8.1 Exercises

See **CalcChat.com** for tutorial help and worked-out solutions to odd-numbered exercises.

Choosing an Antiderivative In Exercises 1–4, select the correct antiderivative.

1. $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{x^2+1}}$

(a) $2\sqrt{x^2+1} + C$ (b) $\sqrt{x^2+1} + C$

(c) $\frac{1}{2}\sqrt{x^2+1} + C$ (d) $\ln(x^2+1) + C$

2. $\dfrac{dy}{dx} = \dfrac{x}{x^2+1}$

(a) $\ln\sqrt{x^2+1} + C$ (b) $\dfrac{2x}{(x^2+1)^2} + C$

(c) $\arctan x + C$ (d) $\ln(x^2+1) + C$

3. $\dfrac{dy}{dx} = \dfrac{1}{x^2+1}$

(a) $\ln\sqrt{x^2+1} + C$ (b) $\dfrac{2x}{(x^2+1)^2} + C$

(c) $\arctan x + C$ (d) $\ln(x^2+1) + C$

4. $\dfrac{dy}{dx} = x\cos(x^2+1)$

(a) $2x\sin(x^2+1) + C$ (b) $-\frac{1}{2}\sin(x^2+1) + C$

(c) $\frac{1}{2}\sin(x^2+1) + C$ (d) $-2x\sin(x^2+1) + C$

Choosing a Formula In Exercises 5–14, select the basic integration formula you can use to find the integral, and identify u and a when appropriate.

5. $\displaystyle\int (5x-3)^4\,dx$

6. $\displaystyle\int \frac{2t+1}{t^2+t-4}\,dt$

7. $\displaystyle\int \frac{1}{\sqrt{x}(1-2\sqrt{x})}\,dx$

8. $\displaystyle\int \frac{2}{(2t-1)^2+4}\,dt$

9. $\displaystyle\int \frac{3}{\sqrt{1-t^2}}\,dt$

10. $\displaystyle\int \frac{-2x}{\sqrt{x^2-4}}\,dx$

11. $\displaystyle\int t\sin t^2\,dt$

12. $\displaystyle\int \sec 5x \tan 5x\,dx$

13. $\displaystyle\int (\cos x)e^{\sin x}\,dx$

14. $\displaystyle\int \frac{1}{x\sqrt{x^2-4}}\,dx$

Finding an Indefinite Integral In Exercises 15–46, find the indefinite integral.

