① $\left\{\frac{\ln n}{n^2}\right\}$ converge or diverge?

$\lim_{n\to\infty} \frac{\ln n}{n^2} = 0$, so $\left\{\frac{\ln n}{n^2}\right\}$ converges to zero

② (a) $2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \frac{6}{25}, \dots = \left\{\frac{n+1}{n^2}\right\}$

$\lim_{n\to\infty} \frac{n+1}{n^2} = 0$, so sequences Converges

(b) $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \dots = \left\{\frac{1}{n!}\right\}$

$\lim_{n\to\infty} \frac{1}{n!} = 0$, so sequence Converges

③ (a) $\sum_{n=1}^{\infty} \frac{1+3n^2+n^3}{4n^3-5n+2}$

$\lim_{n\to\infty} \frac{n^3+3n^2+1}{4n^3-5n+2} = \frac{1}{4} \neq 0$

So series diverges by n^{th} term test

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{1}{n^2} = 0$

So series May or May Not converge (n^{th} term test inconclusive)

(c) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

$\lim_{n\to\infty} \frac{n!}{2\cdot n!+1} = \frac{1}{2} \neq 0$

So series diverges by nth term test

(d) $\sum_{n=1}^{\infty} \frac{(n+2)!}{10n!} = \sum_{n=1}^{\infty} \frac{(n+2)(n+1)\cancel{n!}}{10\cancel{n!}} = \sum_{n=1}^{\infty} \frac{n^2+3n+2}{10}$

$\lim_{n\to\infty} \frac{1}{10}(n^2+3n+2) = \infty$

So series diverges by n^{th} term test

④ $\sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+3}\right)$

(a) Telescoping series

$= \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{6}\right) + \dots$

$= \frac{1}{2} + \frac{1}{3} = \boxed{\frac{5}{6} \approx 0.833}$

(b) $S_{500} = 0.8293532299 < 0.833$

*from calculator

sum(seq(1/(x+1) − 1/(x+3), X, 1, 500)

List/Math/5 List/OPS/5

⑤ (a) $3 + \frac{15}{4} + \frac{75}{16} + \frac{375}{64} + \dots$

$= 3\left(1 + \frac{5}{4} + \frac{25}{16} + \frac{125}{64} + \dots\right)$

$= 3\left(\left(\frac{5}{4}\right)^0 + \left(\frac{5}{4}\right)^1 + \left(\frac{5}{4}\right)^2 + \left(\frac{5}{4}\right)^3 + \dots\right)$

$= 3\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n, \ |r| = \frac{5}{4} > 1$

So series is Divergent Geometric Series

⑤ (b) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \dots$

$= \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots$

$= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n, \ |r| = \frac{1}{2} < 1$

So convergent Geometric Series

Converges to $\frac{1/2}{1-1/2} = \boxed{1}$

5(c) $\sum_{n=1}^{\infty} n^{-2/3} = \sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

$p = \frac{2}{3} < 1$, so series is a divergent p-series

5(d) $\sum_{n=1}^{\infty} \frac{3n}{2n^2+3}$

for all $n > K, \exists K \in \mathbb{Z}^+$, this series is Decreasing, Continuous, and Positive

$3\int_1^{\infty} \frac{n}{2n^2+3}\,dn$

$= \frac{3}{4} \ln|2n^2+3| \Big|_1^{\infty}$

$= \frac{3}{4}\left[\ln(\infty) - \ln 5\right] = \infty \Rightarrow$ Diverges

So the series diverges too!

⑤ (e) $\sum_{n=1}^{\infty} \frac{e^n}{n}$, compare to $\sum_{n=1}^{\infty} \frac{1}{n}$, the divergent harmonic series.

since $\frac{1}{n} \le \frac{e^n}{n}$ $\forall n \ge 1$

$\sum_{n=1}^{\infty} \frac{e^n}{n}$ diverges too!

⑤ (f) $\sum_{n=1}^{\infty} \frac{3^n}{7^n+1}$, compare to $\sum_{n=1}^{\infty} \left(\frac{3}{7}\right)^n = \sum_{n=1}^{\infty} \frac{3^n}{7^n}$, a convergent geometric series.

Since $\frac{3^n}{7^n+1} \le \frac{3^n}{7^n}$ $\forall n \ge 1$,

$\sum_{n=1}^{\infty} \frac{3^n}{7^n+1}$ converges too!

