8.5 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Partial Fraction Decomposition **In Exercises 1–4, write the form of the partial fraction decomposition of the rational expression. Do not solve for the constants.**

1. $\dfrac{4}{x^2 - 8x}$

2. $\dfrac{2x^2 + 1}{(x - 3)^3}$

3. $\dfrac{2x - 3}{x^3 + 10x}$

4. $\dfrac{2x - 1}{x(x^2 + 1)^2}$

Using Partial Fractions **In Exercises 5–22, use partial fractions to find the indefinite integral.**

5. $\displaystyle\int \frac{1}{x^2 - 9}\,dx$

6. $\displaystyle\int \frac{2}{9x^2 - 1}\,dx$

7. $\displaystyle\int \frac{5}{x^2 + 3x - 4}\,dx$

8. $\displaystyle\int \frac{3 - x}{3x^2 - 2x - 1}\,dx$

9. $\displaystyle\int \frac{x^2 + 12x + 12}{x^3 - 4x}\,dx$

10. $\displaystyle\int \frac{x^3 - x + 3}{x^2 + x - 2}\,dx$

11. $\displaystyle\int \frac{2x^3 - 4x^2 - 15x + 5}{x^2 - 2x - 8}\,dx$

12. $\displaystyle\int \frac{x + 2}{x^2 + 5x}\,dx$

13. $\displaystyle\int \frac{4x^2 + 2x - 1}{x^3 + x^2}\,dx$

14. $\displaystyle\int \frac{5x - 2}{(x - 2)^2}\,dx$

15. $\displaystyle\int \frac{x^2 + 3x - 4}{x^3 - 4x^2 + 4x}\,dx$

16. $\displaystyle\int \frac{8x}{x^3 + x^2 - x - 1}\,dx$

17. $\displaystyle\int \frac{x^2 - 1}{x^3 + x}\,dx$

18. $\displaystyle\int \frac{6x}{x^3 - 8}\,dx$

19. $\displaystyle\int \frac{x^2}{x^4 - 2x^2 - 8}\,dx$

20. $\displaystyle\int \frac{x}{16x^4 - 1}\,dx$

21. $\displaystyle\int \frac{x^2 + 5}{x^3 - x^2 + x + 3}\,dx$

22. $\displaystyle\int \frac{x^2 + 6x + 4}{x^4 + 8x^2 + 16}\,dx$

Evaluating a Definite Integral **In Exercises 23–26, evaluate the definite integral. Use a graphing utility to verify your result.**

23. $\displaystyle\int_0^2 \frac{3}{4x^2 + 5x + 1}\,dx$

24. $\displaystyle\int_1^5 \frac{x - 1}{x^2(x + 1)}\,dx$

25. $\displaystyle\int_1^2 \frac{x + 1}{x(x^2 + 1)}\,dx$

26. $\displaystyle\int_0^1 \frac{x^2 - x}{x^2 + x + 1}\,dx$

Finding an Indefinite Integral **In Exercises 27–34, use substitution and partial fractions to find the indefinite integral.**

27. $\displaystyle\int \frac{\sin x}{\cos x + \cos^2 x}\,dx$

28. $\displaystyle\int \frac{5\cos x}{\sin^2 x + 3\sin x - 4}\,dx$

29. $\displaystyle\int \frac{\sec^2 x}{\tan^2 x + 5\tan x + 6}\,dx$

30. $\displaystyle\int \frac{\sec^2 x}{\tan x(\tan x + 1)}\,dx$

31. $\displaystyle\int \frac{e^x}{(e^x - 1)(e^x + 4)}\,dx$

32. $\displaystyle\int \frac{e^x}{(e^{2x} + 1)(e^x - 1)}\,dx$

33. $\displaystyle\int \frac{\sqrt{x}}{x - 4}\,dx$

34. $\displaystyle\int \frac{1}{\sqrt{x} - \sqrt[3]{x}}\,dx$

Verifying a Formula **In Exercises 35–38, use the method of partial fractions to verify the integration formula.**

35. $\displaystyle\int \frac{1}{x(a + bx)}\,dx = \frac{1}{a}\ln\left|\frac{x}{a + bx}\right| + C$

36. $\displaystyle\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a + x}{a - x}\right| + C$

37. $\displaystyle\int \frac{x}{(a + bx)^2}\,dx = \frac{1}{b^2}\left(\frac{a}{a + bx} + \ln|a + bx|\right) + C$

38. $\displaystyle\int \frac{1}{x^2(a + bx)}\,dx = -\frac{1}{ax} - \frac{b}{a^2}\ln\left|\frac{x}{a + bx}\right| + C$

WRITING ABOUT CONCEPTS

39. **Using Partial Fractions** What is the first step when integrating $\displaystyle\int \frac{x^3}{x - 5}\,dx$? Explain.

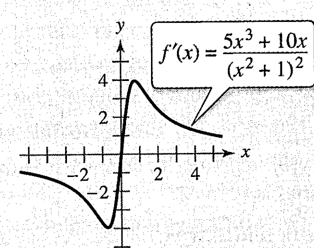
40. **Decomposition** Describe the decomposition of the proper rational function $N(x)/D(x)$ (a) for $D(x) = (px + q)^m$ and (b) for $D(x) = (ax^2 + bx + c)^n$ where $ax^2 + bx + c$ is irreducible. Explain why you chose that method.

