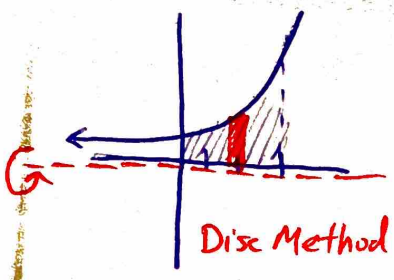
8.2 AP Practice Problems (p.595-596)

1. What is the volume of the solid of revolution generated when the region in the first quadrant bounded by the graph of $y = e^x$, the x-axis, and the line $x = 3$ is revolved about the x-axis?

(A) $2\pi e^6$ **(B)** $\frac{\pi}{2}(e^6 - 1)$ (C) $\pi(e^6 - 1)$ (D) $\frac{\pi}{2}(e^3 - 1)$



Disc Method

$R(x) = e^x - 0 = e^x$

$V = \int_0^3 (e^x)^2 dx$

$\int e^{2x} dx$

$u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$\int e^u \cdot \frac{du}{2}$

$\frac{1}{2}\int e^u du$

$\frac{1}{2}e^{2x}\Big]_0^3$

$\frac{1}{2}e^6 - \frac{1}{2}e^0$

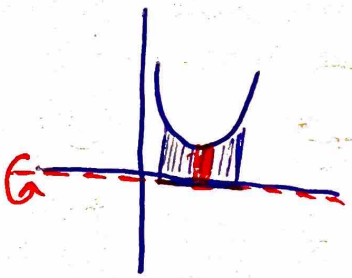
$\frac{1}{2}e^6 - \frac{1}{2}$

$= \boxed{\frac{1}{2}(e^6 - 1)}$

2. Find the volume of the solid of revolution generated when the region bounded by the graph of $y = \csc x$, the x-axis, and the lines $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$ is revolved about the x-axis.

(A) 2 (B) π (C) $\sqrt{2}\pi$ **(D)** 2π

$* \int \csc^2 u\, du = -\cot u + C$



*Disc Method

$R(x) = \csc x - 0$

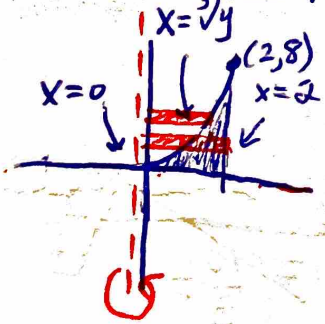
$V = \pi \int_{\pi/4}^{3\pi/4} (\csc x)^2 dx$

$-\cot x\Big]_{\pi/4}^{3\pi/4} = -\cot(3\pi/4) - (-\cot(\pi/4))$

$-(-1) + +(1) = \boxed{2\pi}$

3. The region bounded by the graph of $y = x^3$, the line $x = 2$, and the x-axis is revolved about the y-axis. The volume of the solid of revolution is

(A) 4π **(B)** $\frac{64}{5}\pi$ (C) $\frac{96}{5}\pi$ (D) $\frac{128}{7}\pi$



* Method

$R(y) = 2 - 0$

$r(y) = y^{1/3} - 0$

$V = \int_0^8 [2]^2 - [y^{1/3}]^2 dy$

$V = \int 4 - y^{2/3} dy$

$4y - \frac{y^{5/3}}{5/3}$

$4y - \frac{3}{5}y^{5/3}\Big]_0^8$

$32 - \frac{3}{5}(8)^{5/3} - (0 - 0)$

$32 - \frac{3}{5}(32)$

$= \boxed{\frac{64}{5}\pi}$

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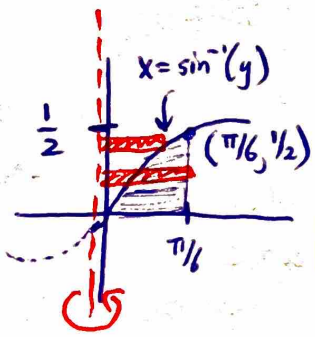
4. The volume of the solid of revolution generated by revolving the region under the graph of $y = \sin x$ from $x = 0$ to $x = \frac{\pi}{6}$ about the y-axis is given by

