8.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Finding an Indefinite Integral Involving Sine and Cosine In Exercises 1–12, find the indefinite integral.

1. $\int \cos^5 x \sin x \, dx$
2. $\int \cos^3 x \sin^4 x \, dx$
3. $\int \sin^7 2x \cos 2x \, dx$
4. $\int \sin^3 3x \, dx$
5. $\int \sin^3 x \cos^2 x \, dx$
6. $\int \cos^3 \frac{x}{3} \, dx$
7. $\int \sin^3 2\theta \sqrt{\cos 2\theta} \, d\theta$
8. $\int \frac{\cos^5 t}{\sqrt{\sin t}} \, dt$
9. $\int \cos^2 3x \, dx$
10. $\int \sin^4 6\theta \, d\theta$
11. $\int x \sin^2 x \, dx$
12. $\int x^2 \sin^2 x \, dx$

Using Wallis's Formulas In Exercises 13–18, use Wallis's Formulas to evaluate the integral.

13. $\int_0^{\pi/2} \cos^7 x \, dx$
14. $\int_0^{\pi/2} \cos^9 x \, dx$
15. $\int_0^{\pi/2} \cos^{10} x \, dx$
16. $\int_0^{\pi/2} \sin^5 x \, dx$
17. $\int_0^{\pi/2} \sin^6 x \, dx$
18. $\int_0^{\pi/2} \sin^8 x \, dx$

Finding an Indefinite Integral Involving Secant and Tangent In Exercises 19–32, find the indefinite integral.

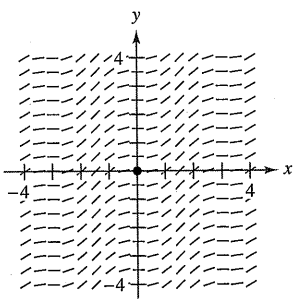
19. $\int \sec 4x \, dx$
20. $\int \sec^4 2x \, dx$
21. $\int \sec^3 \pi x \, dx$
22. $\int \tan^6 3x \, dx$
23. $\int \tan^5 \frac{x}{2} \, dx$
24. $\int \tan^3 \frac{\pi x}{2} \sec^2 \frac{\pi x}{2} \, dx$
25. $\int \tan^3 2t \sec^3 2t \, dt$
26. $\int \tan^5 2x \sec^4 2x \, dx$
27. $\int \sec^6 4x \tan 4x \, dx$
28. $\int \sec^2 \frac{x}{2} \tan \frac{x}{2} \, dx$
29. $\int \sec^5 x \tan^3 x \, dx$
30. $\int \tan^3 3x \, dx$
31. $\int \frac{\tan^2 x}{\sec x} \, dx$
32. $\int \frac{\tan^2 x}{\sec^5 x} \, dx$

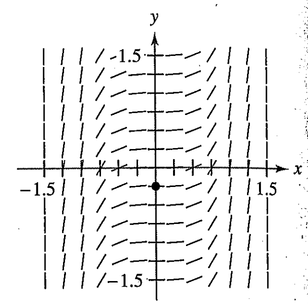
Differential Equation In Exercises 33–36, solve the differential equation.

33. $\frac{dr}{d\theta} = \sin^4 \pi\theta$
34. $\frac{ds}{d\alpha} = \sin^2 \frac{\alpha}{2} \cos^2 \frac{\alpha}{2}$
35. $y' = \tan^3 3x \sec 3x$
36. $y' = \sqrt{\tan x} \sec^4 x$

Slope Field In Exercises 37 and 38, a differential equation, a point, and a slope field are given. (a) Sketch two approximate solutions of the differential equation on the slope field, one of which passes through the given point. (b) Use integration to find the particular solution of the differential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a). To print an enlarged copy of the graph, go to *MathGraphs.com*.

