# 8.6 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

**Integration by Tables** In Exercises 1 and 2, use a table of integrals with forms involving $a + bu$ to find the indefinite integral.

1. $\int \frac{x^2}{5+x}\,dx$
2. $\int \frac{2}{x^2(4+3x)^2}\,dx$

**Integration by Tables** In Exercises 3 and 4, use a table of integrals with forms involving $\sqrt{a^2-u^2}$ to find the indefinite integral.

3. $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$
4. $\int \frac{\sqrt{64-x^4}}{x}\,dx$

**Integration by Tables** In Exercises 5–8, use a table of integrals with forms involving the trigonometric functions to find the indefinite integral.

5. $\int \cos^4 3x\,dx$
6. $\int \frac{\sin^4\sqrt{x}}{\sqrt{x}}\,dx$
7. $\int \frac{1}{\sqrt{x}(1-\cos\sqrt{x})}\,dx$
8. $\int \frac{1}{1+\cot 4x}\,dx$

**Integration by Tables** In Exercises 9 and 10, use a table of integrals with forms involving $e^u$ to find the indefinite integral.

9. $\int \frac{1}{1+e^{2x}}\,dx$
10. $\int e^{-4x}\sin 3x\,dx$

**Integration by Tables** In Exercises 11 and 12, use a table of integrals with forms involving $\ln u$ to find the indefinite integral.

11. $\int x^7 \ln x\,dx$
12. $\int (\ln x)^3\,dx$

**Using Two Methods** In Exercises 13–16, find the indefinite integral (a) using integration tables and (b) using the given method.

| Integral | Method |
|---|---|
| 13. $\int x^2e^{3x}\,dx$ | Integration by parts |
| 14. $\int x^5 \ln x\,dx$ | Integration by parts |
| 15. $\int \frac{1}{x^2(x+1)}\,dx$ | Partial fractions |
| 16. $\int \frac{1}{x^2-36}\,dx$ | Partial fractions |

**Finding an Indefinite Integral** In Exercises 17–38, use integration tables to find the indefinite integral.

17. $\int x\,\text{arccsc}(x^2+1)\,dx$
18. $\int \arcsin 4x\,dx$
19. $\int \frac{1}{x^2\sqrt{x^2-4}}\,dx$
20. $\int \frac{1}{x^2+4x+8}\,dx$
21. $\int \frac{4x}{(2-5x)^2}\,dx$
22. $\int \frac{\theta^3}{1+\sin\theta^4}\,d\theta$
23. $\int e^x \arccos e^x\,dx$
24. $\int \frac{e^x}{1-\tan e^x}\,dx$
25. $\int \frac{x}{1-\sec x^2}\,dx$
26. $\int \frac{1}{t[1+(\ln t)^2]}\,dt$
27. $\int \frac{\cos\theta}{3+2\sin\theta+\sin^2\theta}\,d\theta$
28. $\int x^2\sqrt{2+9x^2}\,dx$
29. $\int \frac{1}{x^2\sqrt{2+9x^2}}\,dx$
30. $\int \sqrt{x}\arctan x^{3/2}\,dx$
31. $\int \frac{\ln x}{x(3+2\ln x)}\,dx$
32. $\int \frac{e^x}{(1-e^{2x})^{3/2}}\,dx$
33. $\int \frac{x}{(x^2-6x+10)^2}\,dx$
34. $\int \sqrt{\frac{5-x}{5+x}}\,dx$
35. $\int \frac{x}{\sqrt{x^4-6x^2+5}}\,dx$
36. $\int \frac{\cos x}{\sqrt{\sin^2 x+1}}\,dx$
37. $\int \frac{e^{3x}}{(1+e^x)^3}\,dx$
38. $\int \cot^4\theta\,d\theta$

**Evaluating a Definite Integral** In Exercises 39–46, use integration tables to evaluate the definite integral.

39. $\int_0^1 xe^{x^2}\,dx$
40. $\int_0^4 \frac{x}{\sqrt{3+2x}}\,dx$
41. $\int_1^2 x^4 \ln x\,dx$
42. $\int_0^{\pi/2} x\sin 2x\,dx$
43. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$
44. $\int_0^5 \frac{x^2}{(5+2x)^2}\,dx$
45. $\int_0^{\pi/2} t^3\cos t\,dt$
46. $\int_0^3 \sqrt{x^2+16}\,dx$

**Verifying a Formula** In Exercises 47–52, verify the integration formula.

47. $\int \frac{u^2}{(a+bu)^2}\,du = \frac{1}{b^3}\left(bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$
48. $\int \frac{u^n}{\sqrt{a+bu}}\,du = \frac{2}{(2n+1)b}\left(u^n\sqrt{a+bu} - na\int \frac{u^{n-1}}{\sqrt{a+bu}}\,du\right)$
49. $\int \frac{1}{(u^2\pm a^2)^{3/2}}\,du = \frac{\pm u}{a^2\sqrt{u^2\pm a^2}} + C$
50. $\int u^n\cos u\,du = u^n\sin u - n\int u^{n-1}\sin u\,du$
51. $\int \arctan u\,du = u\arctan u - \ln\sqrt{1+u^2} + C$
52. $\int (\ln u)^n\,du = u(\ln u)^n - n\int (\ln u)^{n-1}\,du$

