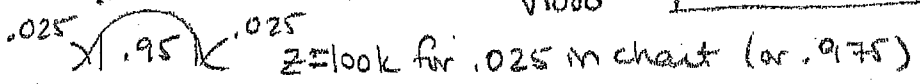
9.09 Homework: Key

1. High school students who take the SAT Mathematics exam a second time generally score higher than on their first try. The change in score has a normal distribution with standard deviation $\sigma = 50$. A random sample of 1000 students gain an average of 22 points on their second try. ($\bar{x}$)

a) Construct a 95% confidence interval for the mean score gain μ in the population.

$95\%\ CI = \bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 22 \pm (1.96)\frac{50}{\sqrt{1000}} = (18.901, 25.099)$



z = look for .025 in chart (or .975)

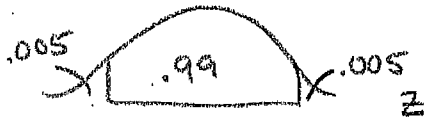
b) Construct a 90% confidence interval for μ.

$90\%\ CI = 22 \pm (1.65)\frac{50}{\sqrt{1000}} = (19.391, 24.609)$

z = look for .05 in chart (or .95)

c) Construct a 99% confidence interval for μ.

$99\%\ CI = 22 \pm (2.58)\frac{50}{\sqrt{1000}} = (17.921, 26.079)$



z = look for .005 in chart (or .995)

d) What is the margin of error for each of the confidence intervals calculated above?

in part a) $\pm 1.96\left(\frac{50}{\sqrt{1000}}\right)$ or $22 - 18.901 = \pm 3.099$

in part b) $\pm 1.65 \cdot \frac{50}{\sqrt{1000}}$ or $22 - 19.391 = \pm 2.609$

in part c) $\pm 2.58 \cdot \frac{50}{\sqrt{1000}}$ or $22 - 17.921 = \pm 4.079$

2. The National Survey of Student Engagement found that 87% of students report that their peers at least "sometimes" copy information from the Internet in their reports without citing the source. Assume that the sample size is 400. Construct an 88% confidence interval and find the margin of error.

$\hat{p} = .87$ ← proportion from a sample

$n = 400$

$88\%\ CI = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = .87 \pm 1.55\sqrt{\frac{.87(.13)}{400}} = .87 \pm .026$

z = look for .06 in chart (or .94)

$1.00 - .88 = .12 \div 2$

(.844, .896)

Margin of Error: ±.026

n2

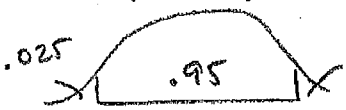
3. A recent survey of 1366 adults found that 1127 of those respondents like hot sauce on their eggs. Construct an 80% confidence interval.

$\hat{p} = \frac{1127}{1366}$ 

z = look for .10 on chart (or .90)

80% CI: $\frac{1127}{1366} \pm 1.28\sqrt{\frac{\frac{1127}{1366}\left(\frac{239}{1366}\right)}{1366}} = .825 \pm .013 = \boxed{(.812, .838)}$

4. A national opinion poll found that 44% of all American adults agree that parents should be given vouchers good for education at any public or private school of their choice. The result was based on a small sample. How large of a random sample is required to obtain a margin of error of 0.03 in a 95% confidence interval?

$ME = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ 

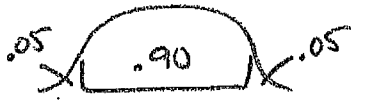
$z = 1.96$

$\frac{.03}{1.96} = \frac{1.96}{1.96}\sqrt{\frac{.44(.56)}{n}} \Rightarrow \left(\frac{.03}{1.96}\right)^2 = \left(\sqrt{\frac{.44(.56)}{n}}\right)^2 \Rightarrow \left(\frac{.03}{1.96}\right)^2 = \frac{.44(.56)}{n}$

$n = \frac{.44(.56)}{\left(\frac{.03}{1.96}\right)^2}$ or $\left(\frac{1.96}{.03}\right)^2(.44)(.56) = 1051.745$

sample 1052 adults

5. A radio talk show invites listeners to enter a dispute about a proposed pay increase for city council members. "What yearly pay do you think council members should get? Call us with your number." In all, 958 people call. The mean pay they suggest is \$8740 per year, and the standard deviation of the responses is \$1125. Calculate a 90% confidence interval for the mean pay μ that all citizens would propose for council members.

$n = 958$ $\bar{x} = \$8740$ $\sigma = \$1125$ 

z @ .05 probability is 1.65

90% CI $= \bar{x} \pm z\frac{\sigma}{\sqrt{n}} = 8740 \pm 1.65\left(\frac{1125}{\sqrt{958}}\right)$

$= 8740 \pm 59.973$

$= \boxed{(\$8680.03, \$8799.97)}$