8.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Setting Up Integration by Parts In Exercises 1–6, identify u and dv for finding the integral using integration by parts. (Do not evaluate the integral.)

1. $\int xe^{2x}\,dx$
2. $\int x^2e^{2x}\,dx$
3. $\int (\ln x)^2\,dx$
4. $\int \ln 5x\,dx$
5. $\int x\sec^2 x\,dx$
6. $\int x^2\cos x\,dx$

Using Integration by Parts In Exercises 7–10, evaluate the integral using integration by parts with the given choices of u and dv.

7. $\int x^3 \ln x\,dx;\ u = \ln x,\ dv = x^3\,dx$

8. $\int (4x + 7)e^x\,dx;\ u = 4x + 7,\ dv = e^x\,dx$

9. $\int x\sin 3x\,dx;\ u = x,\ dv = \sin 3x\,dx$

10. $\int x\cos 4x\,dx;\ u = x,\ dv = \cos 4x\,dx$

Finding an Indefinite Integral In Exercises 11–30, find the indefinite integral. (*Note:* Solve by the simplest method—not all require integration by parts.)

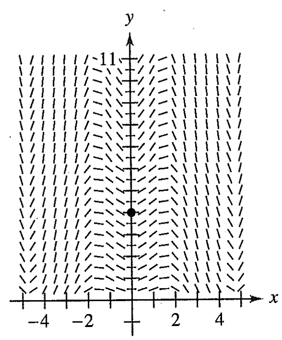
11. $\int xe^{-4x}\,dx$
12. $\int \frac{5x}{e^{2x}}\,dx$
13. $\int x^3e^x\,dx$
14. $\int \frac{e^{1/t}}{t^2}\,dt$
15. $\int t\ln(t + 1)\,dt$
16. $\int x^5 \ln 3x\,dx$
17. $\int \frac{(\ln x)^2}{x}\,dx$
18. $\int \frac{\ln x}{x^3}\,dx$
19. $\int \frac{xe^{2x}}{(2x + 1)^2}\,dx$
20. $\int \frac{x^3e^{x^2}}{(x^2 + 1)^2}\,dx$
21. $\int x\sqrt{x - 5}\,dx$
22. $\int \frac{x}{\sqrt{6x + 1}}\,dx$
23. $\int x\cos x\,dx$
24. $\int t\csc t\cot t\,dt$
25. $\int x^3\sin x\,dx$
26. $\int x^2\cos x\,dx$
27. $\int \arctan x\,dx$
28. $\int 4\arccos x\,dx$
29. $\int e^{-3x}\sin 5x\,dx$
30. $\int e^{4x}\cos 2x\,dx$

Differential Equation In Exercises 31–34, solve the differential equation.

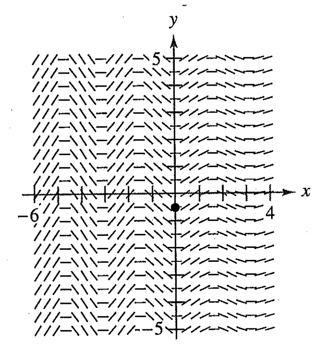
31. $y' = \ln x$
32. $y' = \arctan \frac{x}{2}$
33. $\frac{dy}{dt} = \frac{t^2}{\sqrt{3 + 5t}}$
34. $\frac{dy}{dx} = x^2\sqrt{x - 3}$

Slope Field In Exercises 35 and 36, a differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the given point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a). To print an enlarged copy of the graph, go to *MathGraphs.com.*

35. $\frac{dy}{dx} = x\sqrt{y}\cos x,\ (0, 4)$



36. $\frac{dy}{dx} = e^{-x/3}\sin 2x,\ \left(0, -\frac{18}{37}\right)$



Slope Field In Exercises 37 and 38, use a computer algebra system to graph the slope field for the differential equation and graph the solution through the specified initial condition.

37. $\frac{dy}{dx} = \frac{x}{y}e^{x/8},\quad y(0) = 2$
38. $\frac{dy}{dx} = \frac{x}{y}\sin x,\quad y(0) = 4$

Evaluating a Definite Integral In Exercises 39–48, evaluate the definite integral. Use a graphing utility to confirm your result.

39. $\int_0^3 xe^{x/2}\,dx$
40. $\int_0^2 x^2e^{-2x}\,dx$
41. $\int_0^{\pi/4} x\cos 2x\,dx$
42. $\int_0^{\pi} x\sin 2x\,dx$
43. $\int_0^{1/2} \arccos x\,dx$
44. $\int_0^1 x\arcsin x^2\,dx$
45. $\int_0^1 e^x\sin x\,dx$
46. $\int_0^1 \ln(4 + x^2)\,dx$
47. $\int_2^4 x\,\mathrm{arcsec}\,x\,dx$
48. $\int_0^{\pi/8} x\sec^2 2x\,dx$

Using the Tabular Method In Exercises 49–54, use the tabular method to find the integral.