37. $\frac{dy}{dx} = \sin^2 x, \ (0, 0)$
38. $\frac{dy}{dx} = \sec^2 x \tan^2 x, \ \left(0, -\frac{1}{4}\right)$





Slope Field In Exercises 39 and 40, use a computer algebra system to graph the slope field for the differential equation, and graph the solution through the specified initial condition.

39. $\frac{dy}{dx} = \frac{3 \sin x}{y}, \ y(0) = 2$
40. $\frac{dy}{dx} = 3\sqrt{y} \tan^2 x, \ y(0) = 3$

Using Product-to-Sum Identities In Exercises 41–46, find the indefinite integral.

41. $\int \cos 2x \cos 6x \, dx$
42. $\int \cos 5\theta \cos 3\theta \, d\theta$
43. $\int \sin 2x \cos 4x \, dx$
44. $\int \sin(-7x) \cos 6x \, dx$
45. $\int \sin \theta \sin 3\theta \, d\theta$
46. $\int \sin 5x \sin 4x \, dx$

Finding an Indefinite Integral In Exercises 47–56, find the indefinite integral. Use a computer algebra system to confirm your result.

47. $\int \cot^3 2x \, dx$
48. $\int \tan^5 \frac{x}{4} \sec^4 \frac{x}{4} \, dx$
49. $\int \csc^4 3x \, dx$
50. $\int \cot^3 \frac{x}{2} \csc^4 \frac{x}{2} \, dx$
51. $\int \frac{\cot^2 t}{\csc t} \, dt$
52. $\int \frac{\cot^3 t}{\csc t} \, dt$
53. $\int \frac{1}{\sec x \tan x} \, dx$
54. $\int \frac{\sin^2 x - \cos^2 x}{\cos x} \, dx$
55. $\int (\tan^4 t - \sec^4 t) \, dt$
56. $\int \frac{1 - \sec t}{\cos t - 1} \, dt$

Evaluating a Definite Integral In Exercises 57–64, evaluate the definite integral.

57. $\int_{-\pi}^{\pi} \sin^2 x \, dx$

58. $\int_{0}^{\pi/3} \tan^2 x \, dx$

59. $\int_{0}^{\pi/4} 6 \tan^3 x \, dx$

60. $\int_{0}^{\pi/3} \sec^{3/2} x \tan x \, dx$

61. $\int_{0}^{\pi/2} \frac{\cos t}{1 + \sin t} \, dt$

62. $\int_{\pi/6}^{\pi/3} \sin 6x \cos 4x \, dx$

63. $\int_{-\pi/2}^{\pi/2} 3 \cos^3 x \, dx$

64. $\int_{-\pi/2}^{\pi/2} (\sin^2 x + 1) \, dx$

WRITING ABOUT CONCEPTS

65. **Describing How to Find an Integral** In your own words, describe how you would integrate $\int \sin^m x \cos^n x \, dx$ for each condition.

(a) m is positive and odd. (b) n is positive and odd.

(c) m and n are both positive and even.

66. **Describing How to Find an Integral** In your own words, describe how you would integrate $\int \sec^m x \tan^n x \, dx$ for each condition.

(a) m is positive and even. (b) n is positive and odd.

(c) n is positive and even, and there are no secant factors.

(d) m is positive and odd, and there are no tangent factors.

67. **Comparing Methods** Evaluate $\int \sin x \cos x \, dx$ using the given method. Explain how your answers differ for each method.

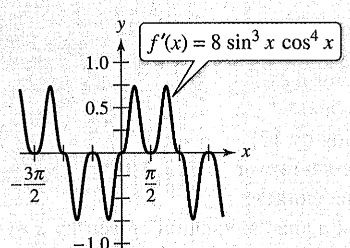
(a) Substitution where $u = \sin x$

(b) Substitution where $u = \cos x$

(c) Integration by parts

(d) Using the identity $\sin 2x = 2 \sin x \cos x$

68. **HOW DO YOU SEE IT?** Use the graph of f' shown in the figure to answer the following.



(a) Using the interval shown in the graph, approximate the value(s) of x where f is maximum. Explain.