⑤ (g) $\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2}$,

compare to $\sum_{n=1}^{\infty} \frac{1}{n}$, the divergent series

$$\lim_{n\to\infty} \left(\frac{3n+6}{7n^2-5n+1} \cdot \frac{n}{1}\right)$$

same as dividing by $\frac{1}{n}$

$$= \lim_{n\to\infty} \frac{3n^2+6n}{7n^2-5n+1} = \frac{3}{7}$$

where $\frac{3}{7}$ is finite & positive, so

$\sum_{n=1}^{\infty} \frac{3n+6}{1-5n+7n^2}$ diverges too!

⑤ (h) $\sum_{n=1}^{\infty} \frac{n+5}{3n(4^n)}$, compare to $\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$

$= \sum_{n=1}^{\infty} \frac{1}{4^n}$, a convergent geom. series.

$$\lim_{n\to\infty} \left(\frac{n+5}{3n(4^n)}\right)\left(\frac{4^n}{1}\right)$$

$$= \lim_{n\to\infty} \frac{\cancel{(4^n)}(n+5)}{\cancel{(4^n)}(3n)} = \frac{1}{3} > 0$$

So $\sum_{n=1}^{\infty} \frac{n+5}{3n(4^n)}$ converges too!

⑤ (i) $\sum_{n=1}^{\infty} \frac{n^3}{n!}$

same as dividing by $\frac{n^3}{n!}$

$$\lim_{n\to\infty} \left| \frac{(n+1)^3}{(n+1)!} \cdot \frac{n!}{n^3} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)^3 \cdot \cancel{n!}}{n^3 \cdot (n+1) \cdot \cancel{n!}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n^3 + \cdots}{n^4 + n^3} \right| = 0 < 1$$

so $\sum_{n=1}^{\infty} \frac{n^3}{n!}$ converges

⑤ (j) $\sum_{n=1}^{\infty} \frac{2}{n^2}$

$$\lim_{n\to\infty} \left| \frac{2}{(n+1)^2} \cdot \frac{n^2}{2} \right|$$

$$= \lim_{n\to\infty} \left| \frac{2n^2}{2n^2 + \cdots} \right| = 1$$

so Ratio Test is inconclusive

* use Limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$ to show it converges.

5 (k) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

*Alternating Series

$\lim_{n\to\infty} \frac{1}{n} = 0$ and

$\{\frac{1}{n}\}$ is decreasing

so $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ converges

(conditionally convergent)

5 (m) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

compare to $\sum_{n=1}^{\infty} \frac{1}{n^2}$, a convergent p-series

$\frac{\sin n}{n^2} \le \frac{1}{n^2}$ since $\sin n \le 1\ \forall n \in \mathbb{R}$,

so $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ converges too!

6 $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

$S_5 = \frac{1}{1^2} - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25}$

$= 0.8386111111 = \frac{3019}{3600}$

$|R_5| \le |a_6| = \left|\frac{-1}{36}\right| = \frac{1}{36}$ ← Abs. value of 1st unused term

So $S \in \left[\frac{3019}{3600} - \frac{1}{36}, \frac{3019}{3600} + \frac{1}{36}\right]$

or $S \in [0.81083, 0.86638]$

(actual sum ≈ 0.822)
$\quad\quad\quad\quad\quad\quad\quad\; \| $
$\quad\quad\quad\quad\quad\quad\; S_{999}$

5 (l) $\sum_{n=1}^{\infty} \frac{(-1)^n (n+3)}{2n}$

*Alternating Series

$\lim_{n\to\infty} \frac{n+3}{2n} = 1 \neq 0$

So diverges by n^{th} term test

(Alt. series test is inconclusive)

5 (n) $\sum_{n=1}^{\infty} \frac{(-1)^n (4^n)}{n!}$ *converges by A.S.T. since $\frac{4^n}{n!}$ is decreasing $\forall n \ge k, \exists k \in \mathbb{Z}^+$

or by Ratio Test (more fun!)

$\lim_{n\to\infty} \left| \frac{4^{n+1}}{(n+1)!} \cdot \frac{n!}{4^n} \right|$ *don't need alternators on Ratio test because of abs. values.

$\lim_{n\to\infty} \left| \frac{\cancel{4^n} \cdot 4 \cdot \cancel{n!}}{\cancel{4^n} \cdot (n+1) \cdot \cancel{n!}} \right|$

$= \lim_{n\to\infty} \left| \frac{4}{n+1} \right| = 0 < 1$ So converges

7 $\sum_{n=0}^{\infty} \left(\frac{2}{7}\right)^n$, convergent geom series

with $|r| = \left|\frac{2}{7}\right| = \frac{2}{7} < 1$.

