8.5 p. 559 7-46 ETQ

12) $3\ln|x| - 2\ln|x-2| + 4\ln|x+2| + C$

18) $2\ln|x-1| + \frac{1}{x-1} + C$

24) $\frac{1}{3}\arctan\left(\frac{x}{3}\right) + \frac{1}{2(x^2+9)} + C$

30) $2\ln\left(\frac{5}{3}\right) - \frac{4}{5}$

36) $\frac{22}{5} - \frac{1}{2}\ln 5$

42) $\ln|1+\sec x| + C$

8.5 p.559 #7-46 · ETQ

7, 10, 13, 16, 19, 22, 25, 28, 31, ~~34~~, ~~37~~, ~~40~~, 43, 46

$$\int \frac{1}{x^2-1}\,dx \qquad \frac{1}{(x-1)(x+1)} = \frac{\frac{1}{2}}{(x-1)} + \frac{-\frac{1}{2}}{(x+1)}$$

$x=1 \qquad x=-1$

$$= \frac{1}{2}\int \frac{1}{x-1}\,dx - \frac{1}{2}\int \frac{1}{x+1}\,dx = \boxed{\frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C}$$

$$= \boxed{\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C}$$

10) $\int \frac{x+1}{(x^2+4x+3)} = \frac{\quad}{x} + \frac{\quad}{C}$

$x=0 \quad \frac{12}{-4} \qquad x=-2 \quad \frac{4-24+12}{8} \qquad x=2 \quad \frac{40}{8}$

13) $\frac{x^2+12x+12}{x(x+2)(x-2)} = \frac{-3}{x} + \frac{-1}{x+2} + \frac{5}{x-2}$

16) $\int \frac{x+2}{x^2-4x}$ $\qquad x=0 \qquad x=4$

$x(x-4)$

$$= \int \frac{-1/2}{x} + \frac{3/2}{x-4}\,dx$$

$$\int \frac{x+2}{x(x-4)}\,dx = \boxed{\frac{3}{2}\ln|x-4| - \frac{1}{2}\ln|x| + C}$$

19) $\int \frac{x^2+3x-4}{x^3-4x^2+4x}\,dx = \frac{A}{x} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$

$x(x^2-4x+4)$

$x(x-2)(x-2)$

$x^2+3x-4 = A(x-2)^2 + Bx(x-2) + Cx$

$x=0 \rightarrow \quad -4 = A(4) + B(0) + C(0)$

$\boxed{A=-1}$

$x=2 \rightarrow \quad 6 = -1(0) + B(0) + C(2)$

$6 = 2C$

$\boxed{3 = C}$

$x=1 \rightarrow$

$1+3-4 = -1(1) + B(1)(-1) + 3(1)$

$0 = -1 - B + 3$

$= -B + 2$

$\boxed{2 = B}$

$$= \int \frac{-1}{x} + \frac{2}{(x-2)} + \frac{3}{(x-2)^2}\,dx$$

$$= \boxed{-\ln|x| + 2\ln|x-2| - \frac{3}{(x-2)} + C}$$

22) $\int \frac{6x}{x^3-8}\,dx$

$(x-2)(x^2+4x+4)$

25) $\int \frac{x}{16x^4-1}\,dx = \frac{A}{(2x-1)} + \frac{B}{(2x+1)} + \frac{Cx+D}{4x^2+1}$

$(2x-1)(2x+1)(4x^2+1)$

$x = A(2x+1)(4x^2+1) + B(2x-1)(4x^2+1) + (Cx+D)(2x-1)(2x+1)$

$x = \frac{1}{2} \rightarrow \frac{1}{2} = A(2)(2)$

$\frac{1}{2} = 4A$

$\frac{1}{8} = A$

$x = -\frac{1}{2} \rightarrow -\frac{1}{2} = 0 - 4B + 0 \quad \frac{1}{8} = B$

$x = 0 \rightarrow 0 = A - B - D \rightarrow D = 0$

$x = 1 \rightarrow 1 = 15A + 5B + 3C + 3D$

$C = -\frac{1}{2}$

$= \int \frac{1}{8}\left(\frac{1}{2x-1}\right)dx + \frac{1}{8}\int \frac{1}{(2x+1)}dx + \int \frac{-\frac{1}{2}x}{4x^2+1}\,dx$

$u = 4x^2+1$

$\frac{du}{dx} = 8x$

$dx = \frac{du}{8x}$

$\frac{1}{8}\cdot\frac{1}{2}\ln|2x-1| + \frac{1}{8}\cdot\frac{1}{2}\ln|2x+1| - \frac{1}{16}\ln|4x^2+1|$

$\frac{1}{16}\ln\left|\frac{(2x-1)(2x+1)}{4x^2+1}\right| = \boxed{\frac{1}{16}\ln\left|\frac{4x^2-1}{4x^2+1}\right| + C}$

28) $\frac{x^2+x+3}{(x^2+3)^2} = \frac{Ax+B}{(x^2+3)} + \frac{Cx+D}{(x^2+3)^2}$

$\int \frac{1}{x^2+3} + \frac{x}{(x^2+3)^2}$

$x^2+x+3 = (Ax+B)(x^2+3) + Cx + D$

$x^2+x+3 = Ax^3 + 3Ax + Bx^2 + 3B + Cx + D$

$x^2+x+3 = Ax^3 + Bx^2 + x(3A+C) + 1(3B+D)$

$A = 0 \quad 3B+D = 3$

$B = 1 \quad 3+D = 3$

$C = 1 \quad D = 0$

$\boxed{\frac{1}{\sqrt{3}}\arctan\left(\frac{1}{\sqrt{3}}\right) - \frac{1}{2(x^2+3)} + C}$

34) $\int \frac{6x^2+1}{x^3(x-1)^3}\,dx$

43) $\int \frac{3\cos x}{\sin^2 x+\sin x-2}\,dx$ $\quad u=\sin x$

$= 3\int \frac{1}{u^2+u-2}\,du$ $\quad u=-2,\ u=1$

$3\int \frac{1}{(u+2)(u-1)}\,du$ $\quad \frac{-1}{u+2}+\frac{1}{u-1}$

$\ln|u-1| - \ln|u+2| = \ln\left|\frac{u-1}{u+2}\right| + C$

$= \ln\left|\frac{\sin x-1}{\sin x+2}\right| + C$

46) $\int \frac{e^x}{(e^{2x}+1)(e^x-1)}\,dx$ $\quad u=e^x$

$\frac{du}{dx}=e^x$

$dx=\frac{du}{e^x}$

$\int \frac{1}{(u^2+1)(u-1)}\,du$

$\frac{A}{u-1}+\frac{Bu+C}{u^2+1}$

$1 = A(u^2+1)+(Bu+C)(u-1)$ → when $u=1$, $A=1/2$

$= Au^2+A+Bu^2-Bu+Cu-C$ when $u=0$, $1=A-C$

$1 = u^2(A+B)+u(-B+C)+1(A-C)$ when $u=-1$, $1=2A+2B-2C$

$A=1/2,\ B=-1/2,\ C=-1/2$

$\frac{1}{2}\int \frac{1}{u-1}\,du - \int \frac{u+1}{u^2+1}\,du$

$\frac{1}{2}\ln|e^x-1| - \frac{1}{4}\ln|e^{2x}+1| - \frac{1}{2}\arctan e^x + C$