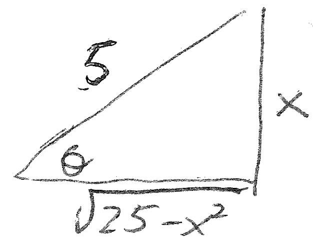
$\#5, 7, 11, 13, \underline{17}, 19, 23, 29, 31, 37, \underline{41}, \underline{43}, \underline{47}$ * Theorem 8.2 p546?

8.4 Trig Substitution p. 549 5-49 Primes:

5) $\displaystyle\int \frac{1\,dx}{\left(\sqrt{25-x^2}\right)^3}$



$\sin\theta = \frac{u}{a}$

$\sin\theta = \frac{x}{5}$

$x = 5\sin\theta$

$\displaystyle\int \frac{5\cos\theta\, d\theta}{(5\cos\theta)^3}$

$\cos\theta = \frac{\sqrt{25-x^2}}{5}$

$5\cos\theta = \sqrt{25-x^2}$

$\frac{dx}{d\theta} = 5\cos\theta$

$dx = 5\cos\theta\, d\theta$

$\frac{dx}{5} = \cos\theta\, d\theta$

$u = 5\cos\theta$

$\frac{du}{d\theta} =$

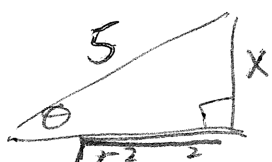
$\displaystyle\int \frac{1}{25\cos^2\theta}\, d\theta$

$\frac{dx}{5\,d\theta} = \frac{\sqrt{25-x^2}}{5}$

$dx = \sqrt{25-x^2}\, d\theta$

$= \frac{1}{25}\displaystyle\int \sec^2\theta\, d\theta = \frac{1}{25}\tan\theta + C = \boxed{\frac{1}{25}\cdot\frac{x}{\sqrt{25-x^2}} + C}$

7) $\displaystyle\int \frac{\overset{5\cos\theta}{\sqrt{25-x^2}}\;\overset{5\cos\theta\, d\theta}{dx}}{x}$



$\sin\theta = \frac{x}{5}$

$x = 5\sin\theta$

$x = 5\sin\theta$

$\frac{dx}{d\theta} = 5\cos\theta$

$\cos\theta = \frac{\sqrt{25-x^2}}{5}$

$\frac{dx}{d\theta} = 5\left(\frac{\sqrt{25-x^2}}{5}\right)$

$\sqrt{25-x^2} = 5\cos\theta$

$dx = \sqrt{25-x^2}\, d\theta$

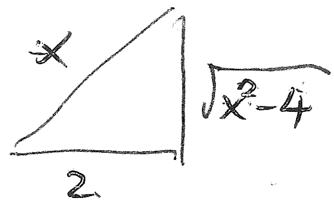
$\displaystyle\int \frac{5^2\cos\theta\, d\theta}{\cancel{5}\sin\theta}$

$= 5\ln\left|\frac{5-\sqrt{25-x^2}}{x}\right| + \sqrt{25-x^2} + C$

$\displaystyle\int \frac{25\cos^2\theta}{5\sin\theta}\, d\theta = 5\int \frac{1-\sin^2\theta}{\sin\theta}\, d\theta = 5\int \csc\theta - \sin\theta\, d\theta$

$= 5\ln|\csc\theta - \cot\theta| + \cos\theta + C$

11) $\int x^3\sqrt{x^2-4}\,dx$



$\sec\theta = \frac{u}{a} = \frac{x}{2}$

$\sec\theta = \frac{x}{2}$

$\boxed{x = 2\sec\theta}$

$\frac{dx}{d\theta} = 2\sec\theta\tan\theta$

$dx = \underline{2\sec\theta\tan\theta\,d\theta}$

$\tan\theta = \frac{\sqrt{x^2-4}}{2}$

$\underline{2\tan\theta} = \sqrt{x^2-4}$

$\int x^3 (2\tan\theta)\cdot 2\sec\theta\tan\theta\,d\theta$ $\quad x^3 \to (2\sec\theta)^3$

$\int (2\sec\theta)^3\, 2\tan\theta \cdot 2\sec\theta\tan\theta\,d\theta$

$\int 32\sec^4\theta\tan^2\theta\,d\theta = \int 32\,\underline{\sec^2\theta}\cdot\sec^2\theta\cdot\tan^2\theta\,d\theta$

