8.13 Ellipses and Hyperbolas Review

Date **Key**

1. Write the equation of the ellipse in standard form. Graph the ellipse and identify requested parts.

$$9x^2 + 16y^2 + 72x - 160y + 400 = 0$$

$$9x^2 + 72x + 16y^2 - 160y = -400$$

$$9(x^2+8x+16) + 16(y^2-10y+25) = -400 + 144 + 400$$

$$\left(\tfrac{8}{2}\right)^2 = 16 \qquad \left(\tfrac{10}{2}\right)^2 = 25$$

$$\frac{9(x+4)^2}{144} + \frac{16(y-5)^2}{144} = \frac{144}{144}$$

$$\frac{(x+4)^2}{16} + \frac{(y-5)^2}{9} = 1$$

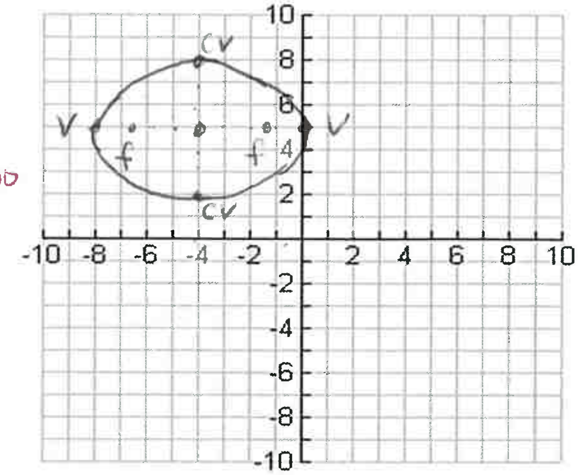
$a = 4 \leftrightarrow$

$b = 3 \updownarrow$

$c^2 = a^2 - b^2$

$c^2 = 16 - 9 = 7$

$c = \sqrt{7} \leftrightarrow$



Center: $(-4, 5)$

vertices: $(-8, 5), (0, 5)$

Foci: $(-4 \pm \sqrt{7}, 5)$

co-vertices: $(-4, 2), (-4, 8)$

Eccentricity = $\frac{\sqrt{7}}{4}$

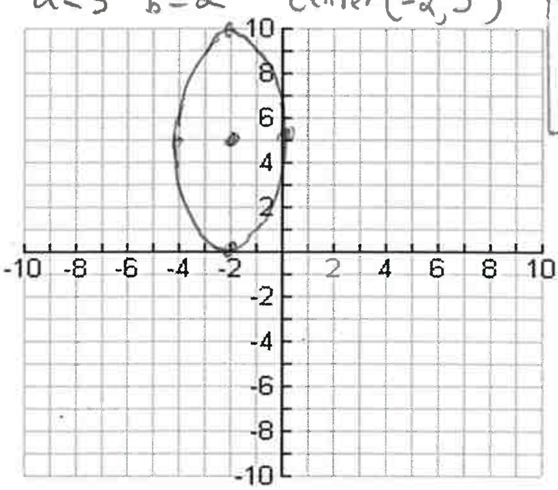
2. Write the equation of the ellipse in standard form which is tangent to the x-axis and the y-axis and the center is (-2, 5) then graph.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Equation:

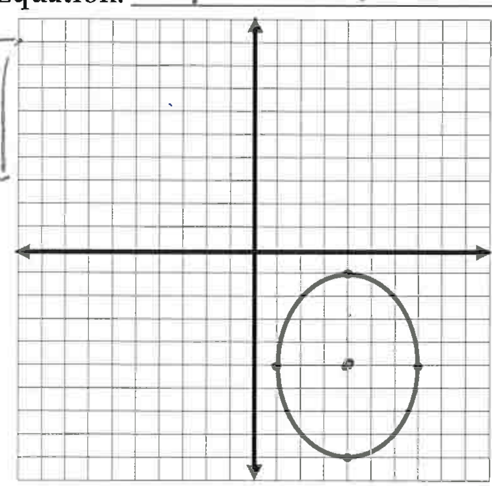
$a = 5 \quad b = 2 \quad$ center $(-2, 5)$

