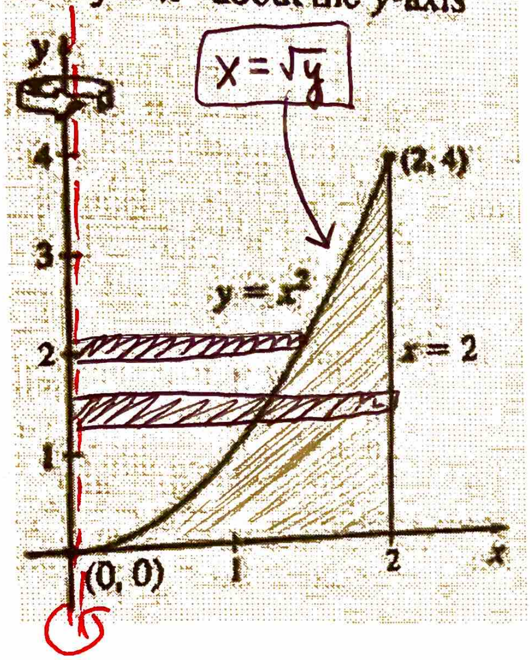
8.2b Washer Method Homework Homework Solutions

10. $y = x^2$ about the y-axis



*Washer Method (Right − Left)

$R(y) = 2 - 0$

$r(y) = \sqrt{y} - 0$

$V = \pi \int_0^4 [2]^2 - [\sqrt{y}]^2 \, dy = \boxed{8\pi}$

10. Since we are revolving around the y-axis, we solve $y = x^2$ for x, giving $x = \sqrt{y}$ (since $x \geq 0$). Then using the washer method, the volume is

$$V = \pi \int_0^4 \left(2^2 - (\sqrt{y})^2\right) dy = \pi \int_0^4 (4 - y)\, dy = \pi \left[4y - \frac{1}{2}y^2\right]_0^4 = \boxed{8\pi}.$$

47. The region is shown below:

In Problems 47–50, find the volume of the solid of revolution generated by revolving the indicated region about each line.

47. The region bounded by $y=x^2$, the x-axis, and $x=3$

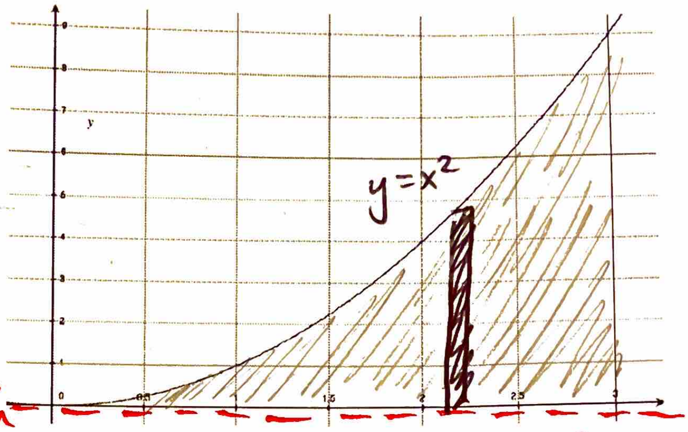
(a) About the x-axis

(b) About the line $y=-1$

(c) About the line $y=10$

(d) About the line $y=a$, $a \geq 9$

AOR
$y=0$



*Disc Method $R(x)=x^2-0$

$$V=\pi\int_0^3 [x^2]^2\,dx$$

(a) The radius of each disk when revolved around the x-axis is $y=x^2$. Therefore the volume is

$$V=\pi\int_0^3 \cancel{x}^2\,dx=\pi\int_0^3 (x^2)^2\,dx=\pi\int_0^3 x^4\,dx=\pi\left[\frac{1}{5}x^5\right]_0^3=\pi\left[\frac{1}{5}(3)^5-0\right]=\boxed{\frac{243}{5}\pi}$$

