$*\ z = \frac{x - \bar{x}}{\sigma}$

9.07 Standard Normal Distribution Applications Date ____________

All data in the following exercises is normally distributed. $\bar{x} = 125$ $\sigma = 15$

1. Jack took a standardized achievement test with a mean of 125 and a standard deviation of 15. Jack's score was 148. $x = 148$
   a. What is the perçentile rank for Jack's score on this test? Percentile rank means that Jack scored as well or better than that percentage of students taking the test.

   $z = \frac{148-125}{15} = 1.53$ $\quad 0.937 \rightarrow \boxed{93.7\%}$

   b. If Jill scored at the 67th percentile, what was her score on the test?

   $67\% \rightarrow z = 0.44$ | $z = \frac{x-\bar{x}}{\sigma}$ | $\frac{0.44}{1} = \frac{x-125}{15}$ | $x - 125 = 15(0.44)$, $x - 125 = 6.6$, $\boxed{x = 131.6}$

2. The average number of absences for 1st graders is 15 with a standard deviation of 6.
   a. What is the probability of a 1st grader having fewer than 6 absences?

   $\bar{x} = 15$, $\sigma = 6$ | $x = 6$ | $z = \frac{x-\bar{x}}{\sigma}$, $z = \frac{6-15}{6}$ | $z = -1.5$ | $\boxed{P(x \le 6) = 0.0668}$

   b. What is the probability of a 1st grader having more than 20 absences?

   $\bar{x} = 15$, $\sigma = 6$, $x = 20$ | $z = \frac{20-15}{6}$, $z = 0.83$ | $P(x \ge 20) = 1 - 0.7967$ | $\boxed{P(x \ge 20) = 0.2033}$

   c. If a student is absent more often than 30.85% of other 1st graders, how many days did she miss?

   $z = -0.5$ | $z = \frac{x-\bar{x}}{\sigma}$ | $\frac{-0.5}{1} = \frac{x-15}{6}$ | $x - 15 = 6(-0.5)$, $\boxed{x = 12 \text{ days}}$

3. On the Scholastic Aptitude Test (SAT), scores have a mean of 500 and a standard deviation of 100.
   a. Mo scored 600 on the math section. What percentile did he achieve?

   $\bar{x} = 500$, $\sigma = 100$ | $x = 600$ | $z = \frac{x-\bar{x}}{\sigma}$ | $z = \frac{600-500}{100} = 1$ | $0.8413 \rightarrow \boxed{84.13\%}$ percentile

   b. Larry scored 750 on the math section. What percentile did he achieve?

   $\bar{x} = 500$, $\sigma = 100$, $x = 750$ | $z = \frac{750-500}{100} = 2.5$ | $0.9938 \rightarrow \boxed{99.38\%}$ percentile

   c. Curley scored better than 97.72% of students on the math section of the SAT. What was his score?

   $0.9772 \rightarrow z = 2.0$ | $z = \frac{x-\bar{x}}{\sigma}$, $2 = \frac{x-500}{100}$ | $200 = x - 500$, $700 = x$, $\boxed{x = 700}$

$\bar{x}=52 \quad \sigma=5 \quad x=45$

4. A patient recently diagnosed with Alzheimer's disease took a cognitive abilities test. The mean score on this test is 52 with a standard deviation of 5.
   a. If the patient scored a 45 on the test, what is his percentile rank?

$z=\frac{x-\bar{x}}{\sigma} \rightarrow \frac{45-52}{5}=-1.4$ | $P(z\leq -1.4)=0.0808 \rightarrow \boxed{8.08\% \text{ percentile}}$

   b. If a patient scored higher than 87.7% of other Alzheimer's patients, what was her score on the test?

0.877, $z=1.16$ | $z=\frac{x-\bar{x}}{\sigma}$ | $1.16=\frac{x-52}{5}$ | $x-52=5(1.16)$, $x=5(1.16)+52$ | $\boxed{x=57.8}$

5. Mrs. Durand's Algebra 2 unit test scores have a mean of 82 with a standard deviation of 5.5.
   a. What is the probability that a student will score at least a 92? $1-P(x\leq 92)$

$\bar{x}=82$, $\sigma=5.5$, $x=92$ | $z=\frac{92-82}{5.5}$, $z=1.82 \rightarrow 0.9656$ | $P(x\geq 92)=1-P(x\leq 92)$ $=1-0.9656$ | $P(x\geq 92)=\boxed{0.0344}$

   b. What is the probability a student will get a B?

$P(79.5\leq x\leq 89.4)$ $=0.9115-0.3264$ $=\boxed{0.5851}$ | $z_{79.5}=\frac{79.5-82}{5.5}=-0.45$, 0.3264 | $z_{89.4}=\frac{89.4-82}{5.5}=1.35$, 0.9115

   c. What is the probability a student will fail?

$P(x\leq 69.4)$ | $z=\frac{69.4-82}{5.5}$, $z=-2.29$ | $P(x\leq 69.4)=P(z\leq -2.29)=\boxed{0.0110}$

6. The diameter of maple trees in a Canadian forest have a mean of 10 inches and standard deviation of 3.2 inches. What is the percentage of trees with a diameter between 8 and 15 inches?

$\bar{x}=10$, $\sigma=3.2$ | $P(8\leq x\leq 15)$ $0.9406-0.2643$ $\boxed{0.6763}$ | $z_8=\frac{8-10}{3.2}=-0.63$, 0.2643 | $z_{15}=\frac{15-10}{3.2}=1.56$, 0.9406

7. The weights of 1800 fish in a lake have a mean of 3 kilograms and standard deviation of .5 kilograms.
   a. What percentage of fish weigh at least 3.75 kilograms? (3.75 kg or more) $\bar{x}=3 \quad \sigma=0.5$

$P(x\geq 3.75)=1-P(x\leq 3.75)$

$z=\frac{3.75-3}{0.5}=1.5 \rightarrow 0.9332$ | $1-0.9332$, $P(x\geq 3.75)=0.0668 \rightarrow \boxed{6.68\%}$

   b. Approximately how many fish in the lake weigh at least 3.75 kilograms?

$0.0668 \times 1800 = \boxed{120 \text{ fish}}$