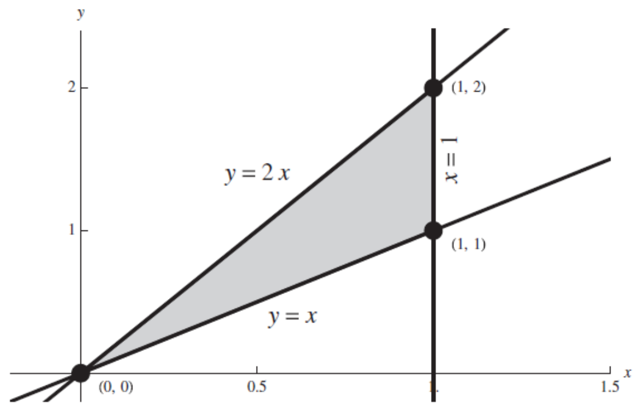
AP Calculus – 8.1 Homework Solutions   pg. 579-580

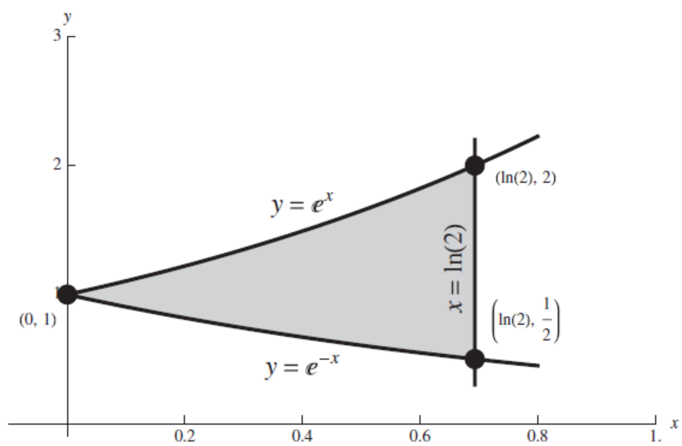
3. The region is shown below:



Partitioning along the x-axis, we have $x \leq 2x$, so the area is

$$\int_0^1 (2x - x)\,dx = \int_0^1 x\,dx = \left[\frac{1}{2}x^2\right]_0^1 = \frac{1}{2} - 0 = \boxed{\frac{1}{2}}.$$

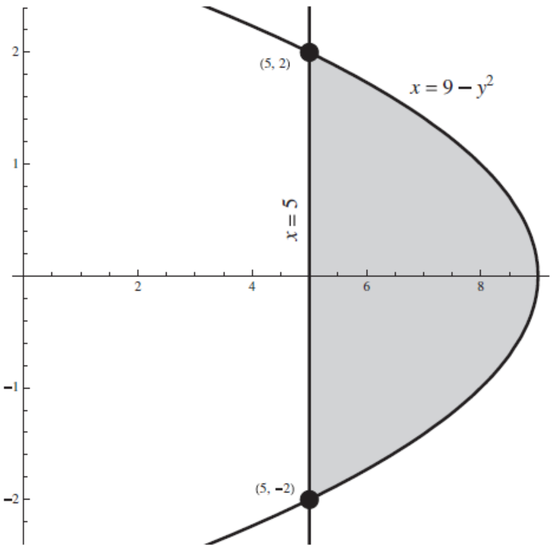
7. The region is shown below:



Partitioning along the x-axis, we have $e^{-x} \leq e^x$, so the area is

$$\int_0^{\ln 2} \left(e^x - e^{-x}\right)\,dx = \left[e^x + e^{-x}\right]_0^{\ln 2} = \left(e^{\ln 2} + e^{-\ln 2}\right) - \left(e^0 + e^0\right) = \left(2 + \frac{1}{2}\right) - (1+1) = \boxed{\frac{1}{2}}.$$

15. The region is shown below:



The two curves intersect where $9 - y^2 = 5$, so when $y = -2$ and $y = 2$. Partitioning along the y-axis, we have $5 \le 9 - y^2$, so the area is

$$\int_{-2}^{2} (9 - y^2 - 5)\, dy = \int_{-2}^{2} (4 - y^2)\, dy = \left[4y - \frac{1}{3}y^3\right]_{-2}^{2} = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) = \boxed{\frac{32}{3}}.$$

21. This region is bounded above by $y = \cos x$ and below by $y = -\sin x$, and it extends from $x = -\frac{\pi}{4}$ to $x = \frac{3\pi}{4}$. So we integrate by subdividing along the x axis, and we get

$$\int_{-\pi/4}^{3\pi/4} (\cos x - (-\sin x))\, dx = \int_{-\pi/4}^{3\pi/4} (\cos x + \sin x)\, dx = [\sin x - \cos x]_{-\pi/4}^{3\pi/4}$$

