Ch. 8.3 p.540 #5-71, EOO

stp #29

9) $\int \sin^5 x \cos^2 x \, dx = \int \sin x \cdot \sin^4 x \cos^2 x \, dx$

$\int \sin x \cdot (1-\cos^2 x)^2 \cos^2 x \, dx = \int \sin x \cdot [1 - 2\cos^2 x + \cos^4 x] \cos^2 x \, dx$

$\int \sin x \cdot [\cos^2 x - 2\cos^4 x + \cos^6 x] dx$ $\quad u = \cos x \quad dx = \frac{du}{-\sin x}$

$\frac{du}{dx} = -\sin x$

$\int \sin x \left[u^2 - 2u^4 + u^6\right] \cdot \frac{du}{-\sin x} = -\left[\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7}\right] + C$

$= \boxed{-\frac{1}{3}\cos^3 x + \frac{2}{5}\cos^5 x - \frac{1}{7}\cos^7 x + C}$

13) $\int \cos^2 3x \, dx = \frac{1}{2}\int 1 + \cos 2(3x) \, dx = \frac{1}{2}\int 1 \, dx + \frac{1}{2}\int \cos 2(3x) \, dx$

$u = 6x$

$\frac{du}{dx} = 6$

$dx = \frac{du}{6}$

$\frac{1}{2}\int 1 \, dx + \frac{1}{2}\int \cos u \cdot \frac{du}{6}$

$\frac{1}{2}x + \frac{1}{12}\sin u + C = \boxed{\frac{1}{2}x + \frac{1}{12}\sin(6x) + C}$

IBP

17) $\int x \sin^2 x \quad = \int x\left[\frac{1-\cos 2x}{2}\right]dx$

u	dv
$+x$	$\frac{1}{2}-\frac{\cos 2x}{2}$
-1	$\frac{1}{2}x-\frac{1}{4}\sin(2x)$
$+0$	$\frac{x^2}{4}+\frac{1}{8}\cos(2x)$

$$\boxed{\frac{1}{2}x^2-\frac{1}{4}x\sin(2x)-\frac{x^2}{4}-\frac{1}{8}\cos(2x)+c}$$

* 21) $\int_0^{\pi/2}\cos^7 x\,dx = \int_0^{\pi/2}\cos^6 x\cos x\,dx$ $\quad u=\cos x \quad \cancel{\frac{du}{dx}=-\sin x} \quad \cancel{dx=\frac{du}{-\sin x}}$

$\int \cancel{(1-\sin^2 x)^3 \cos x\,dx}$ $\quad \cancel{1-\sin^2 x = u} \quad \cancel{\frac{du}{dx}=-2}$

*Wallis Formula

odd degree:

$n=7 \quad \frac{n-1}{n} \to \frac{(2)(4)(6)}{(3)(5)(7)} \quad \boxed{\frac{16}{35}}$

25) $\int \sec(3x)\,dx = \frac{1}{3}\ln\left|\sec 3x+\tan 3x\right|+c$

~~*29)~~

37) $\int \sec^6(4x)\tan(4x)\,dx = \int \sec^5(4x)\sec(4x)\tan(4x)\,dx$

$u=\sec(4x)$

$\frac{du}{dx}=\sec(4x)\tan(4x)\cdot 4$

$dx=\frac{du}{4\cdot\sec(4x)\tan(4x)}$

$\int u^5\cdot\sec(4x)\tan(4x)\cdot\frac{du}{4\sec(4x)\tan(4x)}$

$\frac{1}{4}\frac{u^6}{6}+c = \boxed{\frac{1}{24}\sec^6(4x)+c}$

41) $\int \frac{\tan^2 x}{\sec x}\,dx = \int \frac{\sec^2 x - 1}{\sec x}\,dx = \int \sec x - \cos x\,dx$

$= \boxed{\ln|\sec x + \tan x| - \sin x + c}$

49) $\frac{dy}{dx}$

* product identities

$\sin x \sin y = \frac{1}{2}\left[\cos(x-y) - \cos(x+y)\right]$

* 53) $\int \sin\theta \sin 3\theta \, d\theta$

~~$u = \sin 3\theta$ $dv = \sin\theta$~~

~~$\frac{du}{d\theta} = 3\cos 3\theta$ $v = \cos\theta$~~

~~$-\cos\theta \sin 3\theta -$~~ $\int$

$\frac{1}{2}\int \cos(2\theta) - \cos(4\theta)\,dx$

57) $\int \csc^4\theta \, d\theta = \int \csc^2\theta(1+\cot^2\theta)\,d\theta = \boxed{-\cot\theta - \frac{1}{3}\cot^3\theta + C}$

61) $\int \frac{1}{\sec x \tan x}\,dx = \int \frac{\cos^2 x}{\sin x}\,dx = \int \frac{1-\sin^2 x}{\sin x}\,dx$

$\int \csc x - \sin x\,dx = \boxed{\ln|\csc x - \cot x| + \cos x + C}$

65) $\displaystyle\int_{-\pi}^{\pi} \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} dx = x - \frac{1}{2}\sin 2x \Big]_0^{\pi} = \boxed{\pi}$

69) $\displaystyle\int_0^{\pi/2} \frac{\cos t}{1+\sin t} dt = \ln|1+\sin t| \Big]_0^{\pi/2} = \boxed{\ln 2}$