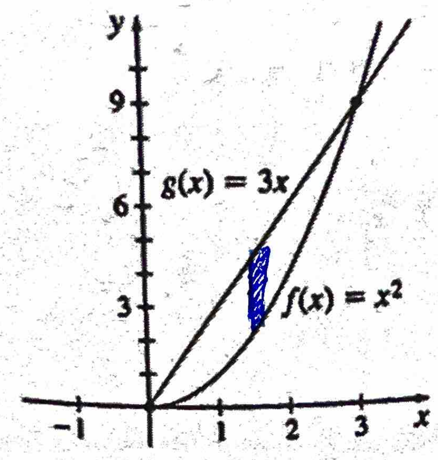
39

Key

8.1 AP Practice Problems (p.581-582)

1. The graphs of the functions $f(x) = x^2$ and $g(x) = 3x$ are shown below.



* Intersection:
$x^2 = 3x$
$x^2 - 3x = 0$
$x(x-3) = 0$
$x = 0,\ x = 3$

Find the area of the region bounded by the two graphs.

(A) $\frac{1}{3}$ (B) $\frac{7}{6}$ $\boxed{\text{(C)}}$ $\frac{9}{2}$ (D) $\frac{27}{2}$

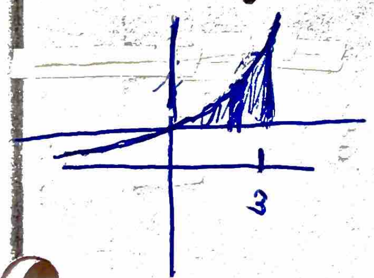
$$\text{Area} = \int_0^3 3x - x^2\,dx$$

$$= \frac{3x^2}{2} - \frac{x^3}{3}\Big]_0^3 = \frac{3(9)}{2} - \frac{27}{3} = \boxed{\frac{9}{2}}$$

2. What is the area of the region bounded by the graphs of $y = e^{x/3}$, $y = 1$, and the line $x = 3$?

(A) $\frac{1}{3}(e - 10)$ (B) $e - 4$

(C) $3e - 3$ $\boxed{\text{(D)}}$ $3e - 6$



$$\text{Area} = \int_0^3 e^{x/3} - 1\,dx$$

$$\int e^{\frac{1}{3}x}dx - \int 1\,dx$$

$-x + C$

u-sub
$u = \frac{1}{3}x$
$\frac{du}{dx} = \frac{1}{3}$

$dx = 3du$

$$\int e^u \cdot 3du$$

$$3 \cdot e^{x/3} - x\Big]_0^3$$

$3e^1 - 3 - (3e^0 - 0)$

$3e - 3 - 3 = \boxed{3e - 6}$

40

8.1

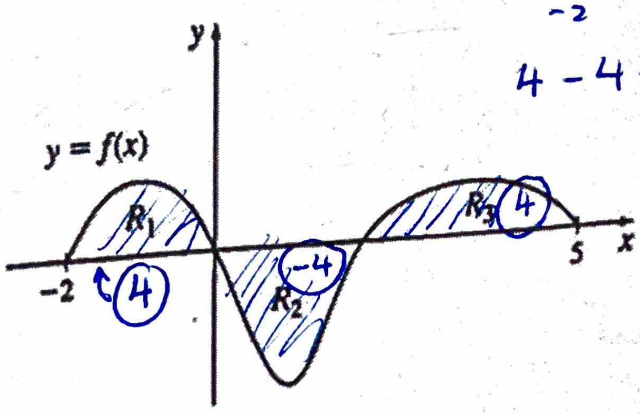
3. The figure below shows the graph of a function f over the closed interval $[-2, 5]$. If each of the regions R_1, R_2, and R_3 bounded by the graph of f and the x-axis has area equal to 4, then what is $\int_{-2}^{5} [f(x) + e^x]\, dx$?