15. $\displaystyle\int 14(x-5)^6\,dx$

16. $\displaystyle\int \frac{5}{(t+6)^3}\,dt$

17. $\displaystyle\int \frac{7}{(z-10)^7}\,dz$

18. $\displaystyle\int t^3\sqrt{t^4+1}\,dt$

19. $\displaystyle\int \left[v + \frac{1}{(3v-1)^3}\right]dv$

20. $\displaystyle\int \left[4x - \frac{2}{(2x+3)^2}\right]dx$

21. $\displaystyle\int \frac{t^2-3}{-t^3+9t+1}\,dt$

22. $\displaystyle\int \frac{x+1}{\sqrt{3x^2+6x}}\,dx$

23. $\displaystyle\int \frac{x^2}{x-1}\,dx$

24. $\displaystyle\int \frac{3x}{x+4}\,dx$

25. $\displaystyle\int \frac{e^x}{1+e^x}\,dx$

26. $\displaystyle\int \left(\frac{1}{2x+5} - \frac{1}{2x-5}\right)dx$

27. $\displaystyle\int (5+4x^2)^2\,dx$

28. $\displaystyle\int x\left(3+\frac{2}{x}\right)^2 dx$

29. $\displaystyle\int x\cos 2\pi x^2\,dx$

30. $\displaystyle\int \csc \pi x \cot \pi x\,dx$

31. $\displaystyle\int \frac{\sin x}{\sqrt{\cos x}}\,dx$

32. $\displaystyle\int \csc^2 x e^{\cot x}\,dx$

33. $\displaystyle\int \frac{2}{e^{-x}+1}\,dx$

34. $\displaystyle\int \frac{2}{7e^x+4}\,dx$

35. $\displaystyle\int \frac{\ln x^2}{x}\,dx$

36. $\displaystyle\int (\tan x)[\ln(\cos x)]\,dx$

37. $\displaystyle\int \frac{1+\cos\alpha}{\sin\alpha}\,d\alpha$

38. $\displaystyle\int \frac{1}{\cos\theta - 1}\,d\theta$

39. $\displaystyle\int \frac{-1}{\sqrt{1-(4t+1)^2}}\,dt$

40. $\displaystyle\int \frac{1}{25+4x^2}\,dx$

41. $\displaystyle\int \frac{\tan(2/t)}{t^2}\,dt$

42. $\displaystyle\int \frac{e^{1/t}}{t^2}\,dt$

43. $\displaystyle\int \frac{6}{\sqrt{10x-x^2}}\,dx$

44. $\displaystyle\int \frac{1}{(x-1)\sqrt{4x^2-8x+3}}\,dx$

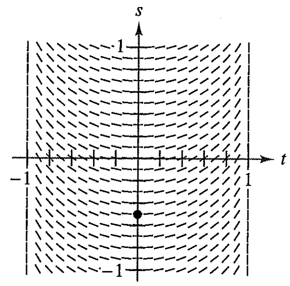
45. $\displaystyle\int \frac{4}{4x^2+4x+65}\,dx$

46. $\displaystyle\int \frac{1}{x^2-4x+9}\,dx$

Slope Field In Exercises 47 and 48, a differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the given point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a). To print an enlarged copy of the graph, go to *MathGraphs.com*.

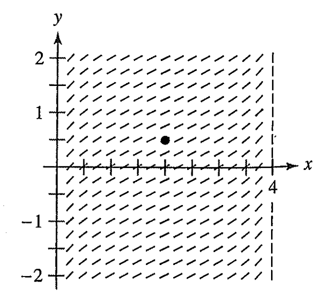
47. $\dfrac{ds}{dt} = \dfrac{t}{\sqrt{1-t^4}}$

$\left(0, -\dfrac{1}{2}\right)$



48. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{4x-x^2}}$

$\left(2, \dfrac{1}{2}\right)$



Slope Field In Exercises 49 and 50, use a computer algebra system to graph the slope field for the differential equation and graph the solution through the specified initial condition.

49. $\frac{dy}{dx} = 0.8y,\ y(0) = 4$

50. $\frac{dy}{dx} = 5 - y,\ y(0) = 1$

Differential Equation In Exercises 51–56, solve the differential equation.

51. $\frac{dy}{dx} = (e^x + 5)^2$

52. $\frac{dy}{dx} = (4 - e^{2x})^2$

53. $\frac{dr}{dt} = \frac{10e^t}{\sqrt{1 - e^{2t}}}$

54. $\frac{dr}{dt} = \frac{(1 + e^t)^2}{e^{3t}}$

55. $(4 + \tan^2 x)y' = \sec^2 x$

56. $y' = \frac{1}{x\sqrt{4x^2 - 9}}$

Evaluating a Definite Integral In Exercises 57–64, evaluate the definite integral. Use the integration capabilities of a graphing utility to verify your result.

57. $\int_0^{\pi/4} \cos 2x\, dx$

58. $\int_0^{\pi} \sin^2 t \cos t\, dt$

59. $\int_0^1 xe^{-x^2}\, dx$

60. $\int_1^e \frac{1 - \ln x}{x}\, dx$

61. $\int_0^8 \frac{2x}{\sqrt{x^2 + 36}}\, dx$

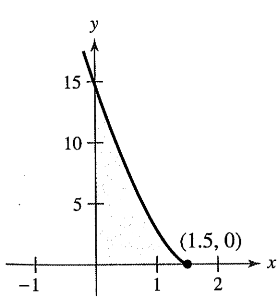
62. $\int_1^3 \frac{2x^2 + 3x - 2}{x}\, dx$

63. $\int_0^{2/\sqrt{3}} \frac{1}{4 + 9x^2}\, dx$

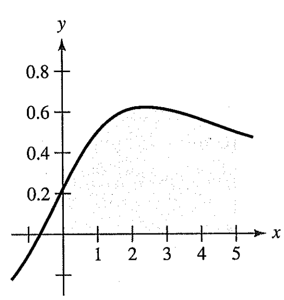
64. $\int_0^7 \frac{1}{\sqrt{100 - x^2}}\, dx$

Area In Exercises 65–68, find the area of the region.