$\int 32(1+\tan^2\theta)\tan^2\theta\cdot\sec^2\theta\,d\theta$

$u = \tan\theta$

$\frac{du}{d\theta} = \sec^2\theta$

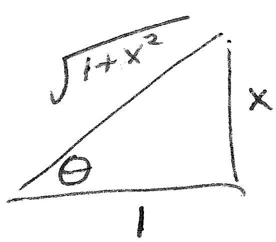
$du = \sec^2\theta\,d\theta$

$32\int [\tan^2\theta + \tan^4\theta]\sec^2\theta\,d\theta$

$32\int u^2+u^4\,du = 32\left[\frac{u^3}{3}+\frac{u^5}{5}\right]+C = \boxed{\frac{32}{3}\tan^3\theta + \frac{32}{5}\tan^5\theta + C}$

13) $\int x\sqrt{1+x^2}\,dx$

*u power rule



$\sqrt{a^2+u^2} \to \tan\theta = \frac{u}{a}$

$\tan\theta = \frac{x}{1}$

$x = \tan\theta$

$dx = \sec^2\theta\,d\theta$

$\sec\theta = \frac{\sqrt{1+x^2}}{1} = \sqrt{1+x^2}$

$\int \tan\theta\cdot\sec\theta\cdot\sec^2\theta\,d\theta$

$\int \tan\theta\sec^3\theta\,d\theta = \int \tan\theta\sec\theta\cdot\sec^2\theta\,d\theta$

$u = \sec\theta$

$\frac{du}{d\theta} = \sec\theta\tan\theta$

$\int u^2\,du = \frac{u^3}{3}+C$

$= \frac{1}{3}\sec^3\theta + C$

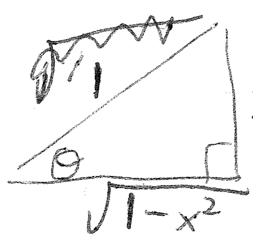
$= \frac{1}{3}(1+x^2)^{3/2}+C$

$* \; 17) \int \sqrt{4+9x^2}\,dx \qquad u=3x,\; a=2,\; du=3dx \qquad * \int \sqrt{u^2+a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2+a^2} + a^2 \ln\left|u+\sqrt{u^2+a^2}\right|\right) + C$

d

$$\left(\frac{1}{3}\right)\frac{1}{2}\left(3x\sqrt{9x^2+4} + 4\ln\left|2+\sqrt{9x^2+4}\right| + C\right)$$

29) $\int \frac{\sqrt{1-x^2}}{x^4}\,dx$



$\sqrt{a^2-u^2} \rightarrow \sin\theta = \frac{u}{a}$

$\sin\theta = \frac{x}{1}$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$\cos\theta = \sqrt{1-x^2}$

$$\int \frac{\cos\theta}{\sin^4\theta}\cos\theta\, d\theta$$

$$\int \frac{\cos^2\theta}{\sin^4\theta}\, d\theta$$

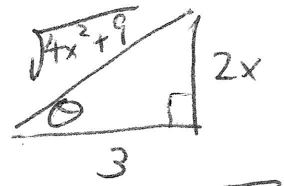
$$\int \cot^2\theta \csc^2\theta\, d\theta \qquad \int u^2 \cdot -du = -\frac{u^3}{3} + C$$

$u = \cot\theta$

$\frac{du}{d\theta} = -\csc^2\theta$

$$= \boxed{-\frac{1}{3}\cot^3\theta + C}$$

31) $\int \frac{1}{x\sqrt{4x^2+9}}\,dx$ $\sqrt{(2x)^2+(3)^2} = \sqrt{u^2+a^2}$ $\tan\theta = \frac{u}{a}$



$\tan\theta = \frac{2x}{3}$

$x = \frac{3}{2}\tan\theta$

$\frac{dx}{d\theta} = \frac{3}{2}\sec^2\theta$

$dx = \frac{3}{2}\sec^2\theta\, d\theta$

$\sec\theta = \frac{\sqrt{4x^2+9}}{3}$

$3\sec\theta = \sqrt{4x^2+9}$

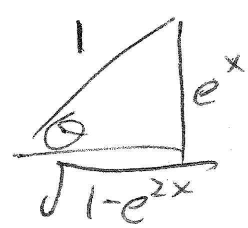
$\int \frac{1 \cdot \cancel{\frac{3}{2}}\sec^2\theta\, d\theta}{\cancel{\frac{3}{2}}\tan\theta \cdot 3\sec\theta}$