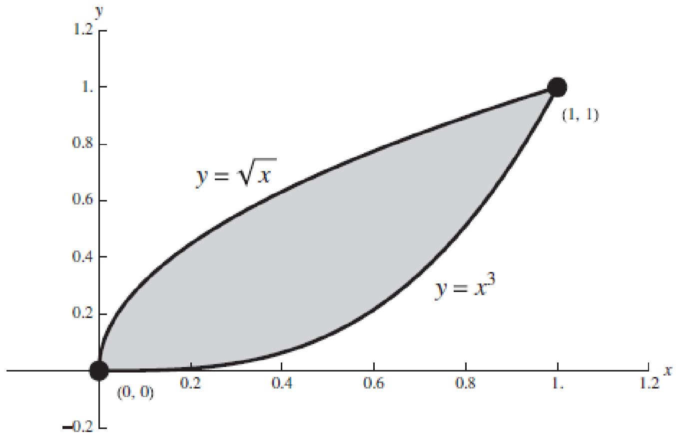
$$= \left(\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)\right) - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) = \boxed{2\sqrt{2}}.$$

23. If we try to partition along the x-axis, we will require two integrals, one for $-5 \le x \le 3$ and the other for $3 \le x \le 4$, since the upper boundof the area changes equations at $x = 3$. So we partition along the y-axis. The equation of the parabola is $x = -y^2 + 4$; solving the linear equation for x gives $x = 2y + 1$. Since $-y^2 + 4 \ge 2y + 1$ throughout the region of integration, the area is

$$\int_{-3}^{1} (-y^2 + 4 - (2y + 1))\, dy = \int_{-3}^{1} (-y^2 - 2y + 3)\, dy = \left[-\frac{1}{3}y^3 - y^2 + 3y\right]_{-3}^{1}$$

$$= \left(-\frac{1}{3} - 1 + 3\right) - (9 - 9 - 9) = \boxed{\frac{32}{3}}.$$

25. The region is shown below:



The two curves intersect when $\sqrt{x} = x^3$, which is at the points $(0,0)$ and $(1,1)$. So:

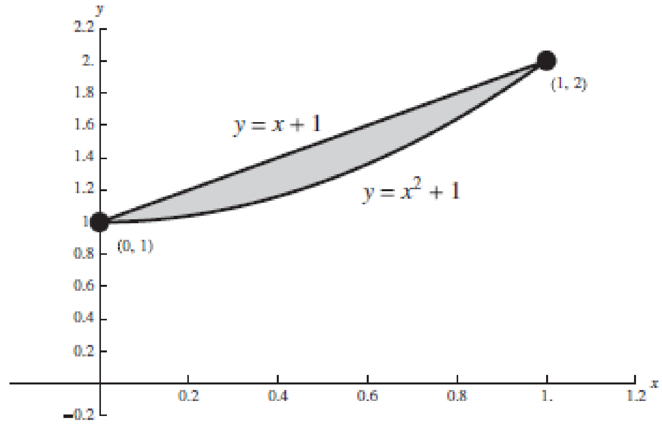
(a) Partitioning the x-axis, the area is

$$\int_0^1 (\sqrt{x} - x^3)\, dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{4}x^4\right]_0^1 = \left(\frac{2}{3} - \frac{1}{4}\right) - (0 - 0) = \boxed{\frac{5}{12}}.$$

(b) Partitioning the y-axis, we must first solve the two equations for x. This gives $x = y^2$ and $x = \sqrt[3]{y}$. Then the area between the curves is

$$\int_0^1 (\sqrt[3]{y} - y^2)\, dy = \left[\frac{3}{4}y^{4/3} - \frac{1}{3}y^3\right]_0^1 = \left(\frac{3}{4} - \frac{1}{3}\right) - (0 - 0) = \boxed{\frac{5}{12}}.$$

27. The region is shown below:



The curves $y = x^2 + 1$ and $y = x + 1$ intersect when $x^2 + 1 = x + 1$, so when $x^2 = x$. As a result, the intersection points are $(0, 1)$ and $(1, 2)$. So:

(a) Partitioning the x-axis, the area is

$$\int_0^1 ((x+1)-(x^2+1))\,dx = \int_0^1 (x-x^2)\,dx = \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1 = \left(\frac{1}{2} - \frac{1}{3}\right) - (0-0) = \boxed{\frac{1}{6}}.$$

(b) Partitioning the y-axis, we must first solve the two equations for x. This gives $x = \sqrt{y-1}$ and $x = y - 1$. Then the area between the curves is

$$\int_1^2 (\sqrt{y-1} - (y-1))\,dy = \int_1^2 (1 - y + \sqrt{y-1})\,dy = \left[y - \frac{1}{2}y^2 + \frac{2}{3}(y-1)^{3/2}\right]_1^2$$
$$= \left(2 - 2 + \frac{2}{3}\right) - \left(1 - \frac{1}{2} + 0\right) = \boxed{\frac{1}{6}}.$$