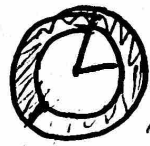
(13)

$\pi R^2 - \pi r^2$

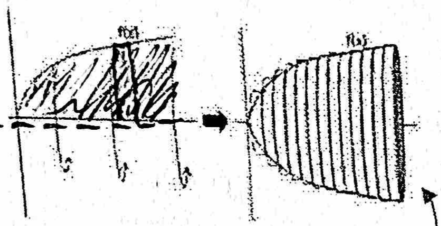


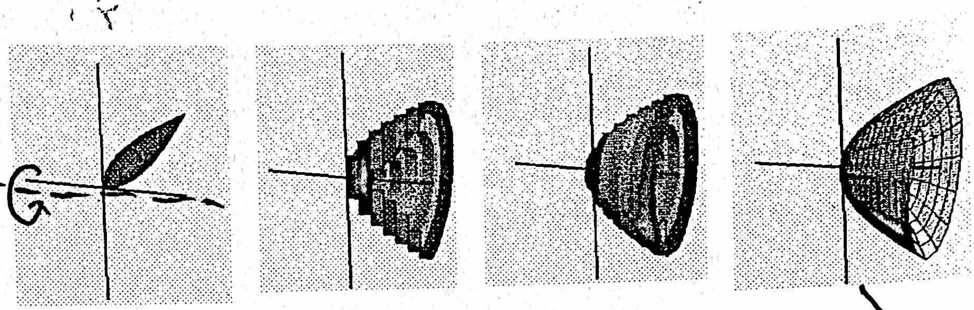
$\pi [R^2 - r^2]$

AP Calculus Ch. 8.2b: Volume by Washer Method Notes

Reviewing Disc Method

Illustration of Washer Method





Disc Method: $V = \pi \int_{x_1}^{x_2} [R(x)]dx$

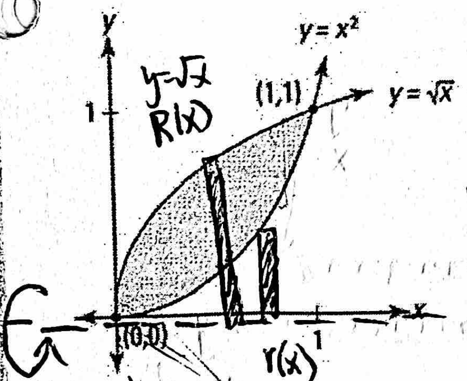
$\cancel{[R - r]^2}$

Washer Method: (Top – Bottom), Vertical Radius (Horizontal AOR)	Washer Method: (Right – Left), Horizontal Radius (Vertical AOR)
$V = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2\, dx$	$V = \pi \int_{y_1}^{y_2} [R(y)]^2 - [r(y)]^2 dy$
(expression(s) used above has form: " y = ___ ")	(expression(s) used above has form: " x = ___ ")

Radius [R(x) or R(y)] - distance from the AOR (Axis of Revolution) to the **outer**(further)curve

radius[r(x) or r(y)] - distance from the AOR (Axis of Revolution) to the **inner**(closer) curve

Example 1: Find the volume of the solid enclosed by the graphs of $y = x^2$ and $y = \sqrt{x}$, and revolving about the x-axis.

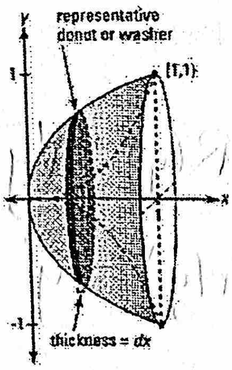


$R(x) = \sqrt{x} - 0$

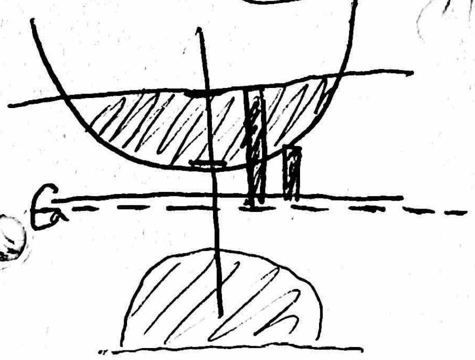
$r(x) = x^2 - 0$

$V = \pi \int_0^1 [\sqrt{x}]^2 - [x^2]^2 dx$

$V = \frac{3}{10}\pi$ units³



Example 2: Find the volume of the solid created by revolving the function y = x² + 1 bounded by the line y = 2 revolved about the x-axis. AOR



$* \; x^2 + 1 = 2$

$x^2 = 1$

$x = \pm 1$

$R(x) = 2 - 0$

$r(x) = x^2 + 1 - 0$

$V = \pi \int_{-1}^{1} [2]^2 - [x^2+1]^2 dx$

$V = \frac{64}{15}\pi$ units³

3)

Radius $[R(x)]$ = *distance from the AOR (Axis of Revolution) to the further graph curve*

radius $[r(x)]$ = *distance from the AOR (Axis of Revolution) to the closer graph curve*

$$Washer\ Method: Volume = \pi \int_{x_1}^{x_2} [R(x)]^2 - [r(x)]^2 dx$$

Example 3: Find the volume of the solid created by revolving the function $y = x^2 + 1$ bounded by the line $y = 2$ and the y-axis about the line $y = 4$

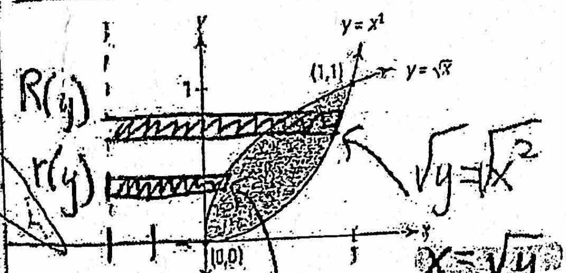
Washer Method (Right - Left)

$$V = \pi \int_{y_1}^{y_2} [R(y)]^2 - [r(y)]^2 dy$$

needs the form "$x =$ ___"

Change Problem

Example 4: Find the volume of the solid created enclosed region of $y = x^2$ and $y = \sqrt{x}$ revolving about the line ~~y=1~~ Revolve about $x=-2$



$\sqrt{y} = \sqrt{x^2}$

$x = \sqrt{y}$

$(y) = (\sqrt{x})^2$

$x = y^2$

AOR $x = -2$

$R(y) = \sqrt{y} - (-2) \rightarrow \sqrt{y} + 2$

$r(y) = y^2 - (-2) \rightarrow y^2 + 2$

$$V = \pi \int_0^1 [\sqrt{y} + 2]^2 - [y^2 + 2]^2 dy$$

$V = 1.633\pi$ units^3

or 5.131 units^3