AP Calculus AB **8.2 Notes: Integration by Parts**

I. **Integration by Parts (I.B.P.)** -a method of integration useful for problems involving the product of two different types of functions . (example: logs and polynomial)

IBP Formula: $\int udv = uv - \int vdu$ *This theorem is derived from the product rule for derivatives

Steps:

1. Determine the u-value by using the acronym L.I.P.E.T.
   a. LIPET shows the priority order for determining u-value
   b. **L**ogs **I**nverse Trig **P**olynomial **E**xponential function **T**rigonometric function
2. Let dv be other function
3. Find u, dv, v, and dv
4. Plug into formula and integrate

Example 1: $\int x\, e^x dx$	Example 2: $\int x\, secxtanxdx$
Example 3: $\int xlnx\, dx$	Example 4: $\int lnx\, dx$

II. Tabular (Tab) Method

Tab method is used whenever the u-value can be assigned to the polynomial

*Use LIPET to determine if polynomial is the appropriate u-value.

*Tab method is especially useful for polynomial of degree higher than 1

Steps:
1) Create 2 columns u | dv
2) u-value must be the polynomial
3) Let dv be the other portion
4) Find the derivative of polynomial (u) until reaching zero
5) Find integral of dv the same number of times
6) Assign alternating signs to each column (+/ –)
7) Add the product of diagonal terms

Example 5: $\int x^4 sinx\, dx$

Example 6: $\int xe^x\, dx$

Example 7: $\int x^3 lnx\, dx$

Example 8: $\int \frac{lnx}{2x} dx$

AP Calculus AB 8.2 Notes: Integration by Parts

I. **Integration by Parts (I.B.P.)** -a method of integration useful for when two different types of functions are multiplied together

IBP Formula: $\int u\,dv = uv - \int v\,du$

Steps:

1. Determine the u-value by using the acronym L.I.P.E.T.
   a. LIPET shows the preference order for determining u-value
   b. Logs Inverse Trig Polynomial Exponential function Trigonometric function
2. Let dv be other function
3. Find u, dv, v, and dv
4. Plug into formula and integrate

Example 1: $\int x e^x dx$

*use LIPET to determine priority order

$u = x$ $\quad \int dv = \int e^x dx$

$\frac{du}{dx} = 1$ $\quad v = e^x$

$du = dx$

$xe^x - \int e^x dx$

$\int e^x dx = e^x$

$\boxed{xe^x - e^x + C}$

Example 2: $\int x \sec x \tan x\, dx$

$u = x$ $\quad \int dv = \int \sec x \tan x\, dx$

$\frac{du}{dx} = 1$ $\quad V = \sec x$

$du = dx$

$x\sec x - \int \sec x\, dx$

$\boxed{x\sec x - \ln|\sec x + \tan x| + C}$

Example 3: $\int x\ln x\, dx$

$u = \ln x$ $\quad \int dv = \int x$

$\frac{du}{dx} = \frac{1}{x}$ $\quad V = \frac{x^2}{2}$

$du = \frac{1}{x}dx$

$\frac{x^2}{2}\ln x - \int \frac{x^2}{2}\left(\frac{1}{x}\right)dx$

$\frac{1}{2}\int x\,dx$

$\frac{1}{2}\left(\frac{x^2}{2}\right)$

$\boxed{\frac{x^2}{2}\ln x - \frac{x^2}{4} + C}$

Example 4: $\int \ln x\, dx$

$\int \ln x \cdot 1\, dx$

$u = \ln x$ $\quad \int dv = \int dx$

$\frac{du}{dx} = \frac{1}{x}$ $\quad V = x$

$du = \frac{1}{x}dx$

$x\ln x - \int \frac{x}{x}dx$

$\int 1\, dx$

x

$\boxed{x\ln x - x + C}$

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Steps:
1) Create 2 columns u | dv
2) u-value must be the polynomial
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5) Find integral of dv the same number of times
6) Assign alternating signs to each column (+/ –)
7) Add the product of diagonal terms

Example 5: $\int x^4 \sin x \, dx$

$u = x^4 \quad dv = \sin x$

$\int \sin u = -\cos u + c$

$\int \cos u = \sin u + c$

	u	dv
+	x^4	$\sin x$
−	$4x^3$	$-\cos x$
+	$12x^2$	$-\sin x$
−	$24x$	$\cos x$
+	24	$\sin x$
−	0	$-\cos x$

$$\boxed{-x^4\cos x + 4x^3\sin x + 12x^2\cos x - 24x\sin x - 24\cos x + c}$$

Example 6: $\int xe^x \, dx$

$u = x \quad dv = e^x$

	u	dv
+	x	e^x
−	1	e^x
+	0	e^x

$$\boxed{xe^x - e^x + C}$$

Example 7: $\int x^3 \ln x \, dx$

$u = \ln x \quad dv = x^3$

$\frac{du}{dx} = \frac{1}{x} \quad v = \frac{x^4}{4}$

$du = \frac{1}{x}dx$

$\frac{x^4}{4}\ln x - \int \frac{x^4}{4} \cdot \frac{1}{x} dx$

$\frac{1}{4}\int x^3 dx$

$\frac{1}{4}\left(\frac{x^4}{4}\right)$

$$\boxed{\frac{x^4}{4}\ln x - \frac{1}{16}x^4 + C}$$

Example 8: $\int \frac{\ln x}{2x} dx$

Integral Method Checklist:
1) Expand/Simplify/Power Rule
2) U-substitution
3) IBP/Tab Method

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$\frac{1}{2}\int \frac{\ln x}{x} dx$