$$\boxed{V=\frac{243}{5}\pi}$$

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*Washer Method

47. The region is shown below:

b) AOR: $y=-1$

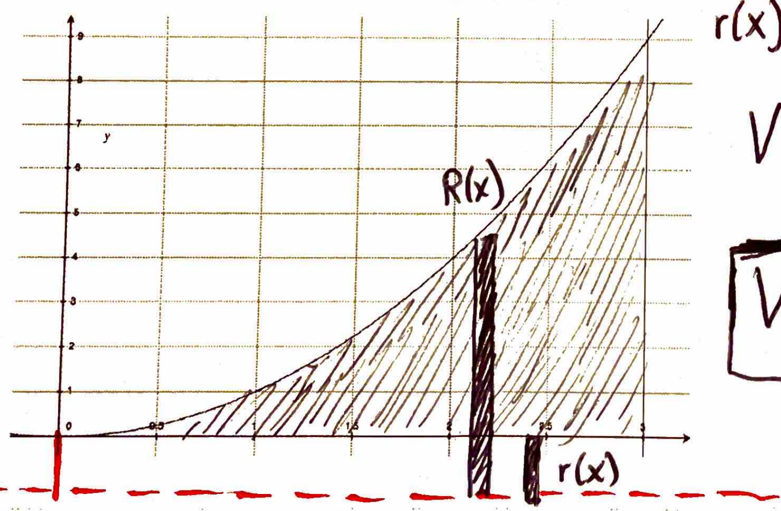
$R(x)=x^2-(-1)=x^2+1$

$r(x)=0-(-1)=1$

$$V=\pi\int_0^3 [x^2+1]^2-[1]^2\,dx$$

$$\boxed{V=\frac{333}{5}\pi}$$

AOR
$y=-1$



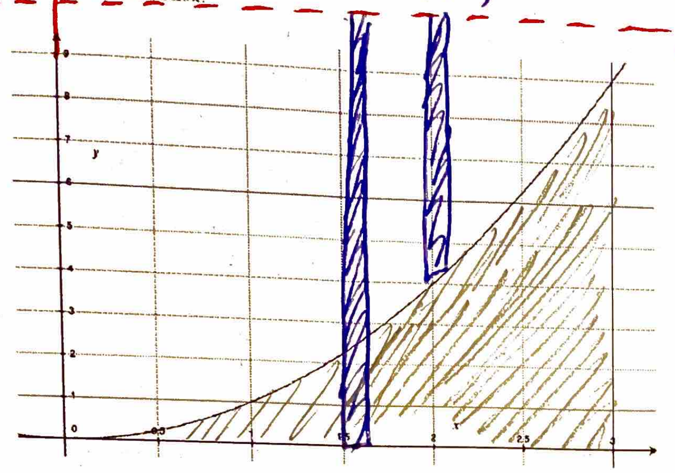
(b) The outer radius of each washer when revolved around the line $y=-1$ is $y-(-1)=x^2+1$, and the inner radius is $0-(-1)=1$. Therefore the volume is

$$V=\pi\int_0^3\left[(x^2+1)^2-1^2\right]dx=\pi\int_0^3\left[(x^4+2x^2+1)-1\right]dx=\pi\int_0^3(x^4+2x^2)\,dx$$

$$=\pi\left[\frac{1}{5}x^5+\frac{2}{3}x^3\right]_0^3=\pi\left\{\left[\frac{1}{5}(3)^5+\frac{2}{3}(3)^3\right]-0\right\}=\boxed{\frac{333}{5}\pi}$$

47. The region is shown below:

R(x) r(x)



47) c) AoR: $y=10$

AoR ($y=10$)

