Key

9.1 AP Practice Problems (p.651) – Parametric Equations

1. Which pair of parametric equations represents a plane curve that is a circle with a radius of 3 and a center at (0, 0)?

(A) $x(t) = 9\cos t,\ y(t) = 9\sin t,\ 0 \le t \le 2\pi$

(B) $x(t) = 3\sin t,\ y(t) = 3\cos t,\ 0 \le t \le \pi$

(C) $x(t) = 3\sin(2t),\ y(t) = 3\cos(2t),\ 0 \le t \le \pi$

(D) $x(t) = \cos(3t),\ y(t) = \sin(3t),\ 0 \le t \le 2\pi$

Circle equation:

$x^2+y^2=r^2$

$x^2+y^2=3^2$

$x^2+y^2=9$

$x=3\sin(2t) \qquad y=3\cos(2t)$

$\frac{x}{3}=\sin(2t) \qquad \frac{y}{3}=\cos(2t)$

$*\ \sin^2\theta+\cos^2\theta=1$

$\sin^2(2t)+\cos^2(2t)=1$

$\left(\frac{x}{3}\right)^2+\left(\frac{y}{3}\right)^2=1$

$\left[\frac{x^2}{9}+\frac{y}{9}=1\right](9)$

$\boxed{x^2+y^2=9}$ ✓

2. A rectangular equation of the curve whose parametric equations are $x(t) = t + 1,\ y(t) = t^2 + 3t$ is

(A) $y = x^2 + x - 2$ (B) $y = x^2 + 3x$

(C) $y = x^2 + 5x + 4$ (D) $y = x^2 + 3x - 2$

$x=t+1$

$x-1=t$

$y=t^2+3t$

$y=(x-1)^2+3(x-1)$

$y=x^2-2x+1+3x-3$

$\boxed{y=x^2+1x-2}$

3. A rectangular equation of the parametric equations $x(t) = 4\cos t$, $y(t) = \frac{1}{2}\sin t$ is

(A) $\frac{x^2}{16} + \frac{y^2}{4} = 1$ (B) $\frac{x^2}{16} + 2y^2 = 1$

(C) $16x^2 + 4y^2 = 1$ (D) $\frac{x^2}{16} + 4y^2 = 1$

$x = 4\cos t \quad y = \frac{1}{2}\sin t$

$\frac{x}{4} = \cos(t) \quad 2y = \sin(t)$

$\cos^2(t) + \sin^2(t) = 1$

$\left(\frac{x}{4}\right)^2 + (2y)^2 = 1$

$\frac{x^2}{16} + 4y^2 = 1$

4. An object is moving along a plane curve according to the parametric equations

$$x(t) = -5\cos\left(\frac{\pi}{2}t\right), y(t) = 2\sin\left(\frac{\pi}{2}t\right), 0 \le t \le 8$$

(a) Find a rectangular equation for the parametric equations.

(b) Describe the motion of the object from $t = 0$ to $t = 8$. Be sure to include where the object begins, where it ends, its path, and its orientation.

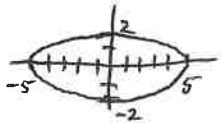
$\frac{x}{-5} = \cos\left(\frac{\pi}{2}t\right) \quad \frac{y}{2} = \sin\left(\frac{\pi}{2}t\right)$

$\cos^2\left(\frac{\pi}{2}t\right) + \sin^2\left(\frac{\pi}{2}t\right) = 1$

$\left(\frac{x}{-5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

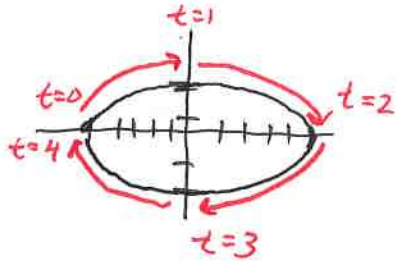
$\frac{x^2}{25} + \frac{y^2}{4} = 1$

a) Ellipse



b)

t	x	y
0	-5	0
1	0	2
2	5	0
3	0	-2
4	-5	0
5	0	2
6	5	0
7	0	-2
8	-5	0



*curve begins at (-5, 0), moves clockwise 2 full revolutions.

*clockwise direction of motion is this curve's orientation.