(b) Using the interval shown in the graph, approximate the value(s) of x where f is minimum. Explain.

Comparing Methods In Exercises 69 and 70, (a) find the indefinite integral in two different ways. (b) Use a graphing utility to graph the antiderivative (without the constant of integration) obtained by each method to show that the results differ only by a constant. (c) Verify analytically that the results differ only by a constant.

69. $\int \sec^4 3x \tan^3 3x \, dx$

70. $\int \sec^2 x \tan x \, dx$

Area In Exercises 71–74, find the area of the region bounded by the graphs of the equations.

71. $y = \sin x, \quad y = \sin^3 x, \quad x = 0, \quad x = \frac{\pi}{2}$

72. $y = \sin^2 \pi x, \quad y = 0, \quad x = 0, \quad x = 1$

73. $y = \cos^2 x, \quad y = \sin^2 x, \quad x = -\frac{\pi}{4}, \quad x = \frac{\pi}{4}$

74. $y = \cos^2 x, \quad y = \sin x \cos x, \quad x = -\frac{\pi}{2}, \quad x = \frac{\pi}{4}$

Volume In Exercises 75 and 76, find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

75. $y = \tan x, \quad y = 0, \quad x = -\frac{\pi}{4}, \quad x = \frac{\pi}{4}$

76. $y = \cos \frac{x}{2}, \quad y = \sin \frac{x}{2}, \quad x = 0, \quad x = \frac{\pi}{2}$

Volume and Centroid In Exercises 77 and 78, for the region bounded by the graphs of the equations, find (a) the volume of the solid formed by revolving the region about the x-axis and (b) the centroid of the region.

77. $y = \sin x, \quad y = 0, \quad x = 0, \quad x = \pi$

78. $y = \cos x, \quad y = 0, \quad x = 0, \quad x = \frac{\pi}{2}$

Verifying a Reduction Formula In Exercises 79–82, use integration by parts to verify the reduction formula.

79. $\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$

80. $\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$

81. $\int \cos^m x \sin^n x \, dx = -\frac{\cos^{m+1} x \sin^{n-1} x}{m+n} + \frac{n-1}{m+n} \int \cos^m x \sin^{n-2} x \, dx$

82. $\int \sec^n x \, dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx$

Using Formulas In Exercises 83–86, use the results of Exercises 79–82 to find the integral.

83. $\int \sin^5 x \, dx$

84. $\int \cos^4 x \, dx$

85. $\int \sec^4 (2\pi x/5)\, dx$ **86.** $\int \sin^4 x \cos^2 x\, dx$

87. Modeling Data The table shows the normal maximum (high) and minimum (low) temperatures (in degrees Fahrenheit) in Erie, Pennsylvania, for each month of the year. *(Source: NOAA)*

Month	Jan	Feb	Mar	Apr	May	Jun
Max	33.5	35.4	44.7	55.6	67.4	76.2
Min	20.3	20.9	28.2	37.9	48.7	58.5

Month	Jul	Aug	Sep	Oct	Nov	Dec
Max	80.4	79.0	72.0	61.0	49.3	38.6
Min	63.7	62.7	55.9	45.5	36.4	26.8

The maximum and minimum temperatures can be modeled by

$$f(t) = a_0 + a_1 \cos \frac{\pi t}{6} + b_1 \sin \frac{\pi t}{6}$$

where $t = 0$ corresponds to January 1 and a_0, a_1, and b_1 are as follows.

$$a_0 = \frac{1}{12}\int_0^{12} f(t)\, dt \qquad a_1 = \frac{1}{6}\int_0^{12} f(t) \cos \frac{\pi t}{6}\, dt$$

$$b_1 = \frac{1}{6}\int_0^{12} f(t) \sin \frac{\pi t}{6}\, dt$$

(a) Approximate the model $H(t)$ for the maximum temperatures. (*Hint:* Use Simpson's Rule to approximate the integrals and use the January data twice.)

