac{5\pi}{2}\left[\frac{x^2}{2}\right]_0^4 = 20\pi$$

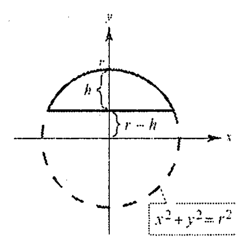
56.
$$y = \frac{hx}{r}$$
$$y' = \frac{h}{r}$$
$$1 + (y')^2 = \frac{r^2 + h^2}{r^2}$$
$$S = 2\pi\int_0^r x\sqrt{\frac{r^2 + h^2}{r^2}}\,dx$$
$$= \left[\frac{2\pi\sqrt{r^2 + h^2}}{r}\left(\frac{x^2}{2}\right)\right]_0^r = \pi r\sqrt{r^2 + h^2}$$

57.
$$y = \sqrt{9 - x^2}$$
$$y' = \frac{-x}{\sqrt{9 - x^2}}$$
$$\sqrt{1 + (y')^2} = \frac{3}{\sqrt{9 - x^2}}$$
$$S = 2\pi\int_0^2 \frac{3x}{\sqrt{9 - x^2}}\,dx$$
$$= -3\pi\int_0^2 \frac{-2x}{\sqrt{9 - x^2}}\,dx$$
$$= \left[-6\pi\sqrt{9 - x^2}\right]_0^2$$
$$= 6\pi\left(3 - \sqrt{5}\right) \approx 14.40$$

See figure in Exercise 58.

58. From Exercise 57 you have:

$$S = 2\pi\int_0^a \frac{rx}{\sqrt{r^2 - x^2}}\,dx$$
$$= -r\pi\int_0^a \frac{-2x\,dx}{\sqrt{r^2 - x^2}}$$
$$= \left[-2r\pi\sqrt{r^2 - x^2}\right]_0^a$$
$$= 2r^2\pi - 2r\pi\sqrt{r^2 - a^2}$$
$$= 2r\pi\left(r - \sqrt{r^2 - a^2}\right)$$
$$= 2\pi rh \text{ (where } h \text{ is the height of the zone)}$$



59. (a) Approximate the volume by summing six disks of thickness 3 and circumference C_i equal to the average of the given circumferences:

$$V \approx \sum_{i=1}^{6} \pi r_i^2(3) = \sum_{i=1}^{6} \pi\left(\frac{C_i}{2\pi}\right)^2(3) = \frac{3}{4\pi}\sum_{i=1}^{6} C_i^2$$

$$= \frac{3}{4\pi}\left[\left(\frac{50+65.5}{2}\right)^2 + \left(\frac{65.5+70}{2}\right)^2 + \left(\frac{70+66}{2}\right)^2 + \left(\frac{66+58}{2}\right)^2 + \left(\frac{58+51}{2}\right)^2 + \left(\frac{51+48}{2}\right)^2\right]$$

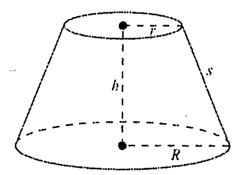
$$= \frac{3}{4\pi}\left[57.75^2 + 67.75^2 + 68^2 + 62^2 + 54.5^2 + 49.5^2\right] = \frac{3}{4\pi}(21813.625) = 5207.62 \text{ in.}^3$$

(b) The lateral surface area of a frustum of a right circular cone is $\pi s(R + r)$. For the first frustum:

$$S_1 \approx \pi\left[3^2 + \left(\frac{65.5-50}{2\pi}\right)^2\right]^{1/2}\left[\frac{50}{2\pi} + \frac{65.5}{2\pi}\right]$$

$$= \left(\frac{50+65.5}{2}\right)\left[9 + \left(\frac{65.5-50}{2\pi}\right)^2\right]^{1/2}.$$

Adding the six frustums together:



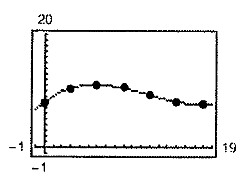
$$S \approx \left(\frac{50+65.5}{2}\right)\left[9+\left(\frac{15.5}{2\pi}\right)^2\right]^{1/2} + \left(\frac{65.5+70}{2}\right)\left[9+\left(\frac{4.5}{2\pi}\right)^2\right]^{1/2}$$

$$+\left(\frac{70+66}{2}\right)\left[9+\left(\frac{4}{2\pi}\right)^2\right]^{1/2} + \left(\frac{66+58}{2}\right)\left[9+\left(\frac{8}{2\pi}\right)^2\right]^{1/2}$$

$$+\left(\frac{58+51}{2}\right)\left[9+\left(\frac{7}{2\pi}\right)^2\right]^{1/2} + \left(\frac{51+48}{2}\right)\left[9+\left(\frac{3}{2\pi}\right)^2\right]^{1/2}$$

$$\approx 224.30 + 208.96 + 208.54 + 202.06 + 174.41 + 150.37 = 1168.64$$

(c) $r = 0.00401y^3 - 0.1416y^2 + 1.232y + 7.943$



(d) $V = \int_0^{18} \pi r^2 dy \approx 5275.9 \text{ in.}^3$

$S = \int_0^{18} 2\pi r(y)\sqrt{1 + r'(y)^2}\, dy \approx 1179.5 \text{ in.}^2$

60. (a) $y = f(x) = 0.0000001953x^4 - 0.0001804x^3 + 0.0496x^2 - 4.8323x + 536.9270$

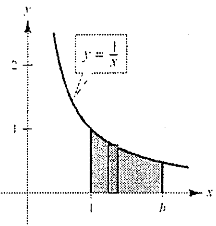
(b) Area $= \int_0^{400} f(x)\,dx \approx 131{,}734.5 \text{ ft}^2 \approx 3.0 \text{ acres} \quad (1 \text{ acre} = 43{,}560 \text{ ft}^2)$

(Answers will vary.)

(c) $L = \int_0^{400} \sqrt{1 + f'(x)^2}\, dx \approx 794.9 \text{ ft}$

(Answers will vary.)

61. (a) $V = \pi\int_1^b \frac{1}{x^2}\,dx = \left[-\frac{\pi}{x}\right]_1^b = \pi\left(1 - \frac{1}{b}\right)$



(b) $$S = 2\pi\int_1^b \frac{1}{x}\sqrt{1 + \left(-\frac{1}{x^2}\right)^2}\,dx$$
$$= 2\pi\int_1^b \frac{1}{x}\sqrt{1 + \frac{1}{x^4}}\,dx$$
$$= 2\pi\int_1^b \frac{\sqrt{x^4+1}}{x^3}\,dx$$

(c) $\lim_{b\to\infty} V = \lim_{b\to\infty} \pi\left(1 - \frac{1}{b}\right) = \pi$

(d) Because

$$\frac{\sqrt{x^4+1}}{x^3} > \frac{\sqrt{x^4}}{x^3} = \frac{1}{x} > 0 \text{ on } [1, b],$$

you have

$$\int_1^b \frac{\sqrt{x^4+1}}{x^3}\,dx > \int_1^b \frac{1}{x}\,dx = \left[\ln x\right]_1^b = \ln b$$

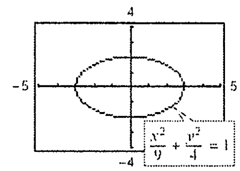
and $\lim_{b\to\infty} \ln b \to \infty$. So,

$$\lim_{b\to\infty} 2\pi\int_1^b \frac{\sqrt{x^4+1}}{x^3}\,dx = \infty.$$

62. (a) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

Ellipse: $y_1 = 2\sqrt{1 - \frac{x^2}{9}}$

$y_2 = -2\sqrt{1 - \frac{x^2}{9}}$



(b) $y = 2\sqrt{1 - \frac{x^2}{9}}, \quad 0 \le x \le 3$

$$y' = 2\left(\frac{1}{2}\right)\left(1 - \frac{x^2}{9}\right)^{-1/2}\left(\frac{-2x}{9}\right)$$
$$= \frac{-2x}{9\sqrt{1-(x^2/9)}} = \frac{-2x}{3\sqrt{9-x^2}}$$
$$L = \int_0^3 \sqrt{1 + \frac{4x^2}{81 - 9x^2}}\,dx$$

(c) You cannot evaluate this definite integral, because the integrand is not defined at $x = 3$. Simpson's Rule will not work for the same reason. Also, the integrand does not have an elementary antiderivative.