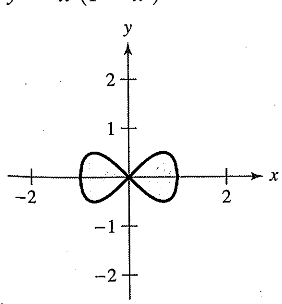
65. $y = (-4x + 6)^{3/2}$



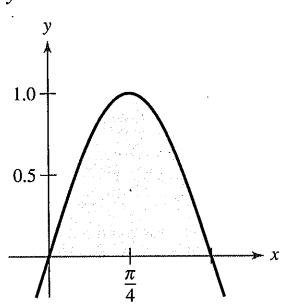
66. $y = \frac{3x + 2}{x^2 + 9}$



67. $y^2 = x^2(1 - x^2)$



68. $y = \sin 2x$



Finding an Integral Using Technology In Exercises 69–72, use a computer algebra system to find the integral. Use the computer algebra system to graph two antiderivatives. Describe the relationship between the graphs of the two antiderivatives.

69. $\int \frac{1}{x^2 + 4x + 13}\, dx$

70. $\int \frac{x - 2}{x^2 + 4x + 13}\, dx$

71. $\int \frac{1}{1 + \sin\theta}\, d\theta$

72. $\int \left(\frac{e^x + e^{-x}}{2}\right)^3 dx$

WRITING ABOUT CONCEPTS

Choosing a Formula In Exercises 73–76, state the integration formula you would use to perform the integration. Explain why you chose that formula. Do not integrate.

73. $\int x(x^2 + 1)^3\, dx$

74. $\int x \sec(x^2 + 1)\tan(x^2 + 1)\, dx$

75. $\int \frac{x}{x^2 + 1}\, dx$

76. $\int \frac{1}{x^2 + 1}\, dx$

77. **Finding Constants** Determine the constants a and b such that

$$\sin x + \cos x = a \sin(x + b).$$

Use this result to integrate

$$\int \frac{dx}{\sin x + \cos x}.$$

78. **Deriving a Rule** Show that

$$\sec x = \frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x}.$$

Then use this identity to derive the basic integration rule

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C.$$

79. **Area** The graphs of $f(x) = x$ and $g(x) = ax^2$ intersect at the points $(0, 0)$ and $(1/a, 1/a)$. Find a $(a > 0)$ such that the area of the region bounded by the graphs of these two functions is $\frac{2}{3}$.

80. **Think About It** When evaluating

$$\int_{-1}^{1} x^2\, dx$$

is it appropriate to substitute

$$u = x^2,\quad x = \sqrt{u},\quad \text{and}\quad dx = \frac{du}{2\sqrt{u}}$$

to obtain

$$\frac{1}{2}\int_1^1 \sqrt{u}\, du = 0?$$

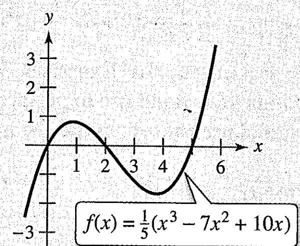
Explain.

81. Comparing Antiderivatives

(a) Explain why the antiderivative $y_1 = e^{x+C_1}$ is equivalent to the antiderivative $y_2 = Ce^x$.

(b) Explain why the antiderivative $y_1 = \sec^2 x + C_1$ is equivalent to the antiderivative $y_2 = \tan^2 x + C$.

