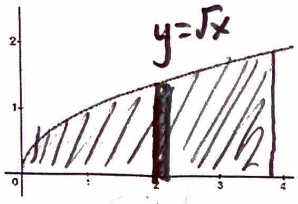
Calculus Ch. 8.2a: Volume by Disc Method

Recall finding area under the curve $y = \sqrt{x}$ between [0, 4]. $Area = \int_a^b (Top\ graph - bottom\ graph)dx$

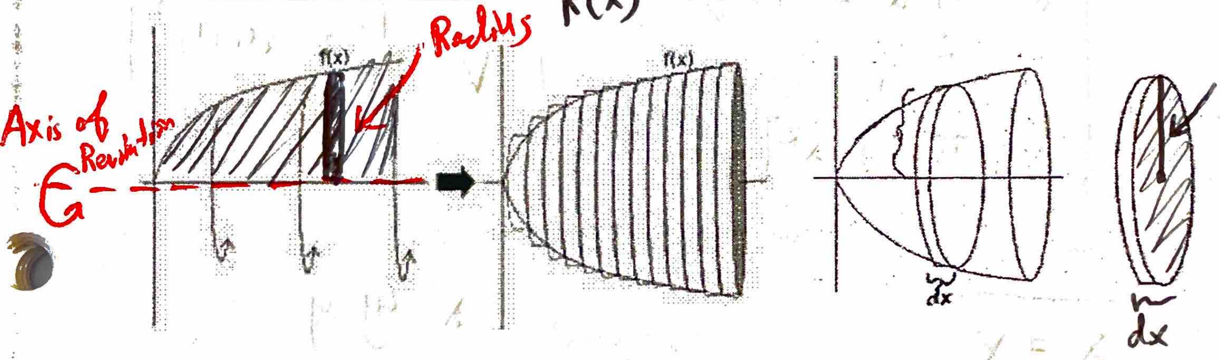


$$A = \int_0^4 \underbrace{\sqrt{x} - 0}_{\text{height}}\ \underbrace{dx}_{\text{width}} = \int x^{1/2} dx = \frac{x^{3/2}}{3/2}\Big]_0^4 = \boxed{\frac{16}{3} \text{ units}^2}$$

*Essentially, the Integral Notation allows us to add infinite numbers of differently sized rectangles to form area calculation.

With **Disc Method**, we are going to take this region created by f(x) and the x-axis and rotate this function 360° around the x-axis. What shapes do you see if we were to separate the resulting object into thin slices?

R(x)



$A = \pi r^2$

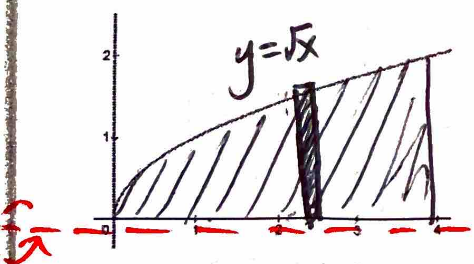
$A = \pi [R(x)]^2$

$V = \int_a^b \pi (R(x))^2 dx$

Disc Method: (Top – Bottom) – Vertical Radius	Disc Method: (Right – Left) – Horizontal Radius
$V = \pi \int_{x_1}^{x_2} [R(x)]^2\, dx$	$V = \pi \int_{y_1}^{y_2} [R(y)]^2\, dy$
(expression(s) used above has form: " y = ___ ")	(expression(s) used above has form: " x = ___ ")

Radius [R(x) or R(y)] - distance from the AOR (Axis of Revolution) to the **boundary** of shaded region

Example 1: Find the volume of the solid formed by rotating the curve $y = \sqrt{x}$ around the x-axis between [0, 4]

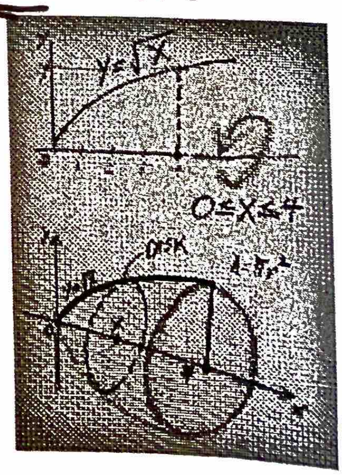


$R(x) = \sqrt{x} - 0 = \sqrt{x}$

$$V = \pi \int_0^4 [\sqrt{x}]^2 dx$$

$$V = \pi \int_0^4 x\, dx \rightarrow \frac{x^2}{2}\Big]_0^4 = \frac{4^2}{2} - \frac{0}{2} =$$

