BC Calculus 9.1-9.3 Review

a) Determine if Sequence converge/diverge b) Determine of the Series converge/diverge/inconclusive (State the appropriate test to justify answer) c) Find the sum of the series if possible (Geometric or Telescoping)

1. $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

2. $\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{2+4n^3-5n}$

3. $\sum_{n=1}^{\infty} \frac{\pi e^2}{n^2}$

4. $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

5. $\sum_{n=1}^{\infty} \frac{(n+2)!}{10n!}$

6. $\sum_{n=1}^{\infty} \frac{1}{n+1} - \frac{1}{n+3}$

7. $\sum_{n=1}^{\infty} n^{-2/3}$

8. $\sum_{n=1}^{\infty} \frac{3n}{2n^2+3}$

9. $\sum_{n=1}^{\infty} 4\left(\frac{2}{7}\right)^n$

10. $\sum_{n=1}^{\infty} \left(\frac{3}{\pi}\right)^{1-n}$

11. $\sum_{n=1}^{\infty} \frac{4}{n^3}$

12. $\sum_{n=1}^{\infty} \frac{5^n}{n^2}$

13. $\sum_{n=1}^{\infty} \frac{3^{1+n}}{2^n}$

14. $\sum_{n=1}^{\infty} \frac{3n^2}{(n+2)(2n+3)}$

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Key

a) Determine if Sequence converge/diverge b) Determine of the Series converge/diverge/inconclusive (State the appropriate test to justify answer) c) Find the sum of the series if possible (Geometric and Telescoping)

1. $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

a) $\lim_{n\to\infty} \frac{\ln n}{n^2} = 0$ sequence converges to zero

b) let $f(x) = \frac{\ln x}{x^2}$ (positive, continuous, decreasing)

$\lim_{b\to\infty} \int_1^b \ln x \cdot x^{-2}\,dx$

IBP $uv - \int v\,du$ | $u = \ln x$ $dv = x^{-2}$ | $du = \frac{1}{x}$ $v = \frac{x^{-1}}{-1}$ | $-\frac{\ln x}{x} - \int -\frac{1}{x^2}dx$

$= -\frac{\ln x}{x} + \int x^{-2}dx = -\frac{\ln x}{x} + \frac{x^{-1}}{-1}$

$\lim_{b\to\infty} \left[-\frac{\ln x}{x} - \frac{1}{x}\right]_1^b = -\frac{\ln b}{b} - \frac{1}{b} - (0-1)$

$= 0 + 1 = \boxed{1}$

Series converge by Integral Test

2. $\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{2+4n^3-5n}$

a) $\lim_{n\to\infty} \frac{1+3n^2+n^3}{2+4n^3-5n} = \frac{1}{4} \neq 0$, sequence converges to $1/4$

b) Series diverge by n^{th} term test

c) cannot determine sum of series

3. $\sum_{n=1}^{\infty} \frac{\pi e^2}{n^2}$

a) $\lim_{n\to\infty} \frac{\pi e^2}{n^2} = 0$ sequence converges to 0. n^{th} term test inconclusive

b) $\pi e^2 \sum \frac{1}{n^2}$

By p-series test, since $p = 2 > 1$, series converge

c) —

4. $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

a) $\lim_{n\to\infty} \frac{n!}{2n!+1} = \frac{1}{2}$ Sequence converges to $1/2$

b) Series diverge by n^{th} term test

c) —

5. $\sum_{n=1}^{\infty} \frac{(n+2)!}{10n!} = \frac{(n+2)(n+1)\cancel{n!}}{10\cancel{n!}}$

a) $\lim_{n\to\infty} \frac{(n+2)(n+1)}{10} = \infty$, sequence diverges.

b) series diverge by n^{th} term test

c) —

6. $\sum_{n=1}^{\infty} \frac{1}{n+1} - \frac{1}{n+3}$

a) sequence converges to 0.

b) $\left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \ldots \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right)$

series converge by telescoping series

c) $\frac{1}{2} + \frac{1}{3} = \boxed{5/6}$

7. $\sum_{n=1}^{\infty} n^{-2/3} = \sum \frac{1}{n^{2/3}}$

a) sequence converges to 0.
∴ n^{th} term test inconclusive

b) By p-series test, $p = 2/3 < 1$, series diverge.

8. $\sum_{n=1}^{\infty} \frac{3n}{2n^2+3}$

a) $\lim_{n\to\infty} \frac{3n}{2n^2+3} = 0$
Sequence converge to 0.

b) $\frac{3n}{2n^2+3}$ is positive, decreasing, continuous

$\int_1^{\infty} \frac{3x}{2x^2+3}\,dx$
$u = 2x^2+3$
$\frac{du}{dx} = 4x$

$\int \frac{3x}{u} \cdot \frac{du}{4x} = \frac{3}{4}\int \frac{1}{u}\,du$

$\lim_{b\to\infty} \frac{3}{4}\ln|2x^2+3|\Big|_1^b = \frac{3}{4}\ln|2b^2+3| - \frac{3}{4}\ln|1|$
$= \infty$

By Integral test, series diverge

9. $\sum_{n=1}^{\infty} 4\left(\frac{2}{7}\right)^n$

a) sequence converges to 0.

b) By Geometric series test, since $r = 2/7 < 1$, series converge

$S = \frac{a_1}{1-r}$ $\frac{8/7}{1-2/7} = \frac{8/7}{5/7} = \boxed{8/5}$

10) $\sum\left(\frac{3}{\pi}\right)^{-n+1} = \frac{3}{\pi}\sum\left(\frac{\pi}{3}\right)^n$ Since $r = \pi/3 > 1$ series Diverge by Geometric series test.

11) $\sum_{n=1}^{\infty} \frac{4}{n^3}$

a) $\lim_{n\to\infty} \frac{4}{n^3} = 0$

b) By p-series test, $n = 3 > 1$, series converge

c) —

12) $\sum_{n=1}^{\infty} \frac{5^n}{n^2}$

a) $\lim_{n\to\infty} \frac{5^n}{n^2} = \infty$
Sequence diverge

b) Series diverge by n^{th} term test

13) $\sum_{n=1}^{\infty} \frac{3^{n+1}}{2^n} = \frac{3^n \cdot 3}{2^n}$

$3\sum\left(\frac{3}{2}\right)^n$

a) sequence diverges

b) Series diverge by n^{th} term test

c) Also, by Geometric Series test, $|r| > 1$, series diverge

14) $\sum_{n=1}^{\infty} \frac{3n^2}{(n+2)(2n+3)}$

a) $\lim_{n\to\infty} \frac{3n^2}{(n+2)(2n+3)} = \frac{3}{2}$

b) Series diverge by n^{th} term test since $\lim_{n\to\infty} a_n \neq 0$.