*washer Method

$R(x)=10-0$

$r(x)=10-x^2$

$V=\pi\int_0^3 (10)^2-(10-x^2)^2\,dx$

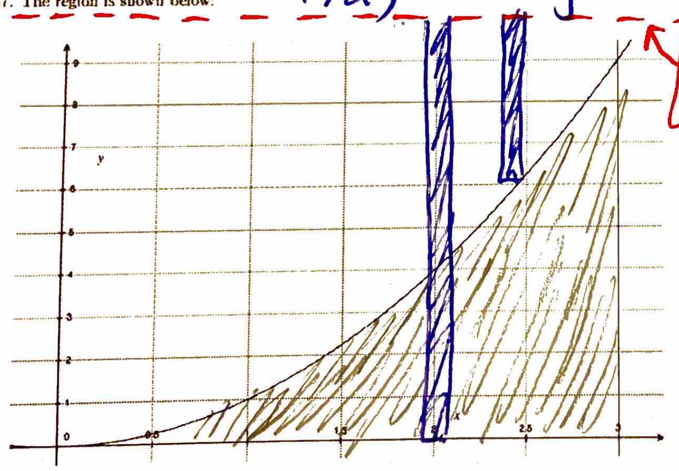
$\boxed{V=\frac{657}{5}\pi}$

(c) The outer radius of each washer when revolved around the line $y=10$ is $10-0=10$ and the inner radius is $10-y=10-x^2$. Therefore the volume is

$$V=\pi\int_0^3\left[10^2-\left(10-x^2\right)^2\right]dx=\pi\int_0^3\left[100-\left(100-20x^2+x^4\right)\right]dx$$
$$=\pi\int_0^3\left(20x^2-x^4\right)dx=\pi\left[\frac{20}{3}x^3-\frac{1}{5}x^5\right]_0^3=\pi\left\{\left[\frac{20}{3}(3)^3-\frac{1}{5}(3)^5\right]-0\right\}=\boxed{\frac{657}{5}\pi}$$

47d) AoR: $y=a$ $(a\geq 9)$

47. The region is shown below:



AoR: $y=a$

$R(x)=a-0$

$r(x)=a-x^2$

$V=\pi\int_0^3 a^2-(a-x^2)^2\,dx$

(d) The outer radius of each washer when revolved around the line $y=a\geq 9$ is $a-0=a$ and the inner radius is $a-y=a-x^2$. Therefore the volume is

$$V=\pi\int_0^3\left[a^2-\left(a-x^2\right)^2\right]dx=\pi\int_0^3\left[a^2-\left(a^2-2ax^2+x^4\right)\right]dx$$
$$=\pi\int_0^3\left(2ax^2-x^4\right)dx=\pi\left[\frac{2a}{3}x^3-\frac{1}{5}x^5\right]_0^3=\pi\left\{\left[\frac{2a}{3}(3)^3-\frac{1}{5}(3)^5\right]-0\right\}$$
$$=\left(18a-\frac{243}{5}\right)\pi=\boxed{\frac{90a-243}{5}\pi}$$

49. The region bounded by $y=x^2$, the y-axis, and $y=4$

(a) About the y-axis

(b) About the line $x=-5$

(c) About the line $x=5$

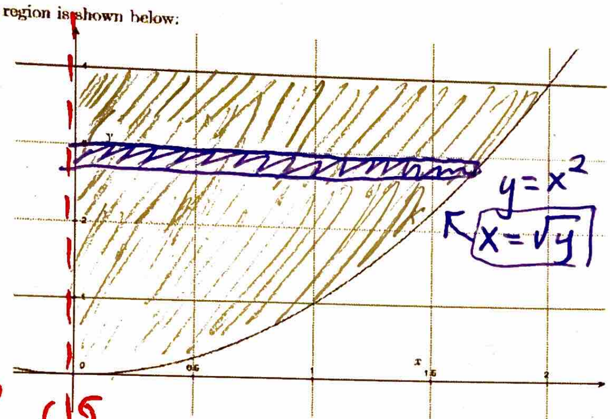
(d) About the line $x=b$, $b\geq 2$

a) *Disc Method
(Right – Left)
$R(y)=\sqrt{y}-0$

$$V=\pi\int_0^4[\sqrt{y}]^2\,dy = \int y\,dy$$

$$\frac{y^2}{2}\Big]_0^4 = \frac{4^2}{2}-0 = \boxed{8\pi}$$

49. The region is shown below:



(a) Solving $y=x^2$ for x gives $x=y^{1/2}$. The radius of each disk when revolved around the y-axis is $x-0=y^{1/2}-0=y^{1/2}$. Therefore the volume is