(b) Repeat part (a) for a model $L(t)$ for the minimum temperature data.

(c) Use a graphing utility to graph each model. During what part of the year is the difference between the maximum and minimum temperatures greatest?

88. Wallis's Formulas Use the result of Exercise 80 to prove the following versions of Wallis's Formulas.

(a) If n is odd ($n \geq 3$), then

$$\int_0^{\pi/2} \cos^n x\, dx = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right) \cdots \left(\frac{n-1}{n}\right).$$

(b) If n is even ($n \geq 2$), then

$$\int_0^{\pi/2} \cos^n x\, dx = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right) \cdots \left(\frac{n-1}{n}\right)\left(\frac{\pi}{2}\right).$$

89. Orthogonal Functions The **inner product** of two functions f and g on $[a, b]$ is given by

$$\langle f, g\rangle = \int_a^b f(x)g(x)\, dx.$$

Two distinct functions f and g are said to be **orthogonal** if $\langle f, g\rangle = 0$. Show that the following set of functions is orthogonal on $[-\pi, \pi]$.

$$\{\sin x, \sin 2x, \sin 3x, \ldots, \cos x, \cos 2x, \cos 3x, \ldots\}$$

Victor Soares/Shutterstock.com

90. Fourier Series The following sum is a *finite Fourier series.*

$$f(x) = \sum_{i=1}^{N} a_i \sin ix$$

$$= a_1 \sin x + a_2 \sin 2x + a_3 \sin 3x + \cdots + a_N \sin Nx$$

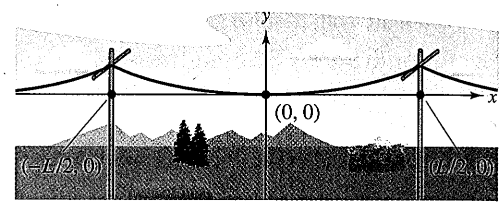
(a) Use Exercise 89 to show that the nth coefficient a_n is given by $a_n = (1/\pi)\int_{-\pi}^{\pi} f(x) \sin nx\, dx$.

(b) Let $f(x) = x$. Find a_1, a_2, and a_3.

SECTION PROJECT

Power Lines

Power lines are constructed by stringing wire between supports and adjusting the tension on each span. The wire hangs between supports in the shape of a catenary, as shown in the figure.

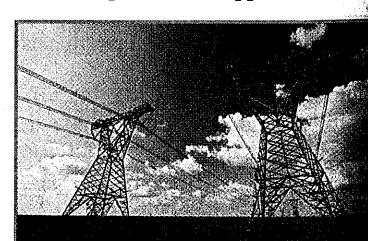


Let T be the tension (in pounds) on a span of wire, let u be the density (in pounds per foot), let $g \approx 32.2$ be the acceleration due to gravity (in feet per second per second), and let L be the distance (in feet) between the supports. Then the equation of the catenary is

$y = \dfrac{T}{ug}\left(\cosh \dfrac{ugx}{T} - 1\right)$, where x and y are measured in feet.

(a) Find the length of the wire between two spans.

(b) To measure the tension in a span, power line workers use the *return wave method.* The wire is struck at one support, creating a wave in the line, and the time t (in seconds) it takes for the wave to make a round trip is measured. The velocity v (in feet per second) is given by $v = \sqrt{T/u}$. How long does it take the wave to make a round trip between supports?



(c) The sag s (in inches) can be obtained by evaluating y when $x = L/2$ in the equation for the catenary (and multiplying by 12). In practice, however, power line workers use the "lineman's equation" given by $s \approx 12.075t^2$. Use the fact that

$$\cosh \frac{ugL}{2T} + 1 \approx 2$$

to derive this equation.

■ **FOR FURTHER INFORMATION** To learn more about the mathematics of power lines, see the article "Constructing Power Lines" by Thomas O'Neil in *The UMAP Journal.*