(A) $\pi \int_0^{\pi/6} \left[\left(\frac{\pi}{6}\right)^2 - \sin^2 x\right] dx$

(B) $\pi \int_0^{1/2} \left[\left(\frac{\pi}{6}\right)^2 - (\arcsin y)^2\right] dy$

(C) $\pi \int_0^{1/2} (\arcsin y)^2 \, dy$

(D) $\pi \int_0^{1/2} \left[\left(\frac{\pi}{6}\right)^2 - \sin^2 y\right] dy$



$R(y) = \pi/6 - 0$

$r(y) = \sin^{-1}(y) - 0$

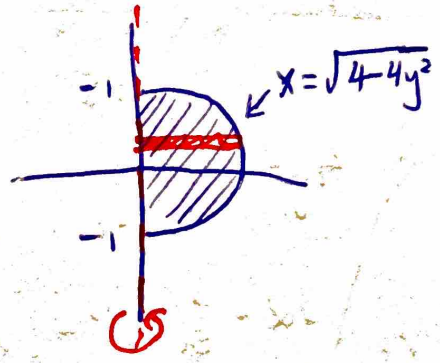
$V = \pi \int_0^{1/2} [\pi/6]^2 - [\sin^{-1}(y)]^2 \, dy$

5. Find the volume of the solid of revolution obtained by revolving the region bounded by the graph of $x = \sqrt{4 - 4y^2}$ and the y-axis about the y-axis.

(A) $\frac{4}{3}\pi$ (B) $\frac{8}{3}\pi$ (C) $\frac{11}{6}\pi$ **(D)** $\frac{16}{3}\pi$

$x^2 = 4 - 4y^2$ ← ellipse

$x^2 + 4y^2 = 4$



*Disc Method

$R(y) = \sqrt{4 - 4y^2} - 0$

$V = \pi \int_{-1}^{1} \left[\sqrt{4 - 4y^2}\right]^2 dy \rightarrow \int 4 - 4y^2 \, dy$

$4y - \frac{4y^3}{3}\Big]_{-1}^{1} = 4 - \frac{4}{3} - \left(-4 + \frac{4}{3}\right)$

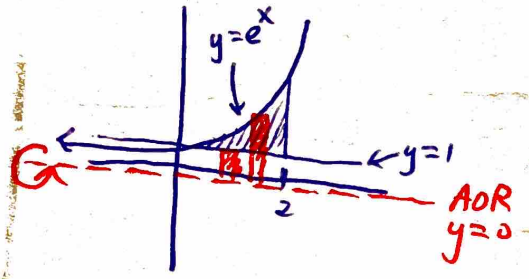
$4 - \frac{4}{3} + 4 - \frac{4}{3} = 8 - 8/3$

$= \frac{16}{3}\pi$

6. The region bounded by the graphs of $y=1$, $y=e^x$, and the line $x=2$ is revolved about the x-axis. The volume of the resulting solid of revolution is

