8.4 AP Practice Problems (p.611-612)

Key

1. A region R is bounded by the graphs of the functions $f(x) = 3x$ and $g(x) = x^2$, $x \geq 0$. If R forms the base of a solid whose slices $\boxed{\text{perpendicular to the } x\text{-axis}}$ are squares, then the volume of the solid is

vertical base

(A) $\frac{9}{2}$ $\boxed{\text{(B)}}$ $\frac{81}{10}$

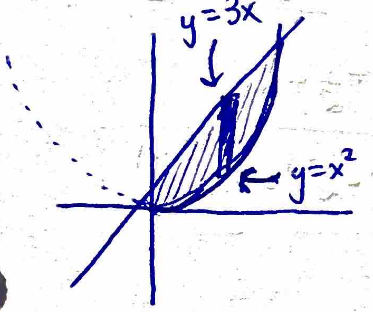
(C) $\frac{45}{2}$ (D) $\frac{162}{5}$

base $= 3x - x^2$

Area (square) $= (\text{base})^2$

$$V = \int_0^3 \left[3x - x^2\right]^2 dx$$

$$\boxed{V = \frac{81}{10} \text{ units}^3}$$



* Intersection

$x^2 = 3x$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, x = 3$

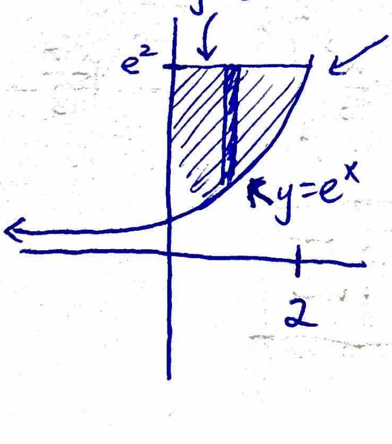
2. A region R in the first quadrant is bounded by the graphs of $y = e^x$, $y = e^2$, and the y-axis. If R forms the base of a solid where slices $\boxed{\text{perpendicular to the } x\text{-axis}}$ are semicircles, then the volume of the solid is given by

(vertical base)

(A) $\frac{1}{2}\int_0^e \pi \left(\frac{e^2 - e^x}{2}\right)^2 dx$ (B) $\int_0^{e^2} \pi \left(\frac{e^2 - e^x}{2}\right)^2 dx$

$\boxed{\text{(C)}}$ $\frac{1}{2}\int_0^2 \pi \left(\frac{e^2 - e^x}{2}\right)^2 dx$ (D) $\frac{1}{2}\int_0^2 \pi (e^2 - e^x)^2 dx$

$y = e^2 \approx 8$



* Intersection

$e^x = e^2$

$x = 2$

base $= e^2 - e^x$

Area (semicircle) $= \frac{\pi}{8}(\text{base})^2$

$$V = \frac{\pi}{8}\int_0^{e^2} \left[e^2 - e^x\right]^2 dx = \boxed{\frac{1}{2}\int_0^{e^2} \pi \left[\frac{e^2 - e^x}{2}\right]^2 dx}$$

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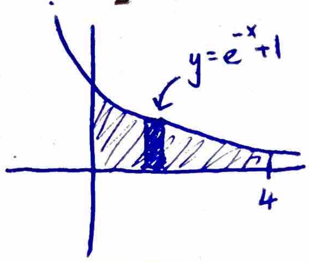
$\boxed{8.4}$

3. The base of a solid is formed by the region bounded by the graphs of $y = e^{-x} + 1$, the x-axis, the y-axis, and the line $x = 4$. If slices perpendicular to the x-axis are squares, what is the volume of the solid? (vertical base)