$\frac{1}{3}\int \frac{\sec\theta}{\tan\theta}\, d\theta$

$\frac{1}{3}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\, d\theta$ $\frac{1}{3}\int \csc\theta\, d\theta = \boxed{-\frac{1}{3}\ln\left|\csc\theta + \cot\theta\right| + C}$

37) $\int e^x\sqrt{1-e^{2x}}\,dx$ $\sqrt{a^2-u^2} \to \sin\theta = \frac{u}{a}$ $\sin\theta = \frac{e^x}{1}$

$\cos\theta\, d\theta = e^x\, dx$



$\cos\theta = \sqrt{1-e^{2x}}$

$\int \cos\theta\, d\theta \cdot \cos\theta\, d\theta$

$\int \cos^2\theta\, d\theta = \int \frac{1+\cos 2\theta}{2}\, d\theta$

$\frac{1}{2}\int 1+\cos 2\theta\, d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$

$\frac{1}{2}\theta + \frac{1}{4}(2\sin\theta\cos\theta)$

$\boxed{= \frac{1}{2}\left[\arcsin e^x + e^x\sqrt{1-e^{2x}}\right] + C}$

$\boxed{8.4}$ p. 549 41, 43, 47

41) $\int \operatorname{arcsec}(2x)\,dx \quad , x > 1/2$

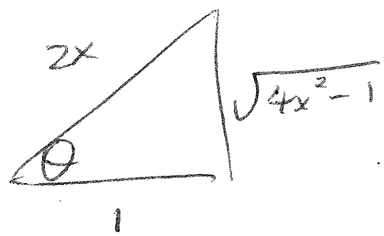
$\int u\,dv = uv - \int v\,du$

$dv = dx \qquad u = \operatorname{arcsec}(2x)$

$v = x \qquad \frac{du}{dx} = \frac{\cancel{2}}{|\cancel{2}x|\sqrt{4x^2-1}}$

$= x\operatorname{arcsec}(2x) - \int \frac{\cancel{x}}{\cancel{x}\sqrt{4x^2-1}}\,dx$

* for $\sqrt{u^2-a^2}$, use $\sec\theta = \frac{u}{a}$



$\sec\theta = \frac{2x}{1}$

$2\frac{dx}{d\theta} = \sec\theta\tan\theta$

$dx = \frac{1}{2}\sec\theta\tan\theta\,d\theta$

$\tan\theta = \sqrt{4x^2-1}$

$$\int \frac{1}{\sqrt{4x^2-1}}\,dx = \int \frac{1}{2}\sec\theta\tan\theta \cdot \frac{1}{\cancel{\tan\theta}} = \frac{1}{2}\int \sec\theta\,d\theta$$

$$= x\operatorname{arcsec}(2x) - \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$

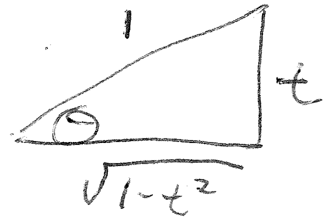
$$\boxed{= x\operatorname{arcsec}(2x) - \frac{1}{2}\ln\left|2x + \sqrt{4x^2-1}\right| + C}$$

*use complete the square

43) $\int \frac{1}{\sqrt{4x-x^2}}\,dx = \int \frac{1}{\sqrt{-(x^2-4x+4)-4}}\,dx = \int \frac{1}{\sqrt{2^2-(x-2)^2}}\,dx$

$$= \boxed{\arcsin\left(\frac{x-2}{2}\right) + C}$$

47) $\displaystyle\int_0^{\sqrt{3}/2} \frac{t^2}{(1-t^2)^{3/2}}\,dt$ * for $\sqrt{a^2-u^2} = \sin\theta = \frac{u}{a}$



$\sin\theta = t$

$dt = \cos\theta\,d\theta$

$$\int \frac{(\sin\theta)^2}{(\cos\theta)^{3/2}}\cos\theta\,d\theta \qquad \cos\theta = \sqrt{1-t^2}$$

$$\cos^2\theta = 1-t^2$$

$$\cancel{\int \frac{(\sin\theta)^2}{(\cos\theta)^{1/2}}\,d\theta} \quad u = \quad \int \frac{(\sin\theta)^2}{(\cos\theta)^3}\cos\theta\,d\theta = \int \tan^2\theta\,d\theta$$

$$= \int \sec^2\theta - 1\,d\theta = \tan\theta - \theta$$

$$= \frac{t}{\sqrt{1-t^2}} - \arcsin t \Big]_0^{\sqrt{3}/2} = \boxed{\sqrt{3} - \pi/3}$$