(A) $\frac{\pi}{2}(e^4-5)$ (B) $2\pi(e^4-2)$

(C) $\frac{\pi}{2}(e^4-2)$ (D) $\frac{\pi}{2}(e^4-4)$



*washer Method

$R(x)=e^x-0$

$r(x)=1-0$

$V=\pi\int_0^2 [e^x]^2-[1]^2\,dx$

u-sub $\int e^{2x}-1\,dx$

$\frac{1}{2}e^{2x}-x\Big]_0^2$

$\frac{1}{2}e^4-2-(\frac{1}{2}e^0-0)$

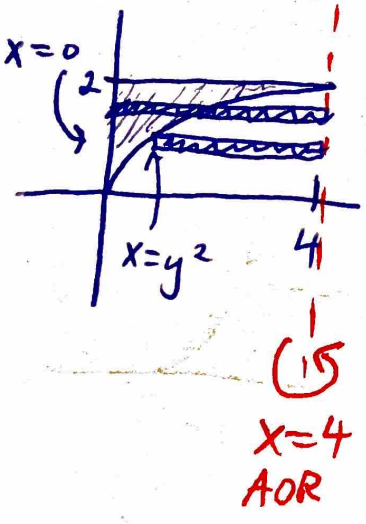
$\frac{1}{2}e^4-2-\frac{1}{2}$

$\frac{1}{2}e^4-\frac{5}{2}$

$\frac{1}{2}(e^4-5)\pi$

7. The volume of the solid of revolution generated by revolving the region bounded by the graphs of $y=\sqrt{x}$, $y=2$, and the y-axis about the line $x=4$ is

(A) $\frac{18}{5}\pi$ (B) $\frac{224}{15}\pi$ (C) $\frac{256}{15}\pi$ (D) $\frac{328}{15}\pi$



*washer Method

$R(y)=4-0$

$r(y)=4-y^2$

$V=\pi\int_0^2 [4]^2-[4-y^2]^2\,dy$

$V=\frac{224}{15}\pi$

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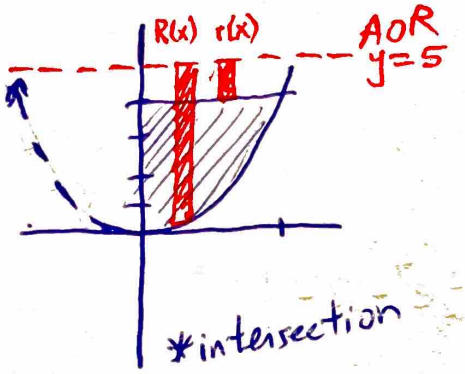
8. The volume of the solid of revolution generated by revolving the region bounded by the graphs of $y = x^2$, $x \geq 0$, the y-axis, and $y = 4$ about the line $y = 5$ is given by

(A) $\pi \int_0^2 (5^2 - x^2)\,dx$ (B) $\pi \int_0^5 [25 - (x^2 - 5)]\,dx$

(C) $\pi \int_0^2 [(5 - x^2)^2 - 1]\,dx$ (D) $\pi \int_0^2 [1 - (5 - x^2)]\,dx$



*intersection

$x^2 = 4$

$x = 2$

*washer Method

$R(x) = 5 - x^2$

$r(x) = 5 - 4 = 1$

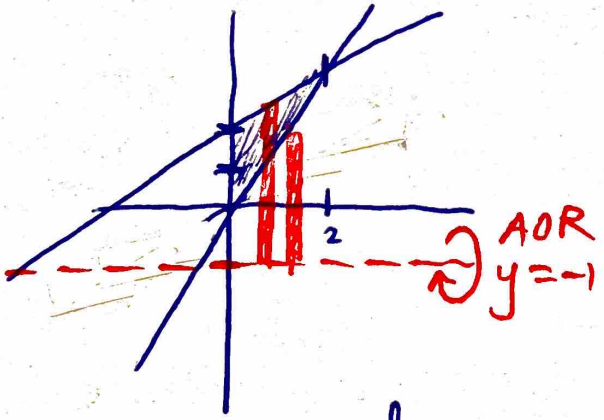
$$V = \pi \int_0^2 [5 - x^2]^2 - [1]^2\,dx$$

9. The region in the first quadrant bounded by the graphs of $y = x + 2$, $y = 2x$, and the y-axis is revolved about the line $y = -1$. Find the volume of the solid generated.

(A) $\frac{52}{3}\pi$ (B) 12π (C) 28π (D) 32π

*intersection: $2x = x + 2$

$x = 2$



*washer Method

$R(x) = x + 2 - (-1) = x + 3$

$r(x) = 2x - (-1) = 2x + 1$

$$V = \pi \int_0^2 [x+3]^2 - [2x+1]^2\,dx$$

$$V = 12\pi$$