82. HOW DO YOU SEE IT? Using the graph, is $\int_0^5 f(x)\,dx$ positive or negative? Explain.



Approximation **In Exercises 83 and 84, determine which value best approximates the area of the region between the x-axis and the function over the given interval. (Make your selection on the basis of a sketch of the region and *not* by integrating.)**

83. $f(x) = \dfrac{4x}{x^2 + 1}$, $[0, 2]$

(a) 3 (b) 1 (c) −8 (d) 8 (e) 10

84. $f(x) = \dfrac{4}{x^2 + 1}$, $[0, 2]$

(a) 3 (b) 1 (c) −4 (d) 4 (e) 10

Interpreting Integrals **In Exercises 85 and 86, (a) sketch the region whose area is given by the integral, (b) sketch the solid whose volume is given by the integral when the disk method is used, and (c) sketch the solid whose volume is given by the integral when the shell method is used. (There is more than one correct answer for each part.)**

85. $\displaystyle\int_0^2 2\pi x^2\,dx$ **86.** $\displaystyle\int_0^4 \pi y\,dy$

87. Volume The region bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = b$ $(b > 0)$ is revolved about the y-axis.

(a) Find the volume of the solid generated when $b = 1$.

(b) Find b such that the volume of the generated solid is $\frac{4}{3}$ cubic units.

88. Volume Consider the region bounded by the graphs of $x = 0$, $y = \cos x^2$, $y = \sin x^2$, and $x = \sqrt{\pi}/2$. Find the volume of the solid generated by revolving the region about the y-axis.

89. Arc Length Find the arc length of the graph of $y = \ln(\sin x)$ from $x = \pi/4$ to $x = \pi/2$.

90. Arc Length Find the arc length of the graph of $y = \ln(\cos x)$ from $x = 0$ to $x = \pi/3$.

91. Surface Area Find the area of the surface formed by revolving the graph of $y = 2\sqrt{x}$ on the interval $[0, 9]$ about the x-axis.

92. Centroid Find the x-coordinate of the centroid of the region bounded by the graphs of

$$y = \frac{5}{\sqrt{25 - x^2}},\quad y = 0,\quad x = 0,\quad \text{and}\quad x = 4.$$

Average Value of a Function **In Exercises 93 and 94, find the average value of the function over the given interval.**

93. $f(x) = \dfrac{1}{1 + x^2}$, $-3 \le x \le 3$

94. $f(x) = \sin nx$, $0 \le x \le \pi/n$, n is a positive integer.

Arc Length **In Exercises 95 and 96, use the integration capabilities of a graphing utility to approximate the arc length of the curve over the given interval.**

95. $y = \tan \pi x$, $\left[0, \frac{1}{4}\right]$ **96.** $y = x^{2/3}$, $[1, 8]$

97. Finding a Pattern

(a) Find $\int \cos^3 x\,dx$.

(b) Find $\int \cos^5 x\,dx$.

(c) Find $\int \cos^7 x\,dx$.

(d) Explain how to find $\int \cos^{15} x\,dx$ without actually integrating.

98. Finding a Pattern

(a) Write $\int \tan^3 x\,dx$ in terms of $\int \tan x\,dx$. Then find $\int \tan^3 x\,dx$.

(b) Write $\int \tan^5 x\,dx$ in terms of $\int \tan^3 x\,dx$.

(c) Write $\int \tan^{2k+1} x\,dx$, where k is a positive integer, in terms of $\int \tan^{2k-1} x\,dx$.

(d) Explain how to find $\int \tan^{15} x\,dx$ without actually integrating.

99. Methods of Integration Show that the following results are equivalent.

Integration by tables:

$$\int \sqrt{x^2 + 1}\,dx = \frac{1}{2}\left(x\sqrt{x^2 + 1} + \ln\left|x + \sqrt{x^2 + 1}\right|\right) + C$$

Integration by computer algebra system:

$$\int \sqrt{x^2 + 1}\,dx = \frac{1}{2}\left[x\sqrt{x^2 + 1} + \operatorname{arcsinh}(x)\right] + C$$

PUTNAM EXAM CHALLENGE

100. Evaluate $\displaystyle\int_2^4 \frac{\sqrt{\ln(9 - x)}\,dx}{\sqrt{\ln(9 - x)} + \sqrt{\ln(x + 3)}}$.