$$\boxed{\frac{(x+2)^2}{4} + \frac{(y-5)^2}{25} = 1}$$



3. Given the graph of the following ellipse, write the standard form of the ellipse equation.

Equation: $\frac{(x-4)^2}{9} + \frac{(y+5)^2}{16} = 1$



$a = 4 \updownarrow$

$b = 3 \leftrightarrow$

center: $(4, -5)$

4. Identify the characteristics of the hyperbola. Then, graph the hyperbola and label all parts.

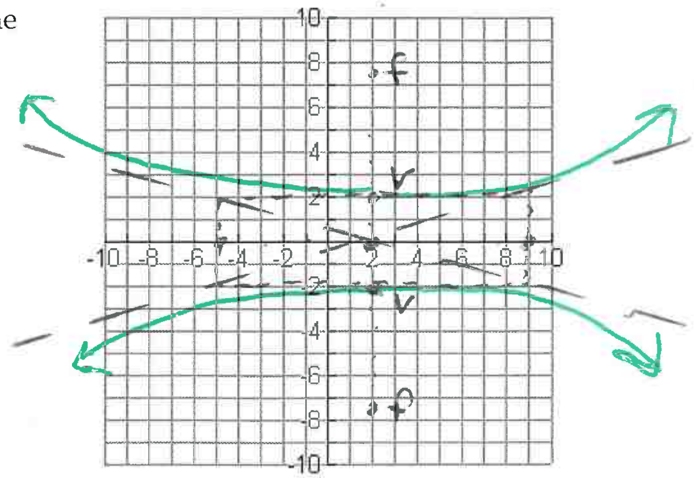
$$\frac{y^2}{4} - \frac{(x-2)^2}{49} = 1$$

$c^2 = a^2 + b^2$

$c^2 = 4 + 49 = 53$

$c = \sqrt{53}$

a = 2 b = 7 c = $\sqrt{53}$



Center: $(2, 0)$

Vertices: $(2, 2)$ $(2, -2)$

Foci: $(2, 0 \pm \sqrt{53})$

Asymptotes: $y - 0 = \pm\frac{2}{7}(x-2)$

Eccentricity: $\frac{c}{a} \rightarrow \frac{\sqrt{53}}{2}$

5. Write the equation of the hyperbola $16x^2 - 9y^2 + 32x - \mathbf{54y} - 209 = 0$ in standard form. Identify the center, vertices, foci, asymptotes, and eccentricity. Graph the **hyperbola** and label all parts.

$16x^2 + 32x - 9y^2 - 54y = 209$

$16(x^2 + 2x + \underline{1}) - 9(y^2 + 6y + \underline{9}) = 209 + \underline{16} + \underline{-81}$

$\frac{16(x+1)^2}{144} - \frac{9(y+3)^2}{144} = \frac{144}{144}$

$\frac{(x+1)^2}{9} - \frac{(y+3)^2}{16} = 1$

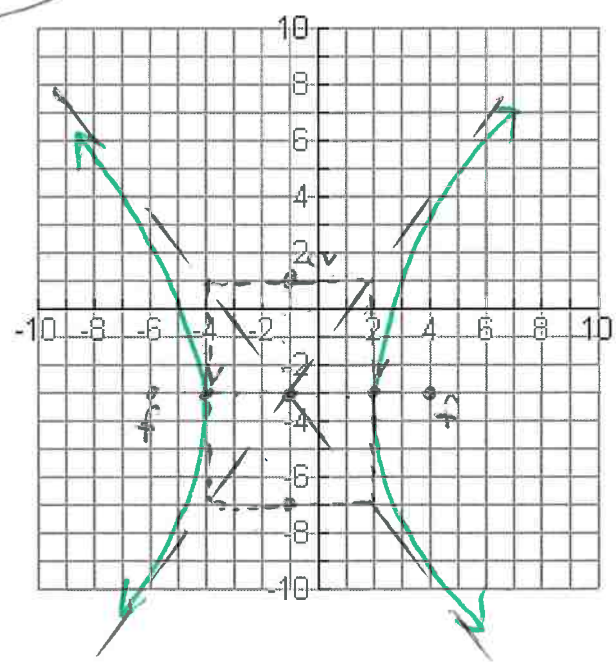
Standard Form:

a = 3 b = 4 c = 5

$c^2 = a^2 + b^2$

$c^2 = 3^2 + 4^2$

$c = 5$



Center: $(-1, -3)$

Vertices: $(2, -3), (-4, -3)$

Foci: $(4, -3)$ $(-6, -3)$

Asymptotes: $y + 3 = \pm\frac{4}{3}(x+1)$

Eccentricity: $\frac{5}{3}$

connects vertex to vertex

6. Write the standard form of the equation for the hyperbola whose transverse axis has endpoints P(–2, 1) and Q(–8, 1) and whose conjugate axis is a length of 6. Identify its characteristics. Then, graph the hyperbola and label all parts.

$b = 3$

$c^2 = a^2 + b^2$

$c^2 = 9 + 9 = 18$

a = 3 b = 3 c = $\sqrt{18}$ or $3\sqrt{2}$

Standard Form: $\frac{(x+5)^2}{9} - \frac{(y-1)^2}{9} = 1$

Center: $(-5, 1)$

Vertices: $(-2, 1)$ $(-8, 1)$

Foci: $(-5 \pm 3\sqrt{2}, 1)$

Asymptotes: $y - 1 = \pm 1(x+5)$

Eccentricity: $\frac{c}{a} \rightarrow \frac{3\sqrt{2}}{3} = \boxed{\sqrt{2}}$