

8.04 Circles Day 1

Date: _____

Recall:

The **Distance Formula** - the distance between the points (x_1, y_1) and (x_2, y_2) is as follows:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The **Midpoint Formula** - the midpoint of the line segment joining (x_1, y_1) and (x_2, y_2) is as follows:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Apply:

Given the two endpoints of the diameter of a circle, find the diameter's length.



$$1. \begin{matrix} x_1, y_1 & x_2, y_2 \\ (1, 7), & (5, 4) \end{matrix}$$

$$2. \begin{matrix} x_1, y_1 & x_2, y_2 \\ (-4, 7), & (0, -1) \end{matrix}$$

$$d = \sqrt{(5-1)^2 + (4-7)^2}$$

$$d = \sqrt{(0-(-4))^2 + (-1-7)^2}$$

$$d = \sqrt{4^2 + 3^2}$$

$$d = \sqrt{4^2 + 8^2} = \sqrt{16 + 64}$$

$$d = \sqrt{25} = \boxed{5}$$

$$\boxed{d = \sqrt{80}}$$

Given the two endpoints of the diameter of a circle, find the center of the circle.

$$3. (1, 7), (5, 5)$$

$$4. (-8, 9), (2, -4)$$

$$M\left(\frac{1+5}{2}, \frac{7+5}{2}\right)$$

$$M\left(\frac{-8+2}{2}, \frac{9-4}{2}\right)$$

$$M\left(\frac{6}{2}, \frac{12}{2}\right) \rightarrow \boxed{M(3, 6)}$$

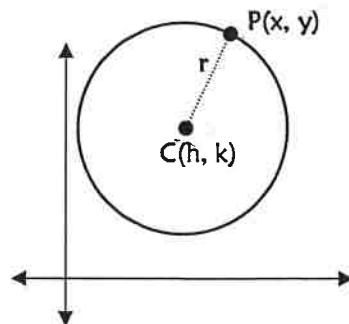
$$M\left(\frac{-6}{2}, \frac{+5}{2}\right)$$

$$\boxed{M\left(-3, \frac{5}{2}\right)}$$

Circle: the set of all points (x, y) that are equidistant from a fixed point, called the center of the circle. The distance r between the center and any point (x, y) on the circle is called the radius.

Suppose the center of this circle is translated from the origin to $C(h, k)$. Let's apply the distance formula to write the equation for the translated circle:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 1, fill in r , $P(x, y)$ and $C(h, k)$:

Step 2, square each side:

This is the standard form of the equation of a circle with radius r and center at (h, k) .

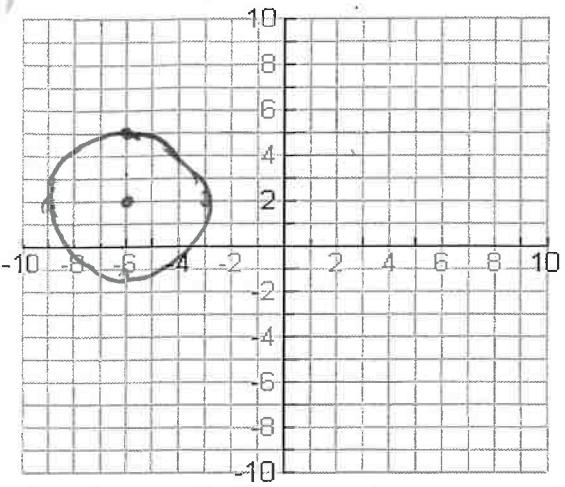
$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

$C(-6, 2)$
 $r=3$

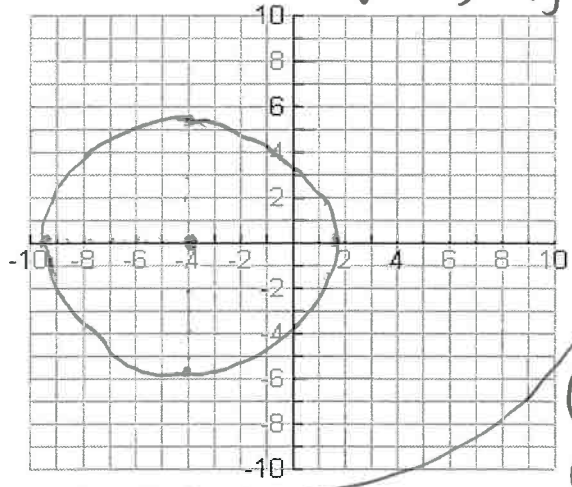
$(x-h)^2 + (y-k)^2 = r^2$

Given the standard form for the circle, identify the center and radius. Then graph.