63. $y = \frac{1}{3}\left(x^{3/2} - 3x^{1/2} + 2\right)$

When $x = 0$, $y = \frac{2}{3}$. So, the fleeing object has traveled $\frac{2}{3}$ unit when it is caught.

$$y' = \frac{1}{3}\left(\frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2}\right) = \left(\frac{1}{2}\right)\frac{x-1}{x^{1/2}}$$
$$1 + (y')^2 = 1 + \frac{(x-1)^2}{4x} = \frac{(x+1)^2}{4x}$$
$$s = \int_0^1 \frac{x+1}{2x^{1/2}}\,dx = \frac{1}{2}\int_0^1 \left(x^{1/2} + x^{-1/2}\right)dx$$
$$= \frac{1}{2}\left[\frac{2}{3}x^{3/2} + 2x^{1/2}\right]_0^1 = \frac{4}{3} = 2\left(\frac{2}{3}\right)$$

The pursuer has traveled twice the distance that the fleeing object has traveled when it is caught.

64.

$$y = \frac{1}{3}x^{1/2} - x^{3/2}$$

$$y' = \frac{1}{6}x^{-1/2} - \frac{3}{2}x^{1/2} = \frac{1}{6}\left(x^{-1/2} - 9x^{1/2}\right)$$

$$1 + (y')^2 = 1 + \frac{1}{36}\left(x^{-1} - 18 + 81x\right) = \frac{1}{36}\left(x^{-1/2} + 9x^{1/2}\right)^2$$

$$S = 2\pi\int_0^{1/3}\left(\frac{1}{3}x^{1/2} - x^{3/2}\right)\sqrt{\frac{1}{36}\left(x^{-1/2} + 9x^{1/2}\right)^2}\,dx = \frac{2\pi}{6}\int_0^{1/3}\left(\frac{1}{3}x^{1/2} - x^{3/2}\right)\left(x^{-1/2} + 9x^{1/2}\right)dx$$

$$= \frac{\pi}{3}\int_0^{1/3}\left(\frac{1}{3} + 2x - 9x^2\right)dx = \frac{\pi}{3}\left[\frac{1}{3}x + x^2 - 3x^3\right]_0^{1/3} = \frac{\pi}{27}\text{ ft}^2 \approx 0.1164\text{ ft}^2 \approx 16.8\text{ in.}^2$$

Amount of glass needed: $V = \frac{\pi}{27}\left(\frac{0.015}{12}\right) \approx 0.00015\text{ ft}^3 \approx 0.25\text{ in.}^3$

65. $x^{2/3} + y^{2/3} = 4$

$$y^{2/3} = 4 - x^{2/3}$$

$$y = \left(4 - x^{2/3}\right)^{3/2}, \quad 0 \le x \le 8$$

$$y' = \frac{3}{2}\left(4 - x^{2/3}\right)^{1/2}\left(-\frac{2}{3}x^{-1/3}\right) = \frac{-\left(4 - x^{2/3}\right)^{1/2}}{x^{1/3}}$$

$$1 + (y')^2 = 1 + \frac{4 - x^{2/3}}{x^{2/3}} = \frac{4}{x^{2/3}}$$

$$S = 2\pi\int_0^8\left(4 - x^{2/3}\right)^{3/2}\sqrt{\frac{4}{x^{2/3}}}\,dx = 4\pi\int_0^8\frac{\left(4 - x^{2/3}\right)^{3/2}}{x^{1/3}}\,dx = \left[-\frac{12\pi}{5}\left(4 - x^{2/3}\right)^{5/2}\right]_0^8 = \frac{384\pi}{5}$$

[Surface area of portion above the x-axis]

66.

$$y^2 = \frac{1}{12}x(4 - x)^2, \quad 0 \le x \le 4$$

$$y = \frac{(4 - x)\sqrt{x}}{\sqrt{12}}$$

$$y' = \frac{(4 - 3x)\sqrt{3}}{12\sqrt{x}}$$

$$1 + (y')^2 = 1 + \frac{(4 - 3x)^2}{48x}$$

$$= \frac{48x + 16 - 24x + 9x^2}{48x} = \frac{(4 + 3x)^2}{48x}, \quad x \ne 0$$

$$S = 2\pi\int_0^4\frac{(4 - x)\sqrt{x}}{\sqrt{12}}\cdot\frac{(4 + 3x)}{\sqrt{48x}}\,dx$$