$\int_{-2}^{5} f(x)dx + \int_{-2}^{5} e^x dx$

$4 - 4 + 4 + e^x \Big]_{-2}^{5}$

$4 + e^5 - e^{-2}$

$= e^5 - e^{-2} + 4$



(A) $e^5 - e^{-2} + 8$ $\boxed{\text{(B)}}$ $e^5 - e^{-2} + 4$

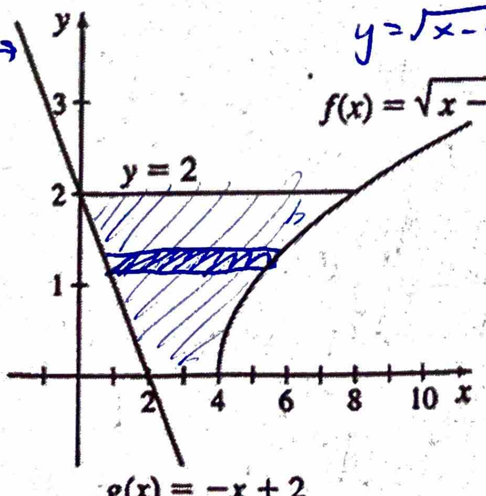
(C) $e^5 - e^{-2} - 4$ (D) $e^5 - e^{-2} + 12$

4. The graphs of the functions $f(x) = \sqrt{x-4}$, $g(x) = -x + 2$, and $y = 2$ are shown in the figure.

$y = -x + 2$

$x = 2 - y$

$y = \sqrt{x-4} \rightarrow y^2 + 4 = x$



$\text{Area} = \int_{y_1}^{y_2} \text{Right} - \text{Left}\, dy$

$\text{Area} = \int_{0}^{2} y^2 + 4 - (2 - y) dy$

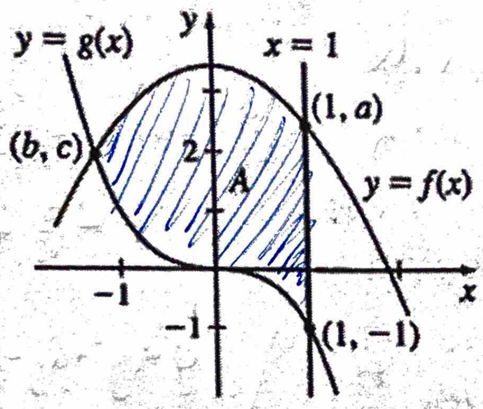
$\int y^2 + 4 - 2 + y\, dy \rightarrow \frac{y^3}{3} + 2y + \frac{y^2}{2} \Big]_{0}^{2}$

$\frac{8}{3} + 4 + \frac{4}{2} = \frac{8}{3} + \frac{18}{3} = \boxed{\frac{26}{3}}$

Find the area of the region bounded by the graphs and the x-axis.

(A) $\frac{4\sqrt{2} - 14}{3}$ $\boxed{\text{(B)}}$ $\frac{26}{3}$ (C) $\frac{86}{3}$ (D) $\frac{112}{3}$

5. The graphs of the functions $y = f(x)$ and $y = g(x)$ intersect at the point (b, c) as shown in the figure below. The area A of the region bounded by the graphs of f, g and the line $x = 1$ is obtained by finding the integral



(A) $\int_1^b [f(x) - g(x)]dx$ $\quad$ $\boxed{\text{(B)}}$ $\int_b^1 [f(x) - g(x)]dx$

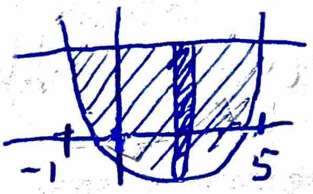
(C) $\int_{-1}^{c} [f(x) - g(x)]dx$

(D) $\int_b^0 [f(x) - g(x)]dx + \int_0^1 [f(x) - |g(x)|]dx$

Intersection: $x^2 - 4x + 2 = 7$ $\quad$ $x^2 - 4x - 5 = 0$ $\quad$ $(x-5)(x+1) = 0$ $\quad$ $x = 5, -1$

6. What is the area of the region bounded by the graphs of $y = x^2 - 4x + 2$ and $y = 7$?

(A) -36 $\quad$ $\boxed{\text{(B)}}$ 36 $\quad$ (C) $\frac{64}{3}$ $\quad$ (D) 60

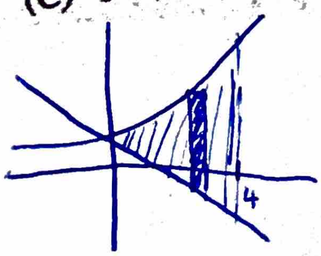


$$\int_{-1}^{5} 7 - (x^2 - 4x + 2)\,dx = \boxed{36}$$

7. Find the area of the region bounded by the graphs of the functions $f(x) = e^x + 2$, $g(x) = -2x + 3$ and the line $x = 4$.