5. $(x+6)^2 + (y-2)^2 = 9$



6. $(x+4)^2 + y^2 = 30$



$(x+4)^2 + (y-0)^2 = 30$

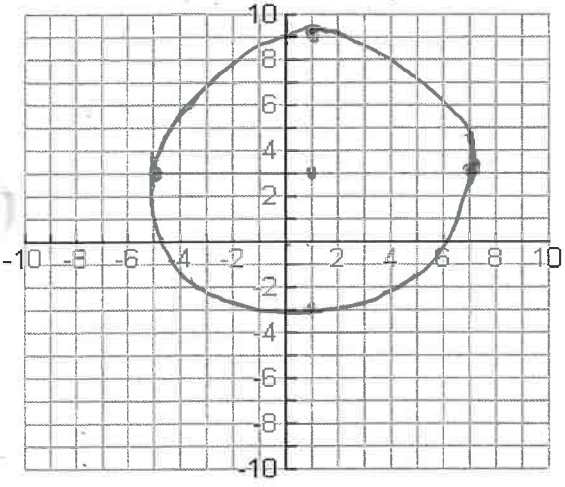
$C(-4, 0)$
 $r = \sqrt{30}$

$(x-h)^2 + (y-k)^2 = r^2$
 $(-4+4)^2 + (0-0)^2 = r^2$

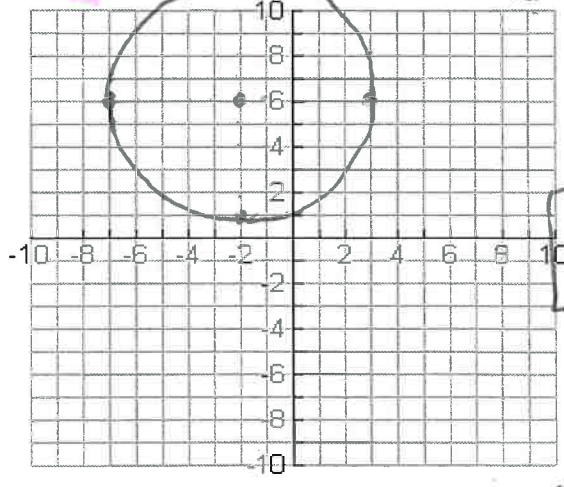
Write the standard form of the equation of each circle described and graph the equation.

7. Center at $(1, 3)$ and radius $= 6$

$(x-1)^2 + (y-3)^2 = 6^2$



8. Center at $(-2, 6)$ passing through $(1, 2)$

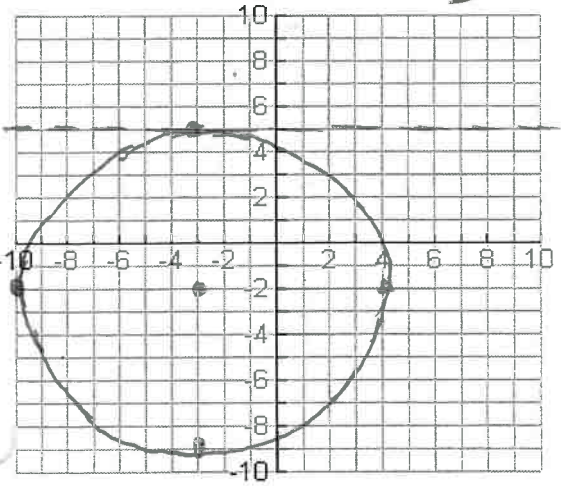


$(x-h)^2 + (y-k)^2 = r^2$
 $(1+2)^2 + (2-6)^2 = r^2$
 $3^2 + (-4)^2 = r^2$
 $9 + 16 = r^2$
 $25 = r^2$
 $r = 5$