$$= 2\pi\int_0^4\frac{(4 - x)(4 + 3x)}{24}\,dx$$

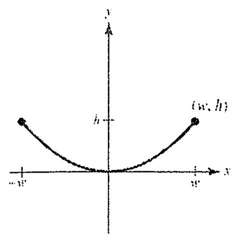
$$= \frac{\pi}{12}\int_0^4\left(16 + 8x - 3x^2\right)dx = \frac{\pi}{12}\left[16x + 4x^2 - x^3\right]_0^4 = \frac{\pi}{12}(64 + 64 - 64) = \frac{16\pi}{3}$$

67. $y = kx^2,\ y' = 2kx$

$1 + (y')^2 = 1 + 4k^2x^2$

$h = kw^2 \Rightarrow k = \frac{h}{w^2} \Rightarrow 1 + (y') = 1 + \frac{4h^2}{w^4}x^2$

By symmetry, $C = 2\int_0^w \sqrt{1 + \frac{4h^2}{w^4}x^2}\, dx.$



68. $C = 2\int_0^w \sqrt{1 + \frac{4h^2}{w^4}x^2}\, dx$

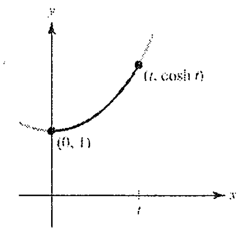
$= 2\int_0^{700} \sqrt{1 + \frac{4(155)^2x^2}{700^4}}\, dx = 1444.5 \text{ m}$

69.
$$y = f(x) = \cosh x$$
$$y' = \sinh x$$
$$1 + (y')^2 = 1 + \sinh^2 x = \cosh^2 x$$
$$\text{Area} = \int_0^t \cosh x\, dx = [\sinh x]_0^t = \sinh t$$
$$\text{Arc length} = \int_0^t \sqrt{1 + (y')^2}\, dx$$
$$= \int_0^t \cosh x\, dx = \sinh x\Big]_0^t$$
$$= \sinh t.$$

Another curve with this property is $g(x) = 1$.

$\text{Area} = \int_0^t dx = t$

$\text{Arc length} = t$



70. Let (x_0, y_0) be the point on the graph of $y^2 = x^3$ where the tangent line makes an angle of 45° with the x-axis.

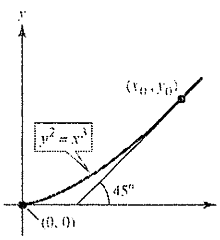
$y = x^{3/2}$

$y' = \frac{3}{2}x^{1/2} = 1$

$x_0 = \frac{4}{9}$

$L = \int_0^{4/9} \sqrt{1 + \frac{9}{4}x}\, dx$

$= \frac{8}{27}\left(2\sqrt{2} - 1\right)$



Section 7.5 Work

1. $W = Fd = 1200(40) = 48{,}000$ ft-lb

2. $W = Fd = 2500(6) = 15{,}000$ ft-lb

3. $W = Fd = (112)(8) = 896$ joules (Newton-meters)

4. $W = Fd = [9(2000)]\left[\frac{1}{2}(5280)\right] = 47{,}520{,}000$ ft-lb

5. $F(x) = kx$

$5 = k(3)$

$k = \frac{5}{3}$

$F(x) = \frac{5}{3}x$

$W = \int_0^7 F(x)\, dx = \int_0^7 \frac{5}{3}x\, dx = \left[\frac{5}{6}x^2\right]_0^7 = \frac{245}{6}$ in.-lb

≈ 40.833 in.-lb ≈ 3.403 ft-lb

6. $F(x) = kx$

$250 = k(30) \Rightarrow k = \frac{25}{3}$

$W = \int_{20}^{50} F(x)\, dx$

$= \int_{20}^{50} \frac{25}{3}x\, dx = \frac{25x^2}{6}\Big]_{20}^{50}$

$= 8750$ n-cm

$= 87.5$ joules or Nm

7. $F(x) = kx$

$20 = k(9)$

$k = \frac{20}{9}$

$W = \int_0^{12} \frac{20}{9}x\, dx = \left[\frac{10}{9}x^2\right]_0^{12} = 160$ in.-lb $= \frac{40}{3}$ ft-lb