41. **Choosing a Method** State the method you would use to evaluate each integral. Explain why you chose that method. Do not integrate.

(a) $\displaystyle\int \frac{x + 1}{x^2 + 2x - 8}\,dx$ (b) $\displaystyle\int \frac{7x + 4}{x^2 + 2x - 8}\,dx$

(c) $\displaystyle\int \frac{4}{x^2 + 2x + 5}\,dx$

42. **HOW DO YOU SEE IT?** Use the graph of f' shown in the figure to answer the following.



(a) Is $f(3) - f(2) > 0$? Explain.

(b) Which is greater, the area under the graph of f' from 1 to 2, or the area under the graph of f' from 3 to 4?

43. **Area** Find the area of the region bounded by the graphs of $y = 12/(x^2 + 5x + 6)$, $y = 0$, $x = 0$, and $x = 1$.

44. **Area** Find the area of the region bounded by the graphs of $y = 7/(16 - x^2)$ and $y = 1$.

45. Modeling Data The predicted cost C (in hundreds of thousands of dollars) for a company to remove $p\%$ of a chemical from its waste water is shown in the table.

P	0	10	20	30	40
C	0	0.7	1.0	1.3	1.7

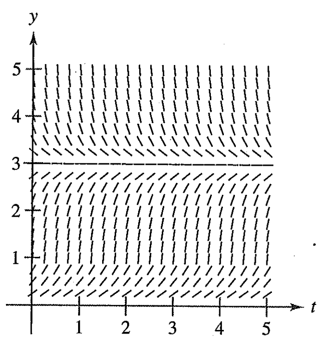
P	50	60	70	80	90
C	2.0	2.7	3.6	5.5	11.2

A model for the data is given by $C = \dfrac{124p}{(10 + p)(100 - p)}$ for $0 \le p < 100$. Use the model to find the average cost of removing between 75% and 80% of the chemical.

46. Logistic Growth In Chapter 6, the exponential growth equation was derived from the assumption that the rate of growth was proportional to the existing quantity. In practice, there often exists some upper limit L past which growth cannot occur. In such cases, you assume the rate of growth to be proportional not only to the existing quantity, but also to the difference between the existing quantity y and the upper limit L. That is, $dy/dt = ky(L - y)$. In integral form, you can write this relationship as

$$\int \frac{dy}{y(L - y)} = \int k\, dt.$$

(a) A slope field for the differential equation $dy/dt = y(3 - y)$ is shown. Draw a possible solution to the differential equation when $y(0) = 5$, and another when $y(0) = \frac{1}{2}$. To print an enlarged copy of the graph, go to *MathGraphs.com.*



(b) Where $y(0)$ is greater than 3, what is the sign of the slope of the solution?

(c) For $y > 0$, find $\lim\limits_{t\to\infty} y(t)$.

(d) Evaluate the two given integrals and solve for y as a function of t, where y_0 is the initial quantity.

(e) Use the result of part (d) to find and graph the solutions in part (a). Use a graphing utility to graph the solutions and compare the results with the solutions in part (a).

(f) The graph of the function y is a **logistic curve.** Show that the rate of growth is maximum at the point of inflection, and that this occurs when $y = L/2$.

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47. Volume and Centroid Consider the region bounded by the graphs of $y = 2x/(x^2 + 1)$, $y = 0$, $x = 0$, and $x = 3$. Find the volume of the solid generated by revolving the region about the x-axis. Find the centroid of the region.

48. Volume Consider the region bounded by the graph of

$$y^2 = \frac{(2 - x)^2}{(1 + x)^2}$$

on the interval $[0, 1]$. Find the volume of the solid generated by revolving this region about the x-axis.

49. Epidemic Model A single infected individual enters a community of n susceptible individuals. Let x be the number of newly infected individuals at time t. The common epidemic model assumes that the disease spreads at a rate proportional to the product of the total number infected and the number not yet infected. So, $dx/dt = k(x + 1)(n - x)$ and you obtain

$$\int \frac{1}{(x + 1)(n - x)}\, dx = \int k\, dt.$$

Solve for x as a function of t.

50. Chemical Reaction

In a chemical reaction, one unit of compound Y and one unit of compound Z are converted into a single unit of compound X. Let x be the amount of compound X formed. The rate of formation of X is proportional to the product of the amounts of unconverted compounds Y and Z. So, $dx/dt = k(y_0 - x)(z_0 - x)$, where y_0 and z_0 are the initial amounts of compounds Y and Z. From this equation, you obtain

$$\int \frac{1}{(y_0 - x)(z_0 - x)}\, dx = \int k\, dt.$$

(a) Perform the two integrations and solve for x in terms of t.

(b) Use the result of part (a) to find x as $t \to \infty$ for (1) $y_0 < z_0$, (2) $y_0 > z_0$, and (3) $y_0 = z_0$.

51. Using Two Methods Evaluate

$$\int_0^1 \frac{x}{1 + x^4}\, dx$$

in two different ways, one of which is partial fractions.

PUTNAM EXAM CHALLENGE

52. Prove $\dfrac{22}{7} - \pi = \displaystyle\int_0^1 \frac{x^4(1 - x)^4}{1 + x^2}\, dx.$

This problem was composed by the Committee on the Putnam Prize Competition.