(A) $e^4 - 1$ $\quad$ $\boxed{\text{(B)}}$ $e^4 + 11$

(C) $e^4 + 12$ $\quad$ (D) $2e^4 + 12$



$$\text{Area} = \int_0^4 e^x + 2 - (-2x + 3)\,dx$$

$$= \int_0^4 e^x + 2 + 2x - 3\,dx$$

$$\int_0^4 e^x + 2x - 1\,dx$$

$$e^x + \frac{2x^2}{2} - x\Big]_0^4$$

$$e^4 + 4^2 - 4 - (e^0 + 0 - 0)$$

$$e^4 + 16 - 4 - 1$$

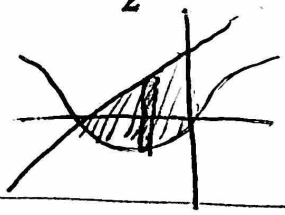
$$\boxed{e^4 + 11}$$

(42) $\boxed{8.1}$

8. The area of the region bounded by the graphs of $y = \sin x$, $y = \frac{1}{\pi}x + 1$, and the y-axis is

$*$intersection
$\frac{1}{\pi}x + 1 = \sin x$
$x \approx -\pi$

(A) $\frac{\pi}{2}$ (B) $1 + \frac{3\pi}{4}$ $\boxed{\text{(C)}}$ $2 + \frac{\pi}{2}$ (D) $\frac{3\pi}{2} - 1$



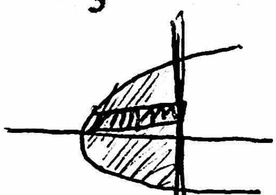
$$\text{Area} = \int_{-\pi}^{0} \frac{1}{\pi}x + 1 - \sin x \, dx$$

$$\frac{1}{\pi}\cdot\frac{x^2}{2} + x - (-\cos x)\Big]_{-\pi}^{0} = \frac{0}{2\pi} + 0 + \cos 0 - \left(\frac{\pi^2}{2\pi} + -\pi + \cos \pi\right)$$

$$1 - \frac{\pi}{2} + \pi + 1 = \boxed{2 + \frac{\pi}{2}}$$

9. Find the area of the region bounded by the graph of $x = y^2 + 2y - 3$ and the y-axis.

(A) $-\frac{32}{3}$ (B) $-\frac{20}{3}$ (C) $\frac{20}{3}$ $\boxed{\text{(D)}}$ $\frac{32}{3}$



$*$ Intersection: $y^2 + 2y - 3 = 0$
$(y+3)(y-1) = 0$
$y = 1, y = -3$

$$\int_{-3}^{1} 0 - (y^2 + 2y - 3)\, dy$$

$$\int -y^2 - 2y + 3\, dy$$

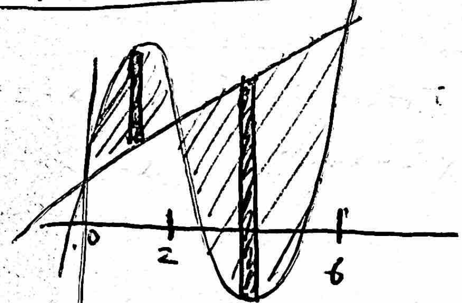
$$-\frac{y^3}{3} - \frac{2y^2}{2} + 3y\Big]_{-3}^{1} = \boxed{\frac{32}{3}}$$

10. Consider the functions $f(x) = x^3 - 8x^2 + 15x + 2$ and $g(x) = 3x + 2$.

(a) Find the points (x, y) where the graphs of f and g intersect.

(b) Write the integral(s) that represent the area of the region(s) bounded by the graphs of f and g.

(c) Find the area of the region bounded by the graphs.



a) Intersection:
$x^3 - 8x^2 + 15x + 2 = 3x + 2$
$x^3 - 8x^2 + 12x = 0$
$x(x^2 - 8x + 12) = 0$
$x(x-6)(x-2) = 0$
$\boxed{x = 0, 6, 2}$

b) $$\int_0^2 x^3 - 8x^2 + 15x + 2 - (3x+2)\, dx + \int_2^6 3x + 2 - (x^3 - 8x^2 + 15x + 2)\, dx$$

c) $\text{Area} = \boxed{\frac{148}{3}}$