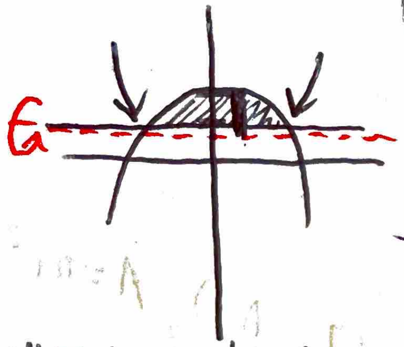
$$\boxed{V = 8\pi \text{ units}^3}$$



Disc Method: (Top – Bottom) – Vertical Radius – Horizontal AOR	**Disc Method: (Right – Left) – Horizontal Radius Vertical AOR**
$V = \pi \int_{x_1}^{x_2} [R(x)]^2\, dx$	$V = \pi \int_{y_1}^{y_2} [R(y)]^2\, dy$
(expression(s) used above has form: " y = ___ ")	(expression(s) used above has form: " x = ___ ")

Radius [R(x) or R(y)] : distance from the AOR(Axis of Revolution) to the **outer boundary** of shaded regi

Example 2: Find the volume of the solid created by $f(x) = 2 - x^2$ revolved $\boxed{\text{about the line y = 1.}}$ AOR



$x^2 = 1$

$x = \pm 1$

$y = -x^2 + 2$

$R(x) = 2 - x^2 - 1 = 1 - x^2$

$\cancel{V = \pi \int_{-1}^{1} (2 - x^2 - (1))\, dx}$

$V = \pi \int_{-1}^{1} [1 - x^2]^2\, dx$

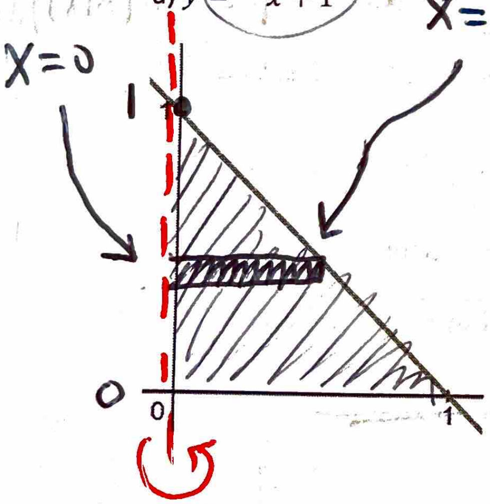
$\boxed{V = \frac{16}{15}\pi \text{ units}^3}$

*Intersection:
$1 = 2 - x^2$

Example 3: Given the region is formed by the function, x-axis, and y-axis. Find the volume of the solid formed by revolving the region $\boxed{\text{about the } \textbf{y-axis}}$

a) $y = -x + 1$

$x = 1 - y$



$R(y) = 1 - y - 0 = 1 - y$

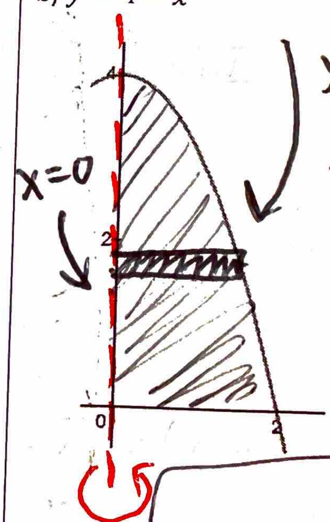
$V = \pi \int_0^1 [1 - y]^2\, dy = \boxed{\frac{\pi}{3} \text{ units}^3}$

b) $y = 4 - x^2$

$x^2 = 4 - y$

$x = \pm\sqrt{4 - y}$

$x = \sqrt{4 - y}$



$R(y) = \sqrt{4 - y} - 0$

$\boxed{V = \pi \int_0^4 [\sqrt{4 - y}]^2\, dy}$

$\boxed{V = 8\pi \text{ units}^3}$