Series converges to $\frac{1}{1 - 2/7} = \boxed{\frac{7}{5}}$

8 $\sum_{n=1}^{\infty} \frac{4}{n^3} = 4\left[\sum_{n=1}^{\infty} \frac{1}{n^3}\right]$ → (or Limit Comparison test) convergent p-series with $p = 3 > 1$

9 $\sum_{n=1}^{\infty} \frac{n^2}{5^n}$, $\lim_{n\to\infty} \left| \frac{(n+1)^2}{5^{n+1}} \cdot \frac{5^n}{n^2} \right|$

$= \lim_{n\to\infty} \left| \frac{(n^2+2n+1)\cancel{5^n}}{n^2 \cdot 5 \cdot \cancel{5^n}} \right| = \frac{1}{5} < 1$

So series converges by Ratio Test.

(10) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5+5}}$

Compare with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{5/3}}$, a Convergent p-series:

since $\displaystyle\frac{1}{\sqrt[3]{n^5+3}} \le \frac{1}{n^{5/3}}$ $\forall n \ge 1$,

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^5+5}}$ converges by Direct Comparison

(Limit Comparison works too!)

(12) $\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \cdots$

$= \displaystyle\sum_{n=5}^{\infty} \frac{1}{n}$ which is the divergent harmonic series

(14) $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2-6n+3}{n^3-7n+8}$, compare with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$, the divergent harmonic series

$\displaystyle\lim_{n\to\infty}\left(\frac{5n^2-6n+3}{n^3-7n+8} \cdot \frac{n}{1}\right) = 5 > 0$

So $\displaystyle\sum_{n=1}^{\infty} \frac{5n^2-6n+3}{n^3-7n+8}$ diverges by the Limit Comparison Test.

(16) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n+4}{2^n}$ *Diverges by nth term test OR

compare with $\displaystyle\sum_{n=1}^{\infty}\left(\frac{3}{2}\right)^n = \sum_{n=1}^{\infty} \frac{3^n}{2^n}$, a divergent geom series.

Since $\displaystyle\frac{3^n+4}{2^n} \ge \frac{3^n}{2^n}$, $\displaystyle\sum \frac{3^n+4}{2^n}$

diverges by Direct Comparison Test

(11) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

$\displaystyle\lim_{n\to\infty} \frac{n^n}{n!} = \infty$

So series diverges by n^{th} term test

(13) $2 + \frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \cdots$

$= 2 + \left(\frac{1}{4}\right)^0\left(\frac{1}{2}\right) + \left(\frac{1}{4}\right)^1\left(\frac{1}{2}\right) + \left(\frac{1}{4}\right)^2\left(\frac{1}{2}\right)$

$= 2 + \frac{1}{2}\underbrace{\displaystyle\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n}_{}$ convergent geometric series with $|r| = \left|\frac{1}{4}\right| = \frac{1}{4} < 1$

converges to $2 + \frac{1}{2}\left[\frac{1}{1-\frac{1}{4}}\right] = 2 + \frac{2}{3} = \boxed{\frac{8}{3}}$

(15) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$, $\cos n\pi = -1, 1, -1, 1$ for $n = 1, 2, 3\ldots$

*Alternating series

$\displaystyle\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ and $\frac{1}{\sqrt{n}}$ is decreasing

So $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$ converges by the Alternating Series Test

(17) $\displaystyle\sum_{n=1}^{\infty} \frac{8n^3-6n^5}{12n^4+9n^5}$

$\displaystyle\lim_{n\to\infty} \frac{8n^3-6n^5}{12n^4+9n^5} = -\frac{6}{9} \ne 0$

So series diverges by n^{th} term test

(18) $\sum_{n=1}^{\infty} \sqrt{\frac{3n+1}{n^5+2}}$ compare with $\sum_{n=1}^{\infty} \sqrt{\frac{1}{n^4}} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ a convergent p-series.

$$\lim_{n\to\infty}\left(\sqrt{\frac{3n+1}{n^5+2}} \cdot \sqrt{\frac{n^4}{1}}\right)$$
$$= \sqrt{\lim_{n\to\infty}\left(\frac{3n^5+n^4}{n^5+2}\right)}$$
$$= \sqrt{3} > 0$$

So $\sum_{n=1}^{\infty} \sqrt{\frac{3n+1}{n^5+2}}$ converges too! by Limit Comparison Test

(19) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{3n+4}}$

the series converges by Alt. Series test.