**Finding or Evaluating an Integral** **In Exercises 53–60, find or evaluate the integral.**

**53.** $\int \frac{1}{2 - 3\sin\theta}\,d\theta$

**54.** $\int \frac{\sin\theta}{1 + \cos^2\theta}\,d\theta$

**55.** $\int_0^{\pi/2} \frac{1}{1 + \sin\theta + \cos\theta}\,d\theta$

**56.** $\int_0^{\pi/2} \frac{1}{3 - 2\cos\theta}\,d\theta$

**57.** $\int \frac{\sin\theta}{3 - 2\cos\theta}\,d\theta$

**58.** $\int \frac{\cos\theta}{1 + \cos\theta}\,d\theta$

**59.** $\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta}}\,d\theta$

**60.** $\int \frac{4}{\csc\theta - \cot\theta}\,d\theta$

**Area** **In Exercises 61 and 62, find the area of the region bounded by the graphs of the equations.**

**61.** $y = \frac{x}{\sqrt{x+3}}, y = 0, x = 6$

**62.** $y = \frac{x}{1 + e^{x^2}}, y = 0, x = 2$

## WRITING ABOUT CONCEPTS

**63. Finding a Pattern**

(a) Evaluate $\int x^n \ln x\,dx$ for $n = 1, 2$, and 3. Describe any patterns you notice.

(b) Write a general rule for evaluating the integral in part (a), for an integer $n \geq 1$.

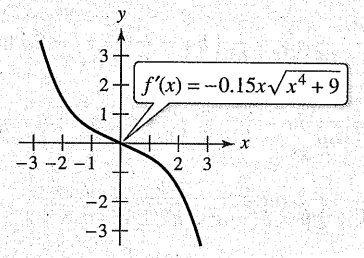
**64. Reduction Formula** Describe what is meant by a reduction formula. Give an example.

**65. Choosing a Method** State (if possible) the method or integration formula you would use to find the antiderivative. Explain why you chose that method or formula. Do not integrate.

(a) $\int \frac{e^x}{e^{2x} + 1}\,dx$ (b) $\int \frac{e^x}{e^x + 1}\,dx$ (c) $\int xe^{x^2}\,dx$

(d) $\int xe^x\,dx$ (e) $\int e^{x^2}\,dx$ (f) $\int e^{2x}\sqrt{e^{2x} + 1}\,dx$

**66. HOW DO YOU SEE IT?** Use the graph of $f'$ shown in the figure to answer the following.




(a) Approximate the slope of $f$ at $x = -1$. Explain.

(b) Approximate any open intervals in which the graph of $f$ is increasing and any open intervals in which it is decreasing. Explain.

**True or False?** **In Exercises 67 and 68, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

**67.** To use a table of integrals, the integral you are evaluating must appear in the table.

**68.** When using a table of integrals, you may have to make substitutions to rewrite your integral in the form in which it appears in the table.

**69. Work** A hydraulic cylinder on an industrial machine pushes a steel block a distance of $x$ feet ($0 \leq x \leq 5$), where the variable force required is $F(x) = 2000xe^{-x}$ pounds. Find the work done in pushing the block the full 5 feet through the machine.

**70. Work** Repeat Exercise 69, using $F(x) = \frac{500x}{\sqrt{26 - x^2}}$ pounds.

**71. Volume** Consider the region bounded by the graphs of

$$y = x\sqrt{16 - x^2}, \quad y = 0, \quad x = 0, \quad \text{and} \quad x = 4.$$

Find the volume of the solid generated by revolving the region about the $y$-axis.

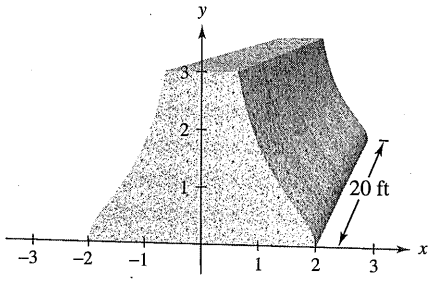
**72. Building Design** The cross section of a precast concrete beam for a building is bounded by the graphs of the equations

$$x = \frac{2}{\sqrt{1 + y^2}}, \quad x = \frac{-2}{\sqrt{1 + y^2}}, \quad y = 0, \quad \text{and} \quad y = 3$$

where $x$ and $y$ are measured in feet. The length of the beam is 20 feet (see figure).

(a) Find the volume $V$ and the weight $W$ of the beam. Assume the concrete weighs 148 pounds per cubic foot.

(b) Find the centroid of a cross section of the beam.




**73. Population** A population is growing according to the logistic model

$$N = \frac{5000}{1 + e^{4.8 - 1.9t}}$$

where $t$ is the time in days. Find the average population over the interval $[0, 2]$.

## PUTNAM EXAM CHALLENGE

**74.** Evaluate $\int_0^{\pi/2} \frac{dx}{1 + (\tan x)^{\sqrt{2}}}$.

This problem was composed by the Committee on the Putnam Prize Competition.