49. $\int x^2e^{2x}\,dx$
50. $\int x^3e^{-2x}\,dx$
51. $\int x^3\sin x\,dx$
52. $\int x^3\cos 2x\,dx$

53. $\int x \sec^2 x \, dx$

54. $\int x^2(x-2)^{3/2} \, dx$

Using Two Methods Together **In Exercises 55–58, find the indefinite integral by using substitution followed by integration by parts.**

55. $\int \sin\sqrt{x} \, dx$

56. $\int 2x^3 \cos x^2 \, dx$

57. $\int x^5 e^{x^2} \, dx$

58. $\int e^{\sqrt{2x}} \, dx$

WRITING ABOUT CONCEPTS

59. Integration by Parts

(a) Integration by parts is based on what differentiation rule? Explain.

(b) In your own words, state how you determine which parts of the integrand should be u and dv.

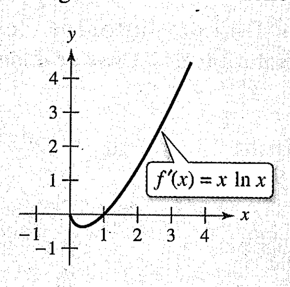
60. Integration by Parts When evaluating $\int x \sin x \, dx$, explain how letting $u = \sin x$ and $dv = x\,dx$ makes the solution more difficult to find.

61. Integration by Parts State whether you would use integration by parts to evaluate each integral. If so, identify what you would use for u and dv. Explain your reasoning.

(a) $\int \frac{\ln x}{x} \, dx$ (b) $\int x \ln x \, dx$ (c) $\int x^2 e^{-3x} \, dx$

(d) $\int 2xe^{x^2} \, dx$ (e) $\int \frac{x}{\sqrt{x+1}} \, dx$ (f) $\int \frac{x}{\sqrt{x^2+1}} \, dx$

62. HOW DO YOU SEE IT? Use the graph of f' shown in the figure to answer the following.



(a) Approximate the slope of f at $x = 2$. Explain.

(b) Approximate any open intervals in which the graph of f is increasing and any open intervals in which it is decreasing. Explain.

63. Using Two Methods Integrate $\int \frac{x^3}{\sqrt{4+x^2}} \, dx$

(a) by parts, letting $dv = \frac{x}{\sqrt{4+x^2}} \, dx$.

(b) by substitution, letting $u = 4 + x^2$.

64. Using Two Methods Integrate $\int x\sqrt{4-x} \, dx$

(a) by parts, letting $dv = \sqrt{4-x} \, dx$.

(b) by substitution, letting $u = 4 - x$.

Finding a General Rule **In Exercises 65 and 66, use a computer algebra system to find the integrals for $n = 0, 1, 2,$ and 3. Use the result to obtain a general rule for the integrals for any positive integer n and test your results for $n = 4$.**

65. $\int x^n \ln x \, dx$

66. $\int x^n e^x \, dx$

Proof **In Exercises 67–72, use integration by parts to prove the formula. (For Exercises 67–70, assume that n is a positive integer.)**

67. $\int x^n \sin x \, dx = -x^n \cos x + n\int x^{n-1} \cos x \, dx$

68. $\int x^n \cos x \, dx = x^n \sin x - n\int x^{n-1} \sin x \, dx$

69. $\int x^n \ln x \, dx = \frac{x^{n+1}}{(n+1)^2}[-1 + (n+1)\ln x] + C$

70. $\int x^n e^{ax} \, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1} e^{ax} \, dx$

71. $\int e^{ax} \sin bx \, dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2} + C$

72. $\int e^{ax} \cos bx \, dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2} + C$

Using Formulas **In Exercises 73–78, find the integral by using the appropriate formula from Exercises 67–72.**

73. $\int x^2 \sin x \, dx$

74. $\int x^2 \cos x \, dx$

75. $\int x^5 \ln x \, dx$

76. $\int x^3 e^{2x} \, dx$

77. $\int e^{-3x} \sin 4x \, dx$

78. $\int e^{2x} \cos 3x \, dx$

Area **In Exercises 79–82, use a graphing utility to graph the region bounded by the graphs of the equations. Then find the area of the region analytically.**

79. $y = 2xe^{-x}, \quad y = 0, \quad x = 3$

80. $y = \frac{1}{10}xe^{3x}, \quad y = 0, \quad x = 0, \quad x = 2$

81. $y = e^{-x} \sin \pi x, \quad y = 0, \quad x = 1$

82. $y = x^3 \ln x, \quad y = 0, \quad x = 1, \quad x = 3$

83. Area, Volume, and Centroid Given the region bounded by the graphs of $y = \ln x$, $y = 0$, and $x = e$, find

(a) the area of the region.

(b) the volume of the solid generated by revolving the region about the x-axis.