*test for Abs convergence (w/o alternator)

$\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{3n+4}}$ compare with $\sum_{n=1}^{\infty} \frac{1}{n^{1/5}}$, a divergent p series.

$$\lim_{n\to\infty} \frac{\sqrt[5]{n}}{\sqrt[5]{3n+4}} = \sqrt[5]{\lim_{n\to\infty}\left(\frac{n}{3n+4}\right)} = \sqrt[5]{\frac{1}{3}} > 0$$

So $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{3n+4}}$ diverges

So $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{3n+4}}$ converges conditionally

(20) (a) $\sum_{n=0}^{\infty} \frac{3}{2^n}$ (convergent geom series)

$$= 3\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$$
$$S = 3\left[\frac{1}{1-1/2}\right] = \boxed{6}$$

(b) $\sum_{n=2}^{\infty}\left(-\frac{3}{2}\right)^{-n}$

$$= \sum_{n=2}^{\infty}\left(-\frac{2}{3}\right)^n$$ (convergent geom/alt series)

$$S = \frac{4/9}{1-(-2/3)} = \left(\frac{4}{9}\right)\left(\frac{3}{5}\right) = \boxed{\frac{4}{15}}$$

1st term

r

(c) $\sum_{n=1}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+3}\right)$ (telescoping series)

$$= \left(\frac{1}{2} - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \dots$$
$$= \frac{1}{2} + \frac{1}{3} = \boxed{\frac{5}{6}}$$

(d) partial fraction decomp.

$$\sum_{n=1}^{\infty} \frac{3}{(2n-1)(2n+1)} = 3\sum_{n=1}^{\infty}\left(\frac{1/2}{2n-1} - \frac{1/2}{2n+1}\right)$$
$$= \frac{3}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$$
$$= \frac{3}{2}\left[\left(1 - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \left(\cancel{\frac{1}{7}} - \frac{1}{9}\right) + \dots\right.$$
$$= \frac{3}{2}(1) = \boxed{\frac{3}{2}}$$

(21) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$

(a) for Abs convergence, $\sum_{n=1}^{\infty} \left|\frac{(-1)^n}{2^n}\right| = \sum_{n=1}^{\infty} \frac{1}{2^n}$ must converge.

$\sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ which is a convergent geometric series with $|r| = \frac{1}{2} < 1$

so $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n}$ converges absolutely (with or without the help of the alternator)

(b) $S_6 = -\frac{1}{2^1} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \frac{1}{2^5} + \frac{1}{2^6} = -0.328125 = -\frac{21}{64}$

$\approx \boxed{-0.328}$

(c) for error to be less than 0.001, the magnitude of the 1st unused term is the partial sum must be < 0.001

trial & error: $\frac{1}{2^6} = 0.0156 \not< 0.001$, $\frac{1}{2^7} = 0.0078 \not< 0.001$, $\frac{1}{2^8} = 0.0039 \not< 0.001$

$\frac{1}{2^9} = 0.00195 \not< 0.001$, $\boxed{\frac{1}{2^{10}} = 0.00097 < 0.001}$

so S_9 approximates S to within 0.001, so 9 terms are needed.

(22) $\sum_{n=1}^{\infty} a_n$ is conditionally convergent, so $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges

(a) $\sum_{n=1}^{\infty} a_n^2$ could converge if $a_n^2 < a_n$

ex) $a_n = \left(\frac{2}{3}\right)^n$

$(a_n)^2 = \left(\frac{4}{9}\right)^n$

(b) $\sum_{n=1}^{\infty} |a_n|$ MUST diverge by def of Abs convergence

(c) $\sum_{n=1}^{\infty} (-1)^{2n} a_n = \sum_{n=1}^{\infty} \left((-1)^2\right)^n a_n = \sum_{n=1}^{\infty} a_n$ which converges

(d) $\sum_{n=1}^{\infty} (-a_n) = -\sum_{n=1}^{\infty} a_n$ which converges

$\Big\}$ so only (b) MUST diverge.

