p.574 7-55 D152 $\boxed{8.7}$

10) a) 0 b) 0

16) 1

22) $\frac{1}{2}$

28) 0

34) 0

40) a) 0 b) 1

46) a) ∞^0 b) 1

52) a) $\infty - \infty$ b) $-1/8$

46) $\lim_{x\to\infty}(1+x)^{1/x} = \infty^0$

$y = \lim_{x\to\infty}(1+x)^{1/x}$

$\ln y = \ln(1+x)^{1/x}$

$= \frac{1}{x}\ln(1+x)$

$\ln y = \lim_{x\to\infty}\frac{\ln(1+x)}{x} \to \frac{\frac{1}{1+x}}{1}$

$\ln y = \lim_{x\to\infty}\frac{1}{1+x}$

$e^{\ln y} = e^0$

$y = e^0$

$\boxed{y=1}$

40) $\lim_{x\to\infty} x\tan(1/x) = \infty \cdot 0$

$\lim_{x\to\infty}\frac{\tan(1/x)}{(1/x)} = \frac{0}{0} \xrightarrow{L'H}$

$\lim_{x\to\infty}\frac{\sec^2(1/x)\cdot \cancel{\frac{1}{x^2}}}{\cancel{\frac{1}{x^2}}} = \sec^2(0) = \boxed{1}$

52) $\lim_{x\to 2^+}\frac{1}{x^2-4} - \frac{\sqrt{x-1}}{x^2-4} = \infty - \infty$

$\lim_{x\to 2^+}\frac{1-\sqrt{x-1}}{x^2-4} \xrightarrow{L'H} \frac{\frac{-1}{2\sqrt{x-1}}}{2x} = \frac{-1}{8}$

$\boxed{\text{or}}$

$\lim_{x\to 2^+}\frac{1-\sqrt{x-1}}{x^2-4}\cdot\frac{1+\sqrt{x-1}}{1+\sqrt{x-1}}$

$\frac{1-(x-1)}{x^2-4\cdot 1+\sqrt{x-1}} = \frac{1-x+1}{} = \frac{(\cancel{2-x})^{-1}}{(\cancel{x-2})(x+2)\cdot(1+\sqrt{x-1})}$

$\lim_{x\to 2^+}\frac{-1}{(x+2)(1+1)} = \frac{-1}{4(2)} = \boxed{-1/8}$

$\boxed{8.8}$ p.585 20-46 D153

20) 4

* 24) $\frac{b}{a^2+b^2}$

28) $\infty - \frac{1}{2}$, diverges

32) Diverges since $\cos x/2$ does not approach a limit.

36) $8\sqrt{6}$

40) Diverges

44) $\infty - 0$ diverges

$\boxed{8.8}$

20) $\int_0^{\infty} xe^{-x/2}\,dx = \lim_{b\to\infty}\int_0^b xe^{-x/2}\,dx$

	u	dv
+	x	$e^{-x/2}$
−	1	$-2e^{-x/2}$
+	0	$4e^{-x/2}$

$\lim_{b\to\infty} -2xe^{-x/2} - 4e^{-x/2}\Big]_0^b$

$\lim_{b\to\infty} \frac{-2x}{e^{x/2}} - \frac{4}{e^{x/2}}\Big]_0^b = \big((0-0)-(0-4)\big) = \boxed{4}$

24) $\int_0^{\infty} e^{-ax}\sin bx\,dx$

28) $\int_0^{\infty} \frac{x^3}{(x^2+1)^2}\,dx = \lim_{b\to\infty}\int_0^b \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2} = \int_0^b \frac{x}{x^2+1} - \frac{x}{(x^2+1)^2}\,dx$

$x^3 = (Ax+B)(x^2+1) + Cx + D$

$x^3 = Ax^3 + Bx^2 + Ax + B + Cx + D$

$x^3 = Ax^3$, $\boxed{A=1}$

$0x^2 = Bx^2$ $\boxed{B=0}$

$0 = Ax + Cx$ $\boxed{C=-1}$

$0 = B + D$, $\boxed{D=0}$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\frac{x}{u^2}\cdot\frac{du}{2x}$

$\frac{1}{2}\int \frac{1}{u}\,du - \frac{1}{2}\int u^{-2}\,du$

$\frac{1}{2}\ln|x^2+1| - \frac{1}{2}\frac{u^{-1}}{-1}$

$\lim_{b\to\infty} \frac{1}{2}\ln|x^2+1| + \frac{1}{2(x^2+1)}\Big]_0^b$

$\infty + 0 - (0 + \tfrac{1}{2})$

$\infty - \tfrac{1}{2} \rightarrow \boxed{\text{Diverges}}$

32) $\int_0^{\infty} \sin\left(\frac{x}{2}\right) dx = \lim_{b \to \infty} -2\cos\left(\frac{x}{2}\right)\Big]_0^b$ $\boxed{\text{diverges}}$

36) $\int_0^6 \frac{4}{\sqrt{6-x}} dx \quad \lim_{b \to 6^-} \int_0^b \frac{4}{\sqrt{6-x}} dx$

$u = 6-x$

$\frac{du}{dx} = -1$

$dx = -du$

$\frac{4}{u^{1/2}} \cdot -du = -4u^{-1/2} = \frac{-4u^{1/2}}{1/2} = -8\left(\sqrt{6-x}\right)\Big]_0^b$

$\lim_{b \to 6^-} -8\sqrt{6-x}\Big]_0^b = 0 - \left(8\sqrt{6}\right) = \boxed{8\sqrt{6}}$

40) $\int_0^{\pi/2} \sec\theta \, d\theta = \lim_{b \to \pi/2} \int_0^b \sec\theta \, d\theta = \ln\left|\sec\theta + \tan\theta\right|\Big]_0^b$ $\sec\left(\frac{\pi}{2}\right)$ at an asymptote

$= \boxed{\infty}$

44) $\int_0^2 \frac{1}{4-x^2} dx \quad \frac{1}{(2+x)(2-x)} = \frac{A}{2+x} + \frac{B}{2-x}$

$A = 1/4 \qquad B = 1/4$

$x = -2 \qquad x = 2$

$= \lim_{b \to 2^-} \int_0^b \frac{1}{4}\left(\frac{1}{2+x}\right) + \frac{1}{4}\left(\frac{1}{2-x}\right) dx$

$\frac{1}{4}\ln|2+x| - \frac{1}{4}\ln|2-x|$

$= \frac{1}{4}\ln\left|\frac{2+x}{2-x}\right|\Big]_0^b$

$\frac{1}{4}\ln\left|\frac{4}{0}\right| - \frac{1}{4}\ln|1|$

$\infty - 0 \quad = \boxed{\text{diverges}}$