

8.5 AP Practice Problems (p. 620) – Arc Length

$$f'(x) = -4x^3$$

Key

1. The arc length of the graph of $f(x) = -x^4 + 2$ from $x = 0$ to $x = 10$ is given by

(A) $\int_0^{10} \sqrt{1 + 16x^3} dx$

(B) $\int_0^{10} \sqrt{1 + 4x^3} dx$

(C) $\int_0^{10} \sqrt{1 + (-x^4 + 2)^2} dx$

(D) $\int_0^{10} \sqrt{1 + 16x^6} dx$

$$L = \int_a^b \sqrt{1 + f'(x)^2} dx$$

$$L = \int_a^b \sqrt{1 + (-4x^3)^2} dx$$

$$L = \int_0^{10} \sqrt{1 + 16x^6} dx$$

2. The arc length of the graph of $y = \tan x$ from $x = a$ to $x = b$, where $-\frac{\pi}{2} < a < b < \frac{\pi}{2}$ is given by

(A) $\int_{-\pi/2}^{\pi/2} \sqrt{1 + \sec^2 x} dx$

(B) $\int_a^b \sqrt{1 + \sec^4 x} dx$

(C) $\int_a^b \sqrt{1 + \sec^2 x} dx$

(D) $\int_a^b \sqrt{1 + \sec^2 x \tan^2 x} dx$

$$y = \tan x$$

$$y' = \sec^2 x$$

$$\int_{-\pi/2}^{\pi/2} \sqrt{1 + [\sec^2 x]^2} dx$$

3. What is the length of the graph of $f(x) = \ln \cos x$ from $x = 0$ to $x = \frac{\pi}{3}$?

(A) $\ln|\sqrt{3} - 2|$

(B) $\ln(2 + \sqrt{3})$

(C) $2\sqrt{3}$

(D) $\ln 4$

$$\ln|\sec x + \tan x| \Big|_0^{\pi/3}$$

$$\ln|\sec \pi/3 + \tan \pi/3| - \ln|\sec 0 + \tan 0|$$

$$\ln|2 + \sqrt{3}| - \ln|1 + 0|$$

$$\ln|2 + \sqrt{3}|$$

$$f(x) = \ln(\cos x)$$

$$f'(x) = \frac{-\sin x}{\cos x}$$

$$f'(x) = -\tan x$$

$$\int_0^{\pi/3} \sqrt{1 + (-\tan x)^2} dx$$

$$\int_0^{\pi/3} \sqrt{1 + \tan^2 x} \rightarrow \sqrt{\sec^2 x}$$

$$* 1 + \tan^2 x = \sec^2 x$$

$$\int_0^{\pi/3} \sec x dx$$

4. The arc length of $y = (x - 8)^{2/3}$ from $x = 0$ to $x = 16$ is given by

(A) $\int_0^{16} \sqrt{1 + \frac{4}{9(x-8)^{2/3}}} dx$ (B) $\int_0^4 \sqrt{1 + \frac{9}{4}y} dy$
 (C) $2 \int_0^4 \sqrt{1 + \frac{9}{4}y} dy$ (D) $\int_0^{16} \sqrt{1 + \frac{2}{3(y-8)^{1/3}}} dy$

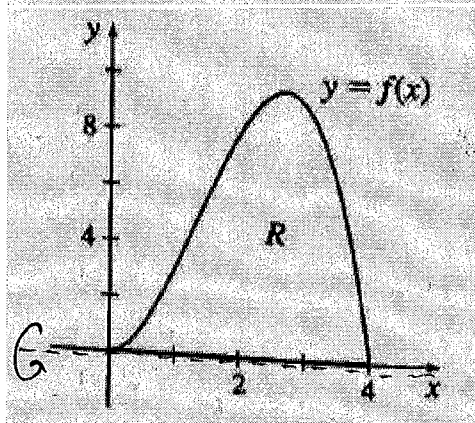
$$\int_0^{16} \sqrt{1 + \frac{4}{9(x-8)^{2/3}}} dx$$

$$y' = \frac{2}{3}(x-8)^{-1/3}(1) \quad \left| \int_0^{16} \sqrt{1 + \left(\frac{2}{3(x-8)^{1/3}}\right)^2} dx \right.$$

$$y' = \frac{2}{3(x-8)^{1/3}}$$

5. The region R bounded by the graph of $f(x) = -x^3 + 4x^2$ and the x -axis is shown in the figure below.

- (a) Find the area under the graph of f from 0 to 4.
 (b) Write, but do not evaluate, an integral for the arc length of the graph of f from $x = 0$ to $x = 4$.
 (c) Write, but do not evaluate, an integral to find the volume of the solid of revolution generated by revolving the region R about the x -axis.



C) Disc Method
 $R(x) = -x^3 + 4x^2 - 0$

$$V = \pi \int R(x)^2 dx$$

$$V = \pi \int_0^4 [-x^3 + 4x^2]^2 dx$$

a) $\int_0^4 -x^3 + 4x^2 dx$
 $\left[-\frac{x^4}{4} + \frac{4x^3}{3} \right]_0^4$
 $= -\frac{4^4}{4} + \frac{4^4}{3}$
 $= \boxed{\frac{64}{3}}$

b) $f'(x) = -3x^2 + 8x$

$$L = \int_0^4 \sqrt{1 + (-3x^2 + 8x)^2} dx$$