Key

7.5 AP Practice Problems (p.566) – Logistic Models

1. A population grows according to the logistic differential equation $\frac{dP}{dt} = 0.01P\left(3 - \frac{P}{2000}\right)$. According to the model, the carrying capacity is

   (A) 2000 (B) 4000 $\boxed{\text{(C)}}$ 6000 (D) 10,000

*Match Form

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

M is the carrying capacity

$$\frac{dP}{dt} = 0.01P \cdot 3\left(\frac{3}{3} - \frac{P}{2000} \cdot \frac{1}{3}\right)$$

$$\frac{dP}{dt} = 0.03P\left(1 - \frac{P}{6000}\right)$$

2. A population P grows according to the logistic differential equation $\frac{dP}{dt} = 0.08P\left(1 - \frac{P}{2000}\right)$, where t is measured in years. What is the population when it is increasing most rapidly?

   (A) 40 (B) 80 $\boxed{\text{(C)}}$ 1000 (D) 2000

*population grows most rapidly at inflection point, when Population $= \frac{M}{2} \rightarrow \frac{2000}{2} = \boxed{1000}$

3. A population P grows according to the logistic function $P(t) = \frac{1200}{1 + 64e^{-0.4t}}$. The maximum population growth rate is

   (A) −0.4 $\boxed{\text{(B)}}$ 0.4 (C) 0.533 (D) 0.64

Logistic Function $P(t) = \frac{L}{1 + Ce^{-kt}}$ or $\frac{M}{1 + Ce^{-kt}}$

k = maximum population growth rate

$\boxed{k = 0.4}$

4. Suppose $P(t)$ is the particular solution of the differential equation $\frac{dP}{dt} = 0.3P(50 - P)$ with the initial condition if $t = 0$, then $P = 5$. Then $\lim_{t\to\infty} P(t)$ equals

(A) ∞ (B) 0.3 (C) 15 $\boxed{\text{(D)}}$ 50

$* P(t) = \frac{L}{1+Ce^{-kt}}$

$* \frac{dP}{dt} = \frac{k}{L}P(L-P)$

L = carrying capacity

$P(0)=5$ $L=50$ $C = \frac{L-P_0}{P_0}$

$C = \frac{50-5}{5} = 9$

$k=0.3$

$P(t) = \frac{50}{1+9e^{-0.3t}}$

$\lim_{t\to\infty} \frac{50}{1+9e^{-0.3t}} = \boxed{50}$

5. A virus spreads among a population of N people at a rate proportional to the product of the number of people infected with the virus and the number of people not infected with the virus. Which differential equation can be used to model the situation with respect to time t? Assume k is positive.

(A) $\frac{dP}{dt} = kP$ (B) $\frac{dP}{dt} = kN(N-P)$

(C) $\frac{dP}{dt} = kP(P-N)$ $\boxed{\text{(D)}}$ $\frac{dP}{dt} = kP(N-P)$

$* \frac{dP}{dt} = \frac{k}{L}P(L-P)$

6. An invasive breed of insect was introduced into a swamp in Florida. Initially 100 insects were present, and their daily maximum population growth rate is 20%. Entomologists have determined that the swamp can support a colony of 600,000 insects and that the population P follows a logistic growth model. Which logistic differential equation models the insect population?

(A) $\frac{dP}{dt} = 20\left(1 - \frac{P}{600{,}000}\right)$ (B) $\frac{dP}{dt} = 100\left(1 - \frac{P}{600{,}000}\right)$

(C) $\frac{dP}{dt} = 0.20P(600{,}000 - P)$ $\boxed{\text{(D)}}$ $\frac{dP}{dt} = 0.20P\left(1 - \frac{P}{600{,}000}\right)$

$\frac{dP}{dt} = \frac{k}{L}P(L-P)$

$k=0.2$
$L=600{,}000$

$P(0)=100$

$\frac{dP}{dt} = \frac{0.2}{600{,}000}P(600{,}000 - P)$

$\frac{dP}{dt} = 0.2P\left(\frac{600{,}000}{600{,}000} - \frac{P}{600{,}000}\right)$

$\boxed{\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{600{,}000}\right)}$

7. In an attempt to regenerate the American bald eagle population, eagles were captured and released in a federal park in Montana. From previous studies, environmentalists know that the population grows according to the logistic function

$P(t) = \dfrac{540}{1 + 89e^{-0.162t}}$, where t is measured in years.

(a) How many American bald eagles were released in Montana? a) $\boxed{6}$

(b) What is the maximum population growth rate of the American bald eagle? b) $\boxed{K=0.162}$

(c) What is the carrying capacity of the park? c) $\boxed{M=540}$

(d) What is the American bald eagle population when it is growing most rapidly? d) $\boxed{270}$

a) $P(t) = \dfrac{L}{1+Ce^{-kt}} \rightarrow$ Find $P(0)$ $\quad P(0) = \dfrac{540}{1+89e^{-0.162(0)}} = \dfrac{540}{90} = \boxed{6}$

b) $K = 0.162$

c) $L = 540$

d) growth most rapid when $\frac{L}{2} \rightarrow \frac{540}{2} \rightarrow \boxed{270}$

8. A population P grows according to the logistic differential equation

$$\frac{dP}{dt} = 0.0005P(800 - P)$$

(a) What is the carrying capacity of the population? $\boxed{800}$

(b) What is the maximum population growth rate of the population? $\boxed{K=0.4}$

(c) How large is the population when it is increasing most rapidly? $\boxed{400}$

$\frac{dP}{dt} = \frac{k}{L}P(L-P)$

a) $L = 800$

b) $\frac{K}{L} = 0.0005$

$\frac{K}{800} = 0.0005$

$\boxed{K = 0.4}$

c) $\frac{L}{2} \rightarrow \frac{800}{2} = \boxed{400}$

9. The population of hyenas in a game preserve increases according to the logistic function $P(t) = \dfrac{150}{1 + 14e^{-0.125t}}$, where t is measured in years.

(a) How many hyenas were originally in the preserve? $\boxed{10}$

(b) How many hyenas can the preserve maintain? $\boxed{M=150}$

(c) What is the maximum population growth rate? $\boxed{K=0.125}$

(d) How many hyenas are there when the population is growing most rapidly? $\boxed{75 \text{ hyenas}}$

a) $P(0) = \dfrac{150}{1+14e^{0}} \rightarrow \dfrac{150}{15} = \boxed{10 \text{ hyenas}}$

b) $P(t) = \dfrac{L}{1+Ce^{-kt}} \rightarrow \boxed{L=150}$

c) $k=0.125$

d) $\dfrac{150}{2} \rightarrow \boxed{75 \text{ hyenas}}$