Key

9.09 Confidence Intervals **Date:** _____

Opener: We plan to meet Saturday morning for a fun day at Six Flags. If I tell you that I will be there at 10:30, what time do you expect me to arrive? 10:30 Would any other times also be reasonable? If so, what are they? 10:25, 10:35, ..10:15 10:30 ± ____ minutes

Would you be more confident that I will arrive "on time" if you make my window of arrival times wider or narrower? Why?

Because there are more options for the reasonable arrival times.

Population vs. Sample:

Population: includes all elements of a set of data
example: all U.S. adults

Sample: includes a portion of a set of data
example: 3500 adults called randomly

Parameter: a number relating to the population
example: N = 252,063,500 US adults
p = 30.11% US adults have college degree

Statistic: a number relating to the sample
example: n = 3500
sample percentage $\hat{p}$ = 28% of adults called have college degree.

Identify the population, sample, and statistic for each of the following scenarios:

A survey of 1300 American households found that 32% of those households have basements.

Population: All American households Sample: 1300 American households surveyed Statistic: $\hat{p}$ = 0.32 have basements.

The average bill from every 6th person getting food at Chipotle in a 3-hour period was $19.61.

Population: all chipotle customers Sample: Every 6th customer in the 3 hr period Statistic: $\bar{x}$ = 19.61 avg. meal cost.

Confidence Intervals are intervals of plausible values for estimating a parameter, with a given percent confidence. We use a sample mean to estimate the population mean. We use a sample proportion to estimate a population proportion.

Consider this: The Milton Parks and Recreation Department wants to build a new park in Crabapple. To allocate funds to build the park, they need to determine if residents in the area want one. They mail a survey to residents within 1 mile of the proposed location and find that 78% of residents who responded are in favor of building the new park. They'll find confidence intervals to project what all residents in the area may think of the new park.

Confidence Interval for Proportion:	**Confidence Interval for Mean:**
$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$
$\hat{p}$ = sample proportion in decimal	$\bar{x}$ = sample mean
z = z-score for probability from $\frac{1-c\%}{2}$	σ = population standard deviation
n = sample size	z = z-score for probability of $\frac{1-c\%}{2}$
Margin of error = $z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ → doubling makes Interval width	n = sample size
p = true population proportion	Margin of error = $z\frac{\sigma}{\sqrt{n}}$
	μ = true population mean

$$\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Examples:

$n = 1150$ $\quad \hat{p} = 0.846$

1. A survey of 1150 people found that 84.6% of respondents believed a toilet paper roll should roll over (not under). Construct the following confidence intervals for the proportion of people whose toilet paper rolls over and state the margin of error for each.

$\frac{1-c\%}{2}$

90%

$\frac{1-0.90}{2} = 0.05 \rightarrow z = -1.65$

$CI = 0.846 \pm 1.65\sqrt{\frac{0.846(1-0.846)}{1150}}$

$CI = 0.846 \pm 0.018$ ← margin of error (M.E.)

$CI = (0.828, 0.864)$

95%

$\frac{1-0.95}{2} \rightarrow 0.025$

$z = -1.96$

$CI = 0.846 \pm 1.96\sqrt{\frac{0.846(1-0.846)}{1150}}$

$= 0.846 \pm 0.021$ (M.E.)

$CI = (0.825, 0.867)$

2. In a sample of 2500 people, 770 people separate their Skittle's by color before eating them. Construct an 85% confidence interval for the proportion of people who "taste the rainbow" with colors separated.

$n = 2500$

$\hat{p} = \frac{770}{2500} = 0.308$

$\frac{1-0.85}{2} = 0.075$

$z = -1.44$

$CI = 0.308 \pm 1.44\sqrt{\frac{0.308(1-0.308)}{2500}}$

$CI = 0.308 \pm 0.01329$

$(0.295, 0.321)$

3. A recent survey of 133 Milton students found their average daily screen time is 5.402 hours. If the population standard deviation is 1.565 hours, construct the following confidence intervals for the average daily screen time for all Milton students and state the margin of error for each.

$\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$

$n = 133$

$\bar{x} = 5.402$

$\sigma = 1.565$

80%

$\frac{1-0.8}{2} = 0.1$ $\quad z = -1.28$

$CI = 5.402 \pm 1.28\left(\frac{1.565}{\sqrt{133}}\right)$

$CI = 5.402 \pm 0.174$

$(5.228, 5.576)$

99%

$\frac{1-0.99}{2} = 0.005$

$z = -2.58$

$CI = 5.402 \pm 2.58\left(\frac{1.565}{\sqrt{133}}\right)$

$CI = 5.402 \pm 0.350$

$(5.052, 5.752)$

4. A recent survey found that Milton students get an average of 6.303 hours of sleep each night. Given the sample size of 540 students and population standard deviation of 0.926 hours, construct an 88% confidence interval for the average amount of sleep by Milton students.

$\bar{x} = 6.303$

$n = 540$

$\sigma = 0.926$

$\frac{1-0.88}{2} = 0.06$

$z = -1.56$

$CI = 6.303 \pm 1.56\left(\frac{0.926}{\sqrt{540}}\right)$

$= 6.303 \pm 0.0621$

$(6.241, 6.365)$