9.10 More Confidence Intervals Practice

$\hat{p} = \frac{130}{1100} \approx 0.118$ $\quad CI = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

1. A town takes a poll of its residents to find out how many people would be willing to pay a new tax to repair the town's sidewalks. Out of 1100 people polled, only 130 said that they would be willing to pay.
   (a) Find a 90% confidence interval for the proportion of the whole town that would be willing to pay the extra tax.

$\frac{1-c\%}{2} \rightarrow \frac{1-0.9}{2} = 0.05$

$z = -1.65$

$CI = 0.118 \pm 1.65\sqrt{\frac{0.118(1-0.118)}{1100}}$

$CI = 0.118 \pm 0.01604$

$\boxed{(0.1020,\ 0.1340)}$

   (b) If 15,000 people live in this town, then we are 90% confident that between 1529 and 2011 will be willing to pay this tax. (Fill in with numbers of people.)

$0.1020(15000) = 1529.26 \qquad 0.1340(15000) = 2010.74$

2. A company that produces white bread is concerned about the distribution of the amount of sodium in its bread. The company takes a simple random sample of 100 slices of bread and computes the sample mean to be 103 milligrams of sodium per slice.
   (a) Construct a 99% confidence interval for the unknown mean sodium level assuming that the population standard deviation is 10 milligrams.

$n = 100$, $\bar{x} = 103$, $\sigma = 10$

$CI = \bar{x} + z\left(\frac{\sigma}{\sqrt{n}}\right)$

$CI = 103 \pm 2.58\left(\frac{10}{\sqrt{100}}\right)$

$CI = 103 \pm 2.58$

$\boxed{(100.42\text{ mg},\ 105.58\text{ mg})}$

$\frac{1-c\%}{2}$, $\frac{1-0.99}{2}$, 0.005, $z = -2.58$

   (b) Interpret the 99% confidence interval found in (a).

We are 99% confident that the mean sodium level per slice is between 100.42mg and 105.58mg.

3. You work for a consumer advocate agency and want to find the mean repair cost of a washing machine. In the past, the standard deviation of the cost of repairs for washing machines has been \$17.50. As part of your study, you randomly select 40 repair costs and find the mean to be \$100.00.
   (a) Calculate a 85% confidence interval for the population mean.

$\sigma = 17.50$, $n = 40$, $\bar{x} = 100.00$

$CI = \bar{x} \pm z\left(\frac{\sigma}{\sqrt{n}}\right)$

$CI = 100 \pm 1.44\left(\frac{17.50}{\sqrt{40}}\right)$

$CI = 100 \pm 3.9844$

$\boxed{(\$96.02,\ \$103.98)}$

$\frac{1-0.85}{2}$, 0.075, $z = -1.44$

   (b) Interpret the interval found in (a).

We are 85% confident that the avg. repair cost for washing machine is between \$96.02 and \$103.98

4. You want to estimate the mean fuel efficiency for all Ford Focus cars with 99% confidence and a margin of error of no more than 1 mile per gallon. Preliminary data suggests that $\sigma = 2.4$ miles per gallon is a reasonable standard deviation for all cars of this make and model. How large a sample do you need?

M.E. $\leq$ 1 mpg

$\sigma = 2.4$

$n = \_\_?$

$z_{99} = -2.58$

$M.E. = z\left(\frac{\sigma}{\sqrt{n}}\right)$

$\cancel{1 = \frac{-2.58(2.4)}{\sqrt{n}}}$

$(\sqrt{n})^2 = (-6.192)^2$

$n = 38.341$

$38.341 \leq n$

$n \geq 38.341$

sample of at least 39 cars of this model.

5. The actual time it takes to cook a 10–pound turkey is a Normal random variable with a mean of 2.8 hours and a standard deviation of 0.24 hours. Suppose that a random sample of 35 10–pound turkeys is taken.

   (a) What is the probability that a randomly selected 10-pound turkey will take less than 3.1 hours to cook?

   $P(x \leq 3.1)$ | $z = \frac{x-\bar{x}}{\sigma}$ | $\bar{x} = 2.8$, $x = 3.1$, $\sigma = 0.24$ | $z = \frac{3.1-2.8}{0.24}$, $z = 1.25$ | $\boxed{P(x \leq 3.1) = 0.8944}$

   (b) What is the probability that the average cooking time of a 10–pound turkey will take between 2.7 and 2.95 hours to cook?

   $z = \frac{2.7-2.8}{0.24}$, $z = -0.42 \rightarrow 0.3372$ | $z = \frac{2.95-2.8}{0.24}$, $z = 0.63 \rightarrow 0.7357$ | $P(2.7 \leq x \leq 2.95) = 0.7357 - 0.3372 = \boxed{0.3985}$

   (c) Given that an average of 2.9 hours was found for a sample of 35 turkeys, calculate an 80% confidence interval for the average cooking time of a 10–pound turkey.

   $\bar{x} = 2.9$, $\sigma = 0.24$, $n = 35$ | $\frac{1-c\%}{2} \rightarrow \frac{1-0.80}{2} \rightarrow 0.1 \rightarrow z = -1.28$ | $CI = \bar{x} \pm z\left(\frac{\sigma}{\sqrt{n}}\right)$, $CI = 2.9 \pm 1.28\left(\frac{0.24}{\sqrt{35}}\right)$ | $CI = 2.9 \pm 0.0519$, $\boxed{(2.848 \text{ hrs}, 2.952 \text{ hrs})}$

6. Weight Watchers takes a poll of 250 members and finds that 95 of them include exercise with their diet program, while the rest do not. Find a 99% confidence interval for the proportion of all members that do exercise.

   $\hat{p} = \frac{95}{250} = 0.38$, $n = 250$ | $\frac{1-0.99}{2} \rightarrow 0.005$, $z = -2.58$ | $CI = \hat{p} \pm z\left(\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$, $CI = 0.38 \pm 2.58\sqrt{\frac{0.38(1-0.38)}{250}}$ | $CI = 0.38 \pm 0.0792$, $\boxed{(0.301, 0.459)}$

7. A magazine polls 395 readers and finds that 95 of them bought the magazine in the store, while the rest had a subscription. Find an 87% confidence interval for the proportion of all readers who have a subscription.

   $\hat{p} = \frac{300}{395} = 0.759$, $n = 395$ | $\frac{1-c\%}{2} \rightarrow \frac{1-0.87}{2}$, 0.065, $z = -1.51$ | $CI = \hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, $CI = 0.759 \pm 1.51\sqrt{\frac{0.759(1-0.759)}{395}}$ | $CI = 0.759 \pm 0.325$, $\boxed{(0.727, 0.791)}$

8. Fill in the blanks with one of the following for how the margin of error is impacted: *increases, decreases,* or *stays the same* where $ME = z\left(\frac{\sigma}{\sqrt{n}}\right)$.

   As the sample size (n) increases, the margin of error (ME) decreases (dividing by larger #'s)

   As the confidence level (C%) increases, the margin of error (ME) increases (wider window, $z\uparrow$)

   As the standard deviation (σ) increases, the margin of error (ME) increases (more variability in the data)