$\frac{1}{2}\int \frac{u}{\cancel{x}} \cdot \cancel{x} du$

$\frac{1}{2}\left(\frac{u^2}{2}\right) + C$

$\frac{1}{4}u^2 + C$

$$= \boxed{\frac{1}{4}(\ln x)^2 + C}$$

8.2 Homework p.531 #11-19odd, 23-33odd
*Integration by Parts

11) $\int x e^{-2x}\,dx$

$\int u\,dv = uv - \int v\,du$
*use LIPET

$u = x \qquad dv = e^{-2x}dx$
$du = 1dx \qquad v = -\frac{1}{2}e^{-2x}$

$= uv - \int v\,du$

$= x\left(-\frac{1}{2}e^{-2x}\right) - \int -\frac{1}{2}e^{-2x}\,dx$

$= \boxed{-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C}$

$\int dv = \int e^{-2x}\,dx \qquad u = -2x$
$= \int e^{u}\cdot\frac{du}{-2} \qquad \frac{du}{dx} = -2$
$= -\frac{1}{2}\int e^{u}\,du \qquad dx = \frac{du}{-2}$
$v = -\frac{1}{2}e^{-2x}$

13) $\int x^3 e^x\,dx$

	u	dv
+	x^3	e^x
−	$3x^2$	e^x
+	$6x$	e^x
−	6	e^x
+	0	e^x

$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

$= e^x(x^3 - 3x^2 + 6x - 6) + C$

15) $\int x^2 e^{x^3}\,dx$

$u = x^3$
$\frac{du}{dx} = 3x^2$
$dx = \frac{du}{3x^2}$

$\int \cancel{x^2}\cdot e^{u}\cdot\frac{du}{3\cancel{x^2}}$

$\frac{1}{3}\int e^{u}\,du$

$\frac{1}{3}e^{u} + C$

$= \boxed{\frac{1}{3}e^{x^3} + C}$

8.2 HW continued

17) $\int t \ln(t+1)\,dt$

$u = \ln(t+1)$ $\quad \int dv = \int t\,dt$

$du = \frac{1}{t+1}dt$ $\quad v = \frac{t^2}{2}$

*LIPET

$\int u\,dv = uv - \int v\,du$

$= \frac{t^2}{2}\cdot \ln(t+1) - \int \frac{t^2}{2}\left(\frac{1}{t+1}\right)dt$

$\frac{1}{2}\int \frac{t^2}{t+1}$

$$\begin{array}{r|rrr} -1 & 1 & 0 & 0 \\ & \downarrow & -1 & 1 \\ \hline & 1 & -1 & \boxed{1} \end{array}$$

$\frac{1}{2}\int t - 1 + \frac{1}{t+1}dt$

$\frac{1}{2}\left(\frac{t^2}{2} - t + \ln|t+1|\right)$

$$= \boxed{\frac{t^2}{2}\ln(t+1) - \frac{1}{2}\left(\frac{t^2}{2} - t + \ln|t+1|\right) + C}$$

19) $\int \frac{(\ln x)^2}{x}dx$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$dx = x\,du$

$\int \frac{u^2}{\cancel{x}}\cdot \cancel{x}\,du$

$= \int u^2\,du = \frac{u^3}{3} + C = \boxed{\frac{1}{3}(\ln x)^3 + C}$

23) $\int (x^2-1)e^x\,dx$

LIPET

$u = x^2-1$

$dv = e^x$

	u	dv
+	x^2-1	e^x
−	$2x$	e^x
+	2	e^x
−	0	e^x

$= e^x(x^2 - 1 - 2x + 2)$

$= e^x(x^2 - 2x + 1)$

$= \boxed{e^x(x-1)^2 + C}$

8.2 HW continued

25) $\int x\sqrt{x-1}\,dx$

	u	dv
+	x	$(x-1)^{1/2}$
−	1	$\frac{2}{3}(x-1)^{3/2}$
+	0	$\frac{2}{3}\cdot\frac{2}{5}(x-1)^{5/2}$

$u=x \quad dv=(x-1)^{1/2}$

$$=\boxed{\frac{2}{3}x(x-1)^{3/2}-\frac{4}{15}(x-1)^{5/2}+C}$$

27) $\int x\cos x\,dx$

	u	dv
+	x	$\cos x$
−	1	$\sin x$
+	0	$-\cos x$

$$=\boxed{x\sin x+\cos x+C}$$

29) $\int x^3\sin x\,dx$

	u	dv
+	x^3	$\sin x$
−	$3x^2$	$-\cos x$
+	$6x$	$-\sin x$
−	6	$\cos x$
+	0	$\sin x$

$$=\boxed{-x^3\cos x+3x^2\sin x+6x\cos x-6\sin x+C}$$

31) $\int t\csc t\cot t\,dt$

	u	dv
+	t	$\csc t\cot t$
−	1	$-\csc t$
+	0	$\ln\lvert\csc t+\cot t\rvert$

$$=\boxed{-t\csc t-\ln\lvert\csc t+\cot t\rvert+C}$$

33) $\int \arctan x\,dx$

$u=\arctan x \quad dv=1\,dx$

$du=\frac{1}{1+x^2}dx \quad v=x$

$\int u\,dv=uv-\int v\,du$

$=x\arctan x-\int x\cdot\frac{1}{1+x^2}dx$

$\int\frac{x}{1+x^2}dx$

$u=1+x^2$

$\frac{du}{dx}=2x$

$dx=\frac{du}{2x}$

$\int\frac{\cancel{x}}{u}\cdot\frac{du}{2\cancel{x}}=\frac{1}{2}\int\frac{1}{u}du$

$=\frac{1}{2}\ln\lvert u\rvert$

$=\frac{1}{2}\ln\lvert 1+x^2\rvert$

$$=\boxed{x\arctan x-\frac{1}{2}\ln\lvert 1+x^2\rvert+C}$$