(c) the volume of the solid generated by revolving the region about the y-axis.

(d) the centroid of the region.

84. **Area, Volume, and Centroid** Given the region bounded by the graphs of $y = x \sin x$, $y = 0$, $x = 0$, and $x = \pi$, find
(a) the area of the region.
(b) the volume of the solid generated by revolving the region about the x-axis.
(c) the volume of the solid generated by revolving the region about the y-axis.
(d) the centroid of the region.

85. **Centroid** Find the centroid of the region bounded by the graphs of $y = \arcsin x$, $x = 0$, and $y = \pi/2$. How is this problem related to Example 6 in this section?

86. **Centroid** Find the centroid of the region bounded by the graphs of $f(x) = x^2$, $g(x) = 2^x$, $x = 2$, and $x = 4$.

87. **Average Displacement** A damping force affects the vibration of a spring so that the displacement of the spring is given by

$$y = e^{-4t}(\cos 2t + 5 \sin 2t).$$

Find the average value of y on the interval from $t = 0$ to $t = \pi$.

88. **Memory Model**

A model for the ability M of a child to memorize, measured on a scale from 0 to 10, is given by

$$M = 1 + 1.6t \ln t, \quad 0 < t \le 4$$

where t is the child's age in years. Find the average value of this model
(a) between the child's first and second birthdays.
(b) between the child's third and fourth birthdays.

Present Value **In Exercises 89 and 90, find the present value P of a continuous income flow of $c(t)$ dollars per year for**

$$P = \int_0^{t_1} c(t)e^{-rt}\, dt$$

where t_1 is the time in years and r is the annual interest rate compounded continuously.

89. $c(t) = 100{,}000 + 4000t$, $r = 5\%$, $t_1 = 10$

90. $c(t) = 30{,}000 + 500t$, $r = 7\%$, $t_1 = 5$

Integrals Used to Find Fourier Coefficients **In Exercises 91 and 92, verify the value of the definite integral, where n is a positive integer.**

91. $$\int_{-\pi}^{\pi} x \sin nx\, dx = \begin{cases} \dfrac{2\pi}{n}, & n \text{ is odd} \\ -\dfrac{2\pi}{n}, & n \text{ is even} \end{cases}$$

92. $$\int_{-\pi}^{\pi} x^2 \cos nx\, dx = \frac{(-1)^n 4\pi}{n^2}$$

93. **Vibrating String** A string stretched between the two points $(0, 0)$ and $(2, 0)$ is plucked by displacing the string h units at its midpoint. The motion of the string is modeled by a **Fourier Sine Series** whose coefficients are given by

$$b_n = h\int_0^1 x \sin \frac{n\pi x}{2}\, dx + h\int_1^2 (-x + 2) \sin \frac{n\pi x}{2}\, dx.$$

Find b_n.

94. **Euler's Method** Consider the differential equation $f'(x) = xe^{-x}$ with the initial condition $f(0) = 0$.
(a) Use integration to solve the differential equation.
(b) Use a graphing utility to graph the solution of the differential equation.
(c) Use Euler's Method with $h = 0.05$, and the recursive capabilities of a graphing utility, to generate the first 80 points of the graph of the approximate solution. Use the graphing utility to plot the points. Compare the result with the graph in part (b).
(d) Repeat part (c) using $h = 0.1$ and generate the first 40 points.
(e) Why is the result in part (c) a better approximation of the solution than the result in part (d)?

Euler's Method **In Exercises 95 and 96, consider the differential equation and repeat parts (a)–(d) of Exercise 94.**

95. $f'(x) = 3x \sin(2x)$
$f(0) = 0$

96. $f'(x) = \cos\sqrt{x}$
$f(0) = 1$

97. **Think About It** Give a geometric explanation of why

$$\int_0^{\pi/2} x \sin x\, dx \le \int_0^{\pi/2} x\, dx.$$

Verify the inequality by evaluating the integrals.

98. **Finding a Pattern** Find the area bounded by the graphs of $y = x \sin x$ and $y = 0$ over each interval.
(a) $[0, \pi]$ (b) $[\pi, 2\pi]$ (c) $[2\pi, 3\pi]$
Describe any patterns that you notice. What is the area between the graphs of $y = x \sin x$ and $y = 0$ over the interval $[n\pi, (n + 1)\pi]$, where n is any nonnegative integer? Explain.

99. **Finding an Error** Find the fallacy in the following argument that $0 = 1$.

$$dv = dx \implies v = \int dx = x$$

$$u = \frac{1}{x} \implies du = -\frac{1}{x^2}\, dx$$

$$0 + \int \frac{dx}{x} = \left(\frac{1}{x}\right)(x) - \int \left(-\frac{1}{x^2}\right)(x)\, dx$$

$$= 1 + \int \frac{dx}{x}$$

So, $0 = 1$.

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