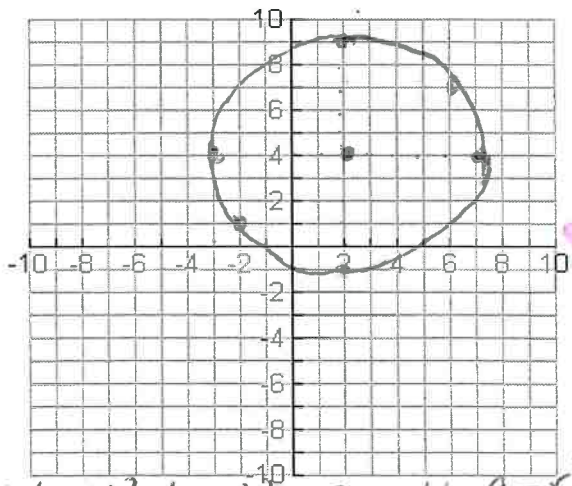
$(x+2)^2 + (y-6)^2 = 25$

9. Center at $(-3, -2)$ and tangent to the line $y=5$

$r=7$



10. The endpoints of the diameter are $(-2, 1)$ and $(6, 7)$



$M(-\frac{2+6}{2}, \frac{1+7}{2})$

$M(\frac{4}{2}, \frac{8}{2})$

$M(2, 4)$

Center $(2, 4)$
 h, k

point $(-2, 1)$
 x, y

$(x-h)^2 + (y-k)^2 = r^2$
 $(-2-2)^2 + (1-4)^2 = r^2$
 $4^2 + 3^2 = r^2$
 $16 + 9 = r^2$
 $r = 5$

$(x-2)^2 + (y-4)^2 = 25$

8.04 Practice

Given the standard form for the circle, identify the center and radius. Then graph.

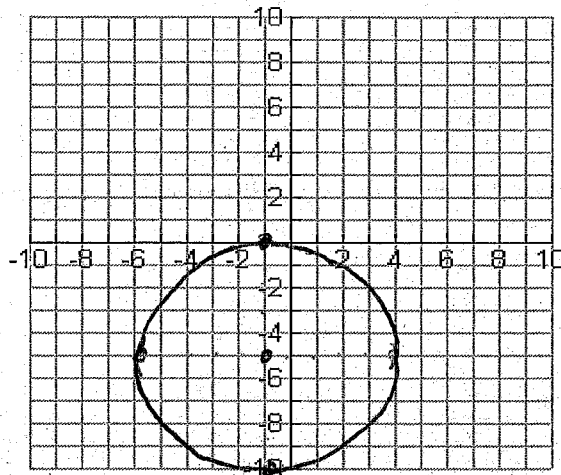
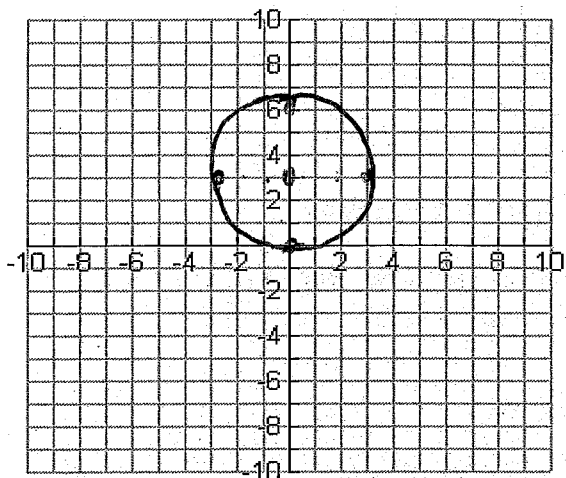
1. $x^2 + (y - 3)^2 = 9$