$$V=\pi\int_0^4\left(y^{1/2}\right)^2dy=\pi\int_0^4 y\,dy=\pi\left[\frac{1}{2}y^2\right]_0^4=\pi\left[\frac{1}{2}(4)^2-0\right]=\boxed{8\pi}$$

AOR
$x=-5$

49b) AOR: $x=-5$

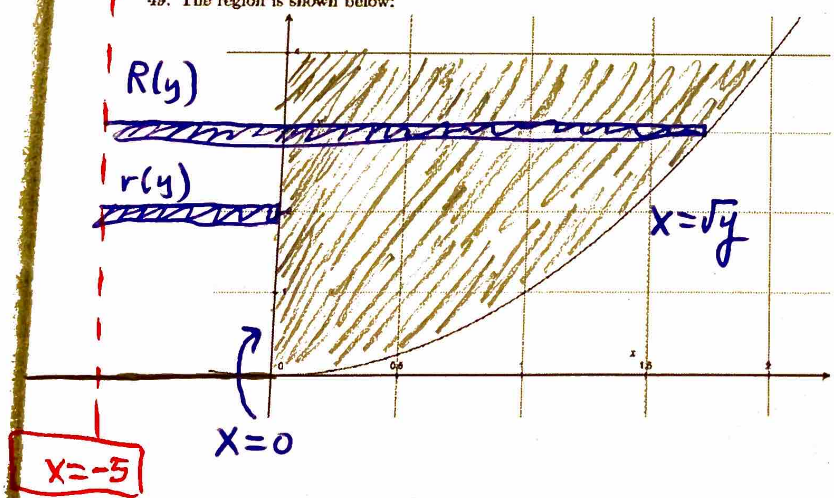
$R(y)=\sqrt{y}-(-5)$

$r(y)=0-(-5)$

$$V=\pi\int_0^4(\sqrt{y}+5)^2-(5)^2\,dy$$

$$\boxed{V=\frac{184}{3}\pi}$$

49. The region is shown below:



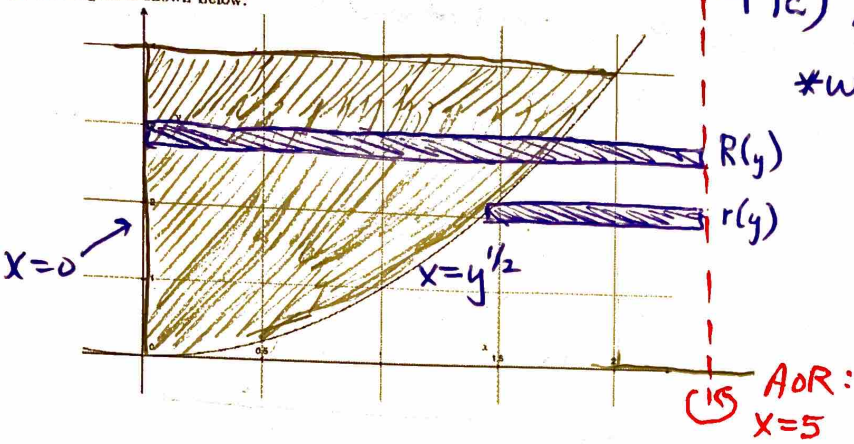
(b) The outer radius of each washer when revolving around the line $x=-5$ is $x-(-5)=y^{1/2}+5$, and the inner radius is $0-(-5)=5$. Therefore the volume is