(23) (a) $\sum_{n=1}^{\infty} \frac{(-1)^n (n-1)}{n\sqrt{n}} = \sum_{n=1}^{\infty} (-1)^n \left(\frac{n-1}{n^{3/2}}\right)$

* Converges conditionally by Alt. series test

* $\sum_{n=1}^{\infty} \left(\frac{n-1}{n^{3/2}}\right)$ Diverges by comparison with $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$, a divergent p-series

(b) $\sum_{n=0}^{\infty} (-1)^n e^{-n}$

* $\sum_{n=0}^{\infty} \frac{1}{e^n}$ Converges by Ratio Test

So $\sum_{n=0}^{\infty} (-1)^n e^{-n}$ converges absolutely

(c) $\sum_{n=2}^{\infty} (-1)^n \frac{\ln n}{n}$

* converges conditionally by Alt. series test

* $\sum_{n=2}^{\infty} \frac{\ln n}{n}$ diverges by comparison to the harmonic series

$\frac{\ln n}{n} > \frac{1}{n} \ \forall n \geq 2$

(23) (d) $\sum_{n=1}^{\infty}\left(-\frac{\pi}{e}\right)^{-n}$

$= \sum_{n=1}^{\infty}(-1)^n\cdot\left(\frac{e}{\pi}\right)^n$

* $\sum_{n=1}^{\infty}\left(\frac{e}{\pi}\right)^n$ converges by the Geom Series Test, $|r| = \frac{e}{\pi} < 1$,

So $\sum_{n=1}^{\infty}(-1)^n\left(\frac{e}{\pi}\right)^n$ converges absolutely (to $\frac{-e/\pi}{1+e/\pi}$)

(e) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{\sqrt{n}}{n+3}\right)$

* converges conditionally by Alt. Series Test

* $\sum_{n=1}^{\infty}\frac{n^{1/2}}{n+3}$ diverges by comparison with $\sum_{n=1}^{\infty}\frac{1}{n^{1/2}}$, a divergent p-series.

(24) $S = \sum_{n=1}^{\infty}\frac{4}{n^2}$ (convergent p-series)

$= 4\sum_{n=1}^{\infty}\frac{1}{n^2}$

$S_2 = 4\left[\frac{1}{1}+\frac{1}{4}\right] = 4+1 = 5$

$R_2 = |a_3| = \frac{4}{3^2} = \frac{4}{9}$

$S \in \left[5-\frac{4}{9}, 5+\frac{4}{9}\right]$

$S \in \left[\frac{41}{9}, \frac{49}{9}\right]$

so $\boxed{\frac{41}{9} \le S \le \frac{49}{9}}$ $\boxed{A}$

(25) which converge?

I. $\sum_{n=1}^{\infty}\frac{n}{n+2}$ → diverges by n^{th} term test since $\lim_{n\to\infty}\frac{n}{n+2} = 1 \neq 0$

✓ II. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$ → conditionally convergent harmonic series by Alt. Series Tst.

III. $\sum_{n=1}^{\infty}\frac{1}{n}$ → divergent harmonic series

So only II converges $\boxed{B}$

(26) $\lim_{b\to\infty}\int_1^b \frac{1}{x^p}dx$ is finite so $\int_1^{\infty}\frac{1}{x^p}dx$ converges,

So $p > 1$. which MUST be true?

✓(A) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges → TRUE by IntegraTest.

$\boxed{\text{So answer is } A}$

(B) $\sum_{n=1}^{\infty}\frac{1}{n^p}$ diverges → False, see (A)

(C) $\sum_{n=1}^{\infty}\frac{1}{n^{p-2}}$ Converges → Not always true. False if $n \le 3$

(D) $\sum_{n=1}^{\infty}\frac{1}{n^{p-1}}$ converges → Not always true. False if $n \le 2$

(E) $\sum_{n=1}^{\infty}\frac{1}{n^{p+1}}$ Diverges, false. by comparison, if $\sum_{n=1}^{\infty}\frac{1}{n^p}$ converges, $\sum_{n=1}^{\infty}\frac{1}{n^{p+1}}$ must too!

27 (Review Question) d/o

$$\lim_{x\to 1} \frac{\int_1^x e^{t^2}\,dt}{x^2-1}$$

by L'Hôp: $\lim_{x\to 1} \frac{e^{x^2}}{2x} = \boxed{\frac{e}{2}}$ $\boxed{C}$

2nd Fun. thm of Calculus

28 For what K, $K>1$ will both $\sum_{n=1}^{\infty} \frac{(-1)^{Kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{K}{4}\right)^n$ converge?

* $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ will only converge (conditionally) if $(-1)^{kn}$ alternates, so K must be an odd integer >1

$\boxed{K = 3, 5, 7, 9, 11, \ldots}$

* $\sum_{n=1}^{\infty} \left(\frac{K}{4}\right)^n$ will only converge by geometric series test if $\frac{K}{4} < 1$, $K < 4$ (but >1)

$\boxed{K = 3, 2}$

The only value that satisfies both scenarios is $\boxed{K=3}$ $\boxed{D}$