2. $(x + 1)^2 + (y + 5)^2 = 25$

$$(x - 0)^2 + (y - 3)^2 = 9$$

$$C(-1, -5) \quad r = 5$$

$$C(0, 3) \quad r = 3$$



3. $(x + 2)^2 + y^2 = \frac{16}{9}$

$$(x + 2)^2 + (y - 0)^2 = \frac{16}{9}$$

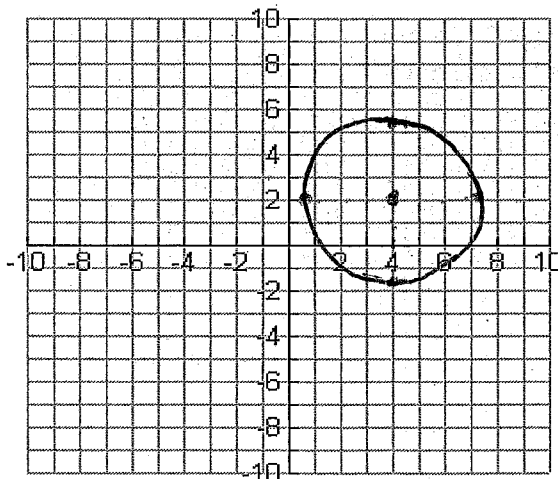
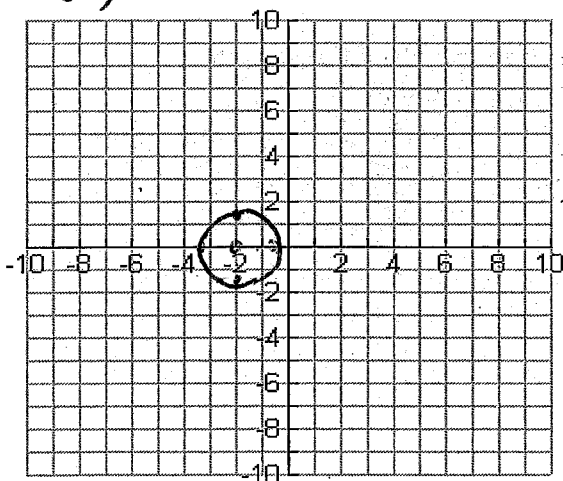
$$r = \sqrt{\frac{16}{9}}$$

$$r = \frac{4}{3}$$

$$C(-2, 0)$$

4. $(x - 4)^2 + (y - 2)^2 = 10$

$$C(4, 2) \quad r = \sqrt{10} \approx 3.1$$



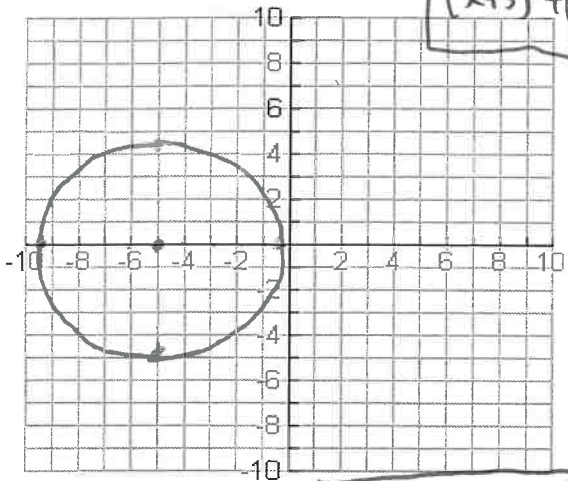
$$(x-h)^2 + (y-k)^2 = r^2 \quad 11$$

Write the standard form of the equation of each circle described and graph the equation.