(A) $\frac{1}{2}(5 - e^{-8} - 4e^{-4})$ (B) $8 - e^{-8} - 2e^{-4}$

(C) $\frac{1}{2}(9 - e^{-8})$ $\boxed{\text{(D)}}$ $\frac{1}{2}(13 - e^{-8} - 4e^{-4})$



base $= e^{-x} + 1 - 0$

Area $= (\text{base})^2$

$V = \int_0^4 (e^{-x}+1)^2\,dx$

$\int (e^{-x}+1)(e^{-x}+1)\,dx$

$\int_0^4 e^{-2x} + 2e^{-x} + 1\,dx$

$-\frac{1}{2}e^{-2x} - 2e^{-x} + x \Big]_0^4$

$\frac{-1}{2e^{2x}} - \frac{2}{e^x} + x \Big]_0^4$

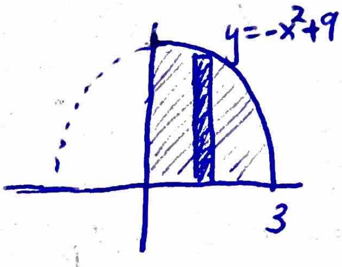
$\frac{-1}{2e^8} - \frac{2}{e^4} + 4 - \left(\frac{-1}{2e^0} - \frac{2}{e^0} + 0\right)$

$\frac{-1}{2e^8} - \frac{2}{e^4} + 4 + \frac{1}{2} + 2$

$= \boxed{\frac{1}{2}(13 - e^{-8} - 4e^{-4})}$

4. The base of a solid is the region in the first quadrant bounded by the coordinate axes and the parabola $y = -x^2 + 9$. Cross sections of the solid perpendicular to the x-axis are squares. What is the volume of the solid?

(A) 18 (B) $\frac{81}{4}$ $\boxed{\text{(C)}}$ $\frac{648}{5}$ (D) $\frac{1458}{5}$



base $= -x^2 + 9 - 0$

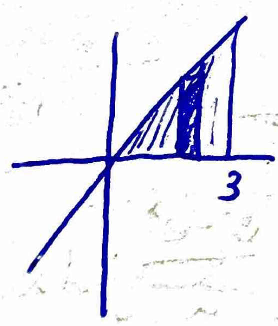
Area $= (\text{base})^2$

$V = \int_0^3 (-x^2+9)^2\,dx$

$\boxed{V = \frac{648}{5}}$

5. The base of a solid is the region in the first quadrant bounded by lines $y = 2x$, $x = 3$, and the x-axis. Every cross section of the solid perpendicular to the x-axis is an equilateral triangle. What is the volume of the solid?

(A) 9 (B) $\frac{9\sqrt{3}}{2}$ $\boxed{\text{(C)}}$ $9\sqrt{3}$ (D) 27



base $= 2x - 0 = 2x$

Area (equil triangle) $= \frac{\sqrt{3}}{4}(\text{base})^2$

$= \frac{\sqrt{3}}{4}(2x)^2$

$V = \int_0^3 \frac{\sqrt{3}}{4} \cdot 4x^2\,dx$

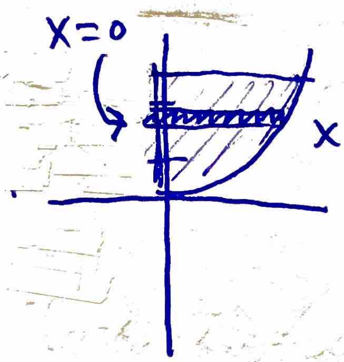
$\sqrt{3} \cdot \frac{x^3}{3}\Big]_0^3 = \sqrt{3} \cdot \frac{27}{3}$

$\boxed{V = 9\sqrt{3}}$

$\boxed{8.4}$ 49

6. Find the volume of the solid whose base is the region in the first quadrant that is bounded by the graphs of $y = x^2$, $y = 4$, and the y-axis if slices perpendicular to the y-axis are squares.

(A) $\frac{32}{5}$ $\boxed{\text{(B)}}$ 8 (C) $\frac{256}{15}$ (D) $\frac{1024}{5}$



$\text{base} = \sqrt{y} - (0)$

$\text{base} = \sqrt{y}$

$\underset{\text{(square)}}{\text{Area}} = (\text{base})^2$

$A = [\sqrt{y}]^2$

$V = \int_0^4 [\sqrt{y}]^2 \, dy$

$= \int_0^4 y \, dy \rightarrow \frac{y^2}{2}\Big]_0^4$

$\frac{4^2}{2} - 0 = \boxed{8}$

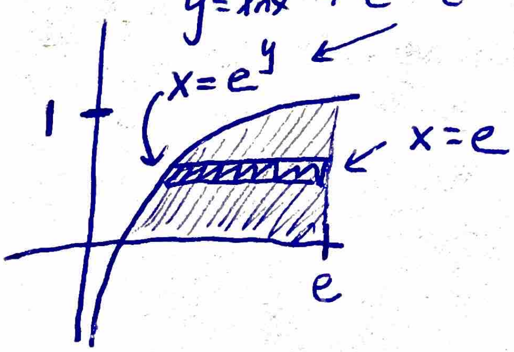
7. The base of a solid is the region bounded by the graph of $y = \ln x$, the line $x = e$, and the x-axis. If slices perpendicular to the y-axis are semicircles, then the volume of the solid is

