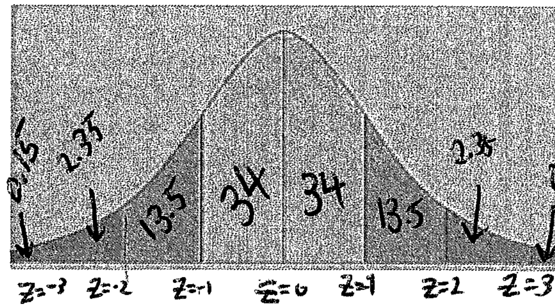
9.06 Standard Normal Distributions Date ______________

What do you do when you are looking for the probability of an x-value in a normal distribution, but that value does not fall on one of the standard deviations?

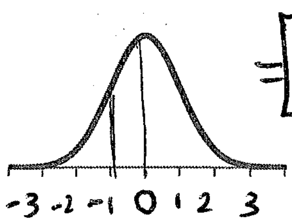
Standard Normal Distribution:



- Formed using a mean of 0 and a standard deviation of 1.
- Used when the x-value does not fall on a standard deviation.
- The Empirical Rule still applies to a standard normal distribution.
- To change an x-value from a normal distribution with mean $\bar{x}$ and standard deviation σ use the z-score formula: $z = \frac{x-\bar{x}}{\sigma}$
- The z-score is the number of standard deviations the x-value lies away from the mean.

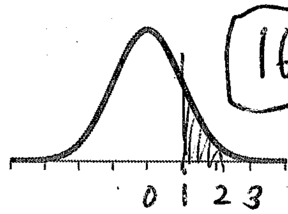
Example 1: For a standard normal distribution, find the following probabilities of a randomly selected x-value from the distribution. If may be helpful to draw a sketch.

a. $P(z \le -1)$



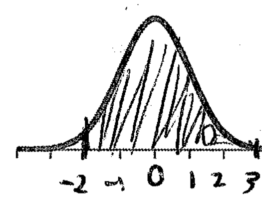
$= \boxed{16\%}$

b. $P(z \ge 1)$



$\boxed{16\%}$

c. $P(-2 \le z \le 3)$



13.5
+34
+34
+13.5
+2.35

$\boxed{97.35\%}$

Converting to a z-score:

Example 2: Consider a normal distribution with a mean of 72 and a standard deviation of 3. Convert the following into z-scores. $\bar{x} = 72$ $\sigma = 3$

a. x = 65

$z = \frac{x-\bar{x}}{\sigma} \rightarrow \frac{65-72}{3} = \boxed{-2.33}$

b. x = 81

$z = \frac{81-72}{3} = \boxed{3.00}$

c. x = 76

$z = \frac{76-72}{3} = \boxed{1.33}$

WHAT'S THE POINT???

Z-scores enable us to find any probability with a table of values. The probability given on the z-score table is the probability that is **less than the z-score**. BE CAREFUL! We are not always looking for less than!!!!

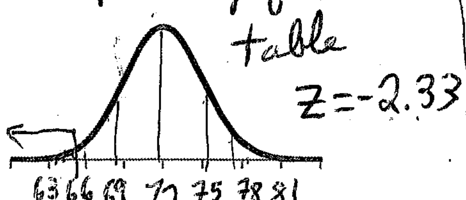
$\bar{x} = 72$ $\sigma = 3$

Example 3: Consider a normal distribution with a mean of 72 and a standard deviation of 3. Sketch a graph and find the following: $\ge$ 1 – probability

* subtract the 2 probabilities from table.

a. $P(x \le 65)$ $\le$ take the probability from the table

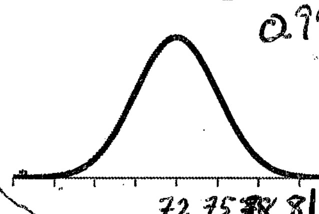
$z = -2.33$



$P(x \le 65) = \boxed{0.0099}$

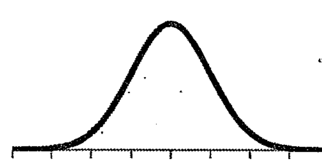
b. $P(x \ge 81)$

z=3
0.9987



$P(x \ge 81) = 1 - 0.9987 = \boxed{0.0013}$

c. $P(76 \le x \le 81)$



0.9987
-0.9082

$\boxed{0.0905}$

9.06 Standard Normal Distribution Homework $\bar{x}=70$ $\sigma=10$ $z=\frac{x-\bar{x}}{\sigma}$

1) A normal distribution has a mean of 70 and a standard deviation of 10. Calculate the z-score and use the z-score table to find the indicated probability.

a. $P(x \le 65)$

$z=\frac{65-70}{10}=-0.5$

$P(x\le 65)=\boxed{0.3085}$

b. $P(x \ge 47)$

$z=\frac{47-70}{10}=-2.3$

$1-P(x\le 47)$

$1-0.0107=\boxed{0.9893}$

c. $P(54 \le x \le 83)$

$z=\frac{54-70}{10}=-1.6$ | $z=\frac{83-70}{10}=1.3$

0.0548 0.9032

$0.9032-0.0548=\boxed{0.8484}$

d. $P(x \le 91)$

$z=\frac{91-70}{10}=2.1$

$P(x\le 91)=\boxed{0.9821}$

e. $P(x \ge 39)$

$z=\frac{39-70}{10}=-3.1$

$1-0.0010=\boxed{0.999}$

f. $P(79 \le x \le 101)$

$z=\frac{79-70}{10}$ $z=\frac{101-70}{10}$

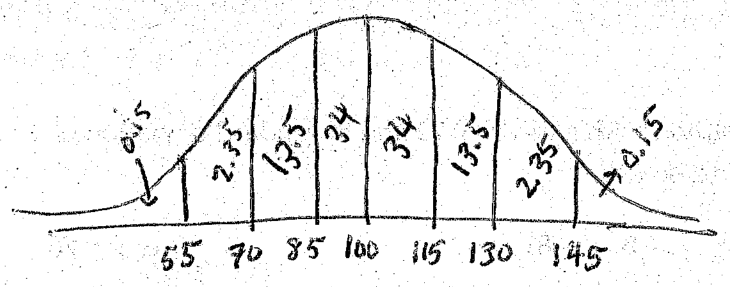
$z=0.9$ $z=3.1$

0.8159 0.9990

$0.9990-0.8159=\boxed{0.1831}$

2) The scores on an Intelligence Quotient (IQ) test are normally distributed with a mean of 100 and a standard deviation of 15.

a. Draw and label the normal distribution.



b. What is the probability that a person will score at least a 121? $P(x\ge 121)$

$z=\frac{121-100}{15}=1.4$

$P(x\ge 121)=1-0.9192=\boxed{0.0808}$

c. What is the probability that a person will score no more than 79? $P(x\le 79)$

$z=\frac{79-100}{15}=-1.4$

$P(x\le 79)=\boxed{0.0808}$

d. What is the probability that a person will score between 92 and 111?

$z_{92}=\frac{92-100}{15}=-0.53\rightarrow 0.2981$

$z_{111}=\frac{111-100}{15}\rightarrow 0.73\rightarrow 0.7673$

$0.7673-0.298$

$=\boxed{0.4692}$

e. What minimum score would someone need to score higher than 80% of those taking an IQ test?

$z=\frac{x-100}{15}$

$\frac{0.84}{1}=\frac{x-100}{15}$

$x-100=15(0.84)$

$x-100=12.6$

$\boxed{x=112.6}$