$$V=\pi\int_0^4\left[\left(y^{1/2}+5\right)^2-5^2\right]dy=\pi\int_0^4\left[\left(y+10y^{1/2}+25\right)-25\right]dy$$

$$=\pi\int_0^4\left(y+10y^{1/2}\right)dy=\left[\frac{1}{2}y^2+\frac{20}{3}y^{3/2}\right]_0^4=\pi\left\{\left[\frac{1}{2}(4)^2+\frac{20}{3}(4)^{3/2}\right]-0\right\}$$

$$=\boxed{\frac{184}{3}\pi}$$

49. The region is shown below:



49c) AOR: $x=5$

*Washer Method (Right−Left)

$R(y)=5-0$

$r(y)=5-y^{1/2}$

$V=\pi\int_0^4 (5)^2-(5-y^{1/2})^2\,dy$

$\boxed{V=\frac{136}{3}\pi}$

(c) The outer radius of each washer when revolving around the line $x=5$ is $5-0=5$, and the inner radius is $5-x=5-y^{1/2}$. Therefore the volume is

$$V=\pi\int_0^4\left[5^2-\left(5-y^{1/2}\right)^2\right]dy=\pi\int_0^4\left[25-\left(25-10y^{1/2}+y\right)\right]dy$$

$$=\pi\int_0^4\left(10y^{1/2}-y\right)dy=\left[\frac{20}{3}y^{3/2}-\frac{1}{2}y^2\right]_0^4=\pi\left\{\left[\frac{20}{3}(4)^{3/2}-\frac{1}{2}(4)^2\right]-0\right\}$$

$$=\boxed{\frac{136}{3}\pi}$$

49d) AOR: $x=b$

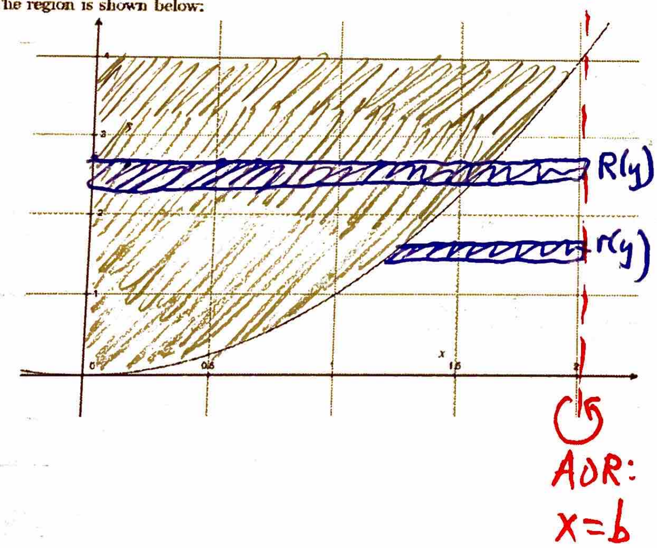
$(b\geq 2)$

*Washer Method (Right−Left)

$R(y)=b-0$

$r(y)=b-y^{1/2}$

49. The region is shown below:



$V=\pi\int_0^4 (b)^2-(b-y^{1/2})^2\,dy$

(d) The outer radius of each washer when revolving around the line $x=b\geq 2$ is $b-0=b$ and the inner radius is $b-x=b-y^{1/2}$. Therefore the volume is

$$V=\pi\int_0^4\left[b^2-\left(b-y^{1/2}\right)^2\right]dy=\pi\int_0^4\left[b^2-\left(b^2-2by^{1/2}+y\right)\right]dy$$

$$=\pi\int_0^4\left(2by^{1/2}-y\right)dy=\left[\frac{4b}{3}y^{3/2}-\frac{1}{2}y^2\right]_0^4=\pi\left\{\left[\frac{4b}{3}(4)^{3/2}-\frac{1}{2}(4)^2\right]-0\right\}$$

$$=\frac{32b}{3}-8=\boxed{\frac{32b-24}{3}\pi}$$