(A) $\frac{\pi}{16}\left[7e^2 - 4e - 1\right]$ $\boxed{\text{(B)}}$ $\frac{\pi}{16}\left[-e^2 + 4e - 1\right]$

(C) $\frac{\pi}{8}\left[7e^2 - 4e - 1\right]$ (D) $\frac{\pi}{8}\left[-e^2 + 4e - 1\right]$

$y = \ln x \rightarrow e^y = e^{\ln x}$

$x = e^y$



*Right-Left

$\text{base} = e - e^y$

$\underset{\text{(semicircle)}}{\text{Area}} = \frac{\pi}{8}(\text{base})^2$

$= \frac{\pi}{8}(e - e^y)^2$

$V = \frac{\pi}{8}\int_0^1 (e - e^y)^2 \, dy$

$\frac{\pi}{8}\int e^2 - 2ee^y + e^{2y} \, dy$

$\frac{\pi}{8}\int_0^1 e^2 - 2e^{y+1} + e^{2y} \, dy$

$e^2 y - 2e^{y+1} + \frac{1}{2}e^{2y}\Big]_0^1$

$e^2 - 2e^2 + \frac{1}{2}e^2 - (0 - 2e + \frac{1}{2}e^0)$

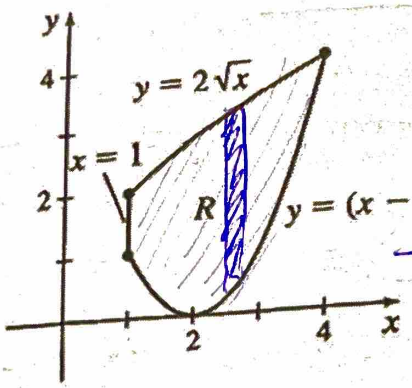
$\frac{\pi}{8}\left[-\frac{1}{2}e^2 + 2e - \frac{1}{2}\right]$

$\boxed{\frac{\pi}{16}\left[-e^2 + 4e - 1\right]}$

(50)

$\boxed{8.4}$

8. The region R bounded by the graphs of $y = (x-2)^2$, $y = 2\sqrt{x}$, and the line $x = 1$ is shown in the figure below.



(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid of revolution that is generated when R is revolved about the x-axis.

~~(c)~~ Write, but do not evaluate, an integral expression that gives the volume of the solid of revolution that is generated when R is revolved about the y-axis. (OMIT)

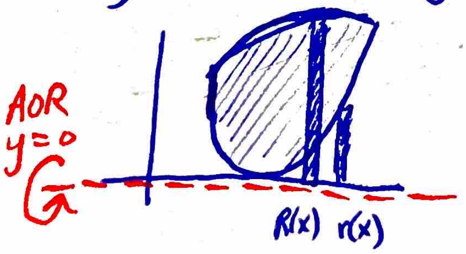
(d) The region forms the base of a solid. If cross sections of the solid perpendicular to the x-axis are squares, write, but do not evaluate, an expression that gives the volume of the solid.

a) *Top − Bottom

$$A = \int_1^4 2\sqrt{x} - (x-2)^2\,dx$$

$$\boxed{A = \frac{19}{3}}$$

b) AoR: x-axis $(y=0)$



b) * Washer Method

$R(x) = 2\sqrt{x} - 0$

$r(x) = (x-2)^2 - 0$

$$V = \pi\int_1^4 [2\sqrt{x}]^2 - [(x-2)^2]^2\,dx$$

$$\boxed{V = \pi\int_1^4 4x - (x-2)^4\,dx}$$

d) base $= 2\sqrt{x} - (x-2)^2$

Area $= (\text{base})^2$

$$\boxed{V = \int_1^4 \left[2\sqrt{x} - (x-2)^2\right]^2 dx}$$