9.02 Standard Deviation Notes Date ______________

Statisticians use the Standard Deviation to discuss dispersion (spread) of data rather than the Mean Absolute Deviation (MAD).

The average of the squared differences of the mean is the Variance.
The Standard deviation is the average distance from the mean. It tells us how tightly the data values are clustered around the mean.

MORE MEASURES OF DISPERSION (SPREAD)		
Variance	**Standard Deviation for the whole population**	**Standard Deviation for a sample**
$\sigma^2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$	$\sigma = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}}$	$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$

Example 1: Compare the spread of the data for the two sets.

Falcons Football
Points Scored in 2008
Regular Season Games

n=8

Home Games	Away Games
34	9
38	9
22	27
34	14
20	24
45	22
13	25
31	24

a. Calculate the range of each set: Home: 32
$45-13=$

Away: 18
$27-9$

b. The range of the home games is > **the range of the away games.**
Explain what this means: The difference b/t the largest and least points scored is greater in Home games than away games

c. Calculate the Mean of each set:

$\bar{x} = \frac{237}{8} = 29.625$ | $\frac{154}{8} = 19.25$

$\bar{x}$ (Home games) = 29.625

$\bar{x}$ (Away games) = 19.25

d. Calculate the IQR for each set:

Q1 (21) Q3 (36)

$\underbrace{13 \quad 20 \quad 22 \quad 31}$ | $\underbrace{34 \quad 34 \quad 38 \quad 45}$

$\overbrace{9 \quad 9 \quad 14 \quad 22}$ $\overbrace{24 \quad 24 \quad 25 \quad 27}$

Q3-Q1 → 36-21

Home: = 15

Q1=11.5
Q3=24.5

Away: 13

e. The IQR of the home games is > **the IQR of the away games. Explain what this means:**
The spread of the middle half of points scored is greater for Home games than for Away Games.

To calculate the Standard Deviation of a data set, find how far (the difference) each data value ($x_1, x_2, x_3, \ldots, x_n$) is from the mean ($\bar{x}$). These are the deviations from the mean. Square the differences and then find the average by adding them together and dividing by the number of data values for a population (n) or by (n – 1) for a sample. This number is the variance: σ^2. The Standard Deviation, σ, is the square root of the variance.

Ex 1 (cont'd) Falcons Points Scored in 2008 Regular Season Games

Home: $\bar{x}$ = 29.625	Away: $\bar{x}$ = 19.25
$(34 - 29.625)^2$ = 19.141	$(9 - 19.25)^2$ = 105.063
$(38 - 29.625)^2$ = 70.141	$(9 - 19.25)^2$ = 105.063
$(22 - \quad)^2$ = 58.141	$(27 - \quad)^2$ = 60.063
$(34 - \quad)^2$ = 19.141	$(14 - \quad)^2$ = 27.563
$(20 - \quad)^2$ = 92.641	$(24 - \quad)^2$ = 22.563
$(45 - \quad)^2$ = 236.391	$(22 - \quad)^2$ = 7.563
$(13 - \quad)^2$ = 276.391	$(25 - \quad)^2$ = 33.063
$(31 - \quad)^2$ = 1.891	$(24 - \quad)^2$ = 22.563
Sum of Dev²: 773.878	Sum of Dev²: 383.504
To calculate the variance, σ^2, divide the sum of deviation² by n (since this is the whole population and not sample)	
σ^2 = 96.73475	σ^2 = 47.938
To calculate the standard deviation, σ, take the square root of the variance.	
σ = 9.835	σ = 6.924

$\frac{773.878}{8}$

f. σ of home games:

9.835

g. σ of away games:

6.924

σ (home) > σ (away)

h. Explain the difference in σ:

The spread of the data around the mean is greater for Home games than for Away games.

Example 2: Use test scores (from the previous lesson): **90, 89, 78, 81, 68, 100, 84, 83, 83, 74, 88, 80, 73, 89, 32** to calculate the following:

Show calculations for standard deviation here:

$\bar{x}$ = ________ σ = ________

Median = ________ IQR = ________

Which measure of center and spread should be used to describe the data? Justify your response.

9.02 Standard Deviation Homework **Date** ______________

Mrs. Durand has a problem and needs your input. She has to give one math award this year to a deserving student, but she can't make a decision. Here are the test grades for her two best students:

Emily: 90, 90, 80, 100, 99, 81, 98, 82 **Jacob: 90, 90, 91, 89, 91, 89, 90, 90**

1. Which of the two students should get the math award? Discuss why he/she should be the recipient.

Maybe Jacob since he is the more consistent performer

2. Calculate the *mean deviation, variance,* and *standard deviation* of Emily's distribution. The formulas are below. Fill out the table to help calculate by hand.

Mean deviation: $\frac{\sum |x_i - \bar{x}|}{n}$ variance: $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$ standard deviation: $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

$\frac{720}{8} =$

The mean of Emily's test scores: 90

$n = 8$

x_i for Emily	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$	$(x_i - \bar{x})^2$
90	0	0	0
90	0	0	0
80	-10	10	100
100	10	10	100
99	9	9	81
81	-9	9	81
98	8	8	64
82	-8	8	64

a. Mean deviation for Emily: $\frac{54}{8} = 6.75$

b. Variance for Emily: $\frac{490}{8} = 61.25$

c. Standard deviation for Emily: $\sqrt{61.25} = \boxed{7.826}$
(square root of variance)

d. What do these measures of spread tell you?

Emily performs well on tests, but her scores vary from the mean quite a bit.

3. Calculate the *mean deviation, variance,* and *standard deviation* of Jacob's distribution. The formulas are below. Fill out the table to help calculate by hand.

Mean deviation: $\frac{\sum |x_i - \bar{x}|}{n}$ variance: $\sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n}$ standard deviation: $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

The mean of Jacob's test scores: 90

x_i for Jacob	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$	$(x_i - \bar{x})^2$
90	0	0	0
90	0	0	0
91	1	1	1
89	-1	1	1
91	1	1	1
89	-1	1	1
90	0	0	0
90	0	0	0

a. Mean deviation for Jacob: $\frac{4}{8} = 0.5$

b. Variance for Jacob: $\frac{4}{8} = 0.5$

c. Standard deviation for Jacob: $\sqrt{0.5} = \boxed{0.707}$

d. What do these measures of spread tell you?

Jacob is the more consistent low A student.

4. Based on this information about the spread of test scores for Emily and Jacob, which of the two students should get the math award and discuss why he/she should be the one to receive it.

Even though they have the same mean (average), Jacob is more consistent in his performance. (High B/Low A)