5. Center at $(-5, 0)$, radius $\frac{9}{2}$

$$(x+5)^2 + (y-0)^2 = \left(\frac{9}{2}\right)^2$$

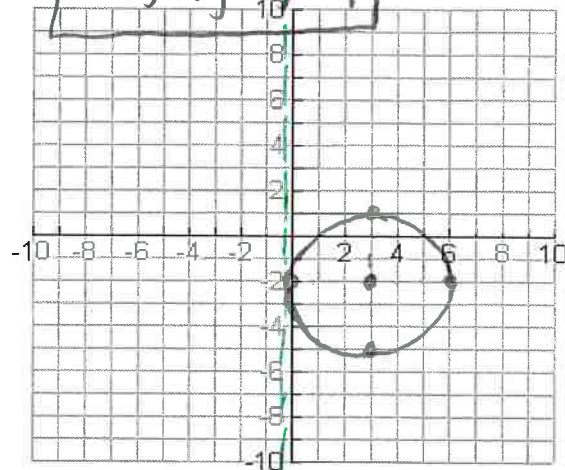
$$(x+5)^2 + (y-0)^2 = \frac{81}{4}$$



6. Center at $(3, -2)$ and tangent to the y-axis.

$$(x-3)^2 + (y+2)^2 = 9$$

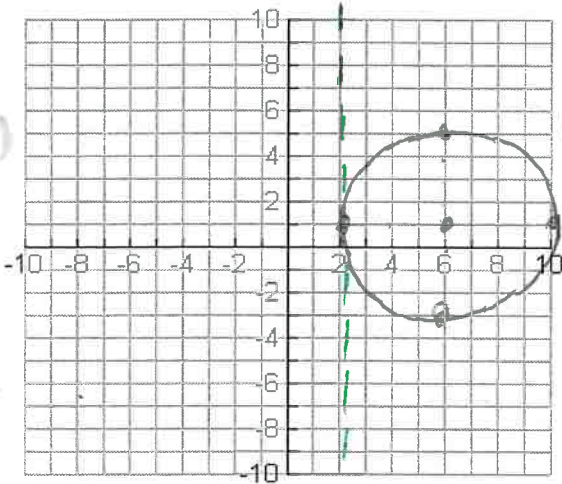
$$r=3$$



7. Center at $(6, 1)$ and tangent to the line $x=2$.

$$(x-6)^2 + (y-1)^2 = 16$$

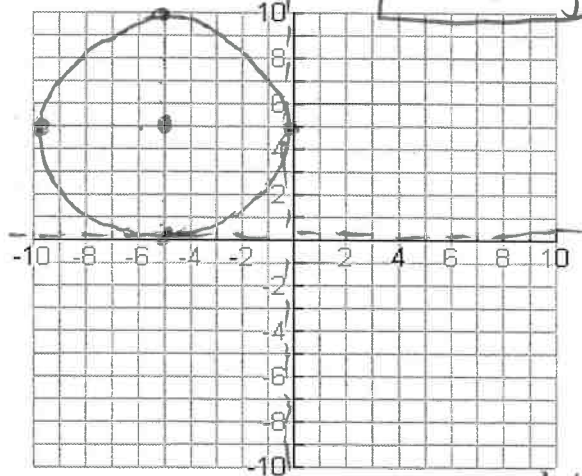
$$r=4$$



8. Center at $(-5, 5)$ and tangent to both the x and y-axes.

$$r=5$$

$$(x+5)^2 + (y-5)^2 = 25$$



Write the equation of the circle that satisfies each set of conditions.

9. The circle passes through the origin and has its center at $(-4, 3)$.

$$\begin{matrix} (0,0) \\ x & y \end{matrix}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(0+4)^2 + (0-3)^2 = r^2$$

$$4^2 + 3^2 = r^2$$

$$16 + 9 = r^2$$

$$25 = r^2, r=5$$

$$(x+4)^2 + (y-3)^2 = 25$$

10. The circle passes through the point $(5, 6)$ and has its center at $(2, 3)$.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(5-2)^2 + (6-3)^2 = r^2$$

$$3^2 + 3^2 = r^2$$

$$9 + 9 = r^2$$

$$18 = r^2$$

$$(x-2)^2 + (y-3)^2 = 18$$

11. The endpoints of a diameter are at $(2, 3)$ and at $(-6, -5)$.

$$M\left(\frac{2-6}{2}, \frac{3-5}{2}\right)$$

$$M(-2, -1)$$

$$C(-2, -1)$$

$$\text{point}(2, 3)$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(2-(-2))^2 + (3-(-1))^2 = r^2$$

$$4^2 + 4^2 = r^2$$

$$32 = r^2$$

$$(x+2)^2 + (y+1)^2 = 32$$