$\boxed{9.1}$

9.1: Sequences

p.602 #1-82 D2S2

Find first 5 terms of sequence

2) $a_n = \dfrac{3^n}{n!}$ $\quad a_1 = \dfrac{3}{1}$ $\quad a_2 = \dfrac{3^2}{2!} = \dfrac{9}{2}$

$a_3 = \dfrac{3^3}{3!} = \dfrac{27}{6} = \dfrac{9}{2}$

$a_4 = \dfrac{3^4}{4!} = \dfrac{81}{24} = \dfrac{27}{8}$

$a_5 = \dfrac{3^5}{5!} = \dfrac{243}{120} = \dfrac{81}{40}$

Write first 5 terms of recursively defined sequence.

13) $a_1 = 32 \quad a_{k+1} = \frac{1}{2}a_k$

$a_2 = \frac{1}{2}a_1 = \frac{1}{2}(32) = 16$

$a_3 = \frac{1}{2}a_2 = \frac{1}{2}(16) = 8$

$a_4 = \frac{1}{2}a_3 = \frac{1}{2}(8) = 4$

$a_5 = \frac{1}{2}a_4 = \frac{1}{2}(4) = 2$

14) $a_1 = 6 \quad a_{k+1} = \frac{1}{3}a_k^2$

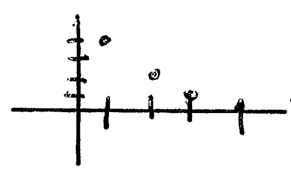
$a_2 = \frac{1}{3}(a_1)^2 = \frac{1}{3}(6)^2 = \frac{36}{3} = 12$

$a_3 = \frac{1}{3}(a_2)^2 = \frac{1}{3}(12)^2 = \frac{144}{3} = 48$

$a_4 = \frac{1}{3}(a_3)^2 = \frac{1}{3}(48)^2 = 768$

$a_5 = \frac{1}{3}(a_4)^2 = \frac{1}{3}(768)^2 = 196{,}608$

17) $a_n = 4(0.5)^{n-1} = 4(\frac{1}{2})^{n-1}$



$\boxed{\text{graph (e)}}$

$a_1 = 4(\frac{1}{2})^0 = 4$

$a_2 = 4(\frac{1}{2})^1 = 2$

$a_3 = 4(\frac{1}{2})^2 = 1$

$a_4 = 4(\frac{1}{2})^3 = \frac{1}{2}$

18) $a_n = \dfrac{4^n}{n!}$

$a_1 = \dfrac{4}{1} = 4$

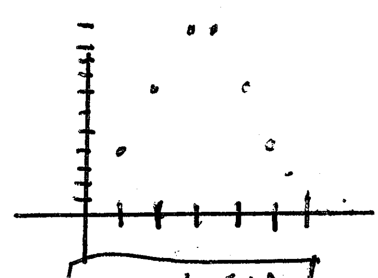
$a_2 = \dfrac{4^2}{2} = 8$

$a_3 = \dfrac{4^3}{3!} = \dfrac{64}{6}$

$a_4 = \dfrac{4^4}{4!} = \dfrac{32}{3}$

$a_5 = \dfrac{4^5}{5!} = 8.53$

$a_6 = \dfrac{4^6}{6!} = 5.683$



$\boxed{\text{graph (b)}}$

Write next 2 terms. Describe pattern:

25) $2, 5, 8, 11, \ldots$ arithmetic sequence: $a_n = a_1 + (n-1)d$

$a_n = 2 + (n-1)(3) = 2 + 3n - 3 = 3n - 1$

$a_n = 3n - 1$

$a_5 = 15 - 1 = 14$

$a_6 = 3(6) - 1 = 17$

* Add 3 to the previous term.

26) $\frac{7}{2}, 4, \frac{9}{2}, 5 \rightarrow \frac{7}{2}, \frac{8}{2}, \frac{9}{2}, \frac{10}{2}, \underline{\frac{11}{2}}, \underline{\frac{12}{2}}$ $a_5 = \frac{11}{2}$ $a_6 = 6$

$a_n = \frac{n+6}{2}$ * Add 1 to numerator, divide by 2

27) $3, \frac{-3}{2}, \frac{3}{4}, \frac{-3}{8}, \underline{\frac{3}{2^4}}, \underline{\frac{-3}{2^5}}$ $a_5 = \frac{3}{16}$

$a_6 = {}^{-3}/_{32}$

$a_n = \frac{3(-1)^{n+1}}{2^{n-1}}$ or $\frac{3}{(-2)^{n-1}}$

*multiply previous term by $-1/2$

30) $1, -3/2, 9/4, -27/8$

$a_n = \left(\frac{-3}{2}\right)^{n-1}$

$a_5 = \left(\frac{-3}{2}\right)^4 = \frac{81}{16}$

$a_6 = \left(\frac{-3}{2}\right)^5 = \frac{-243}{32}$

*multiply each term by $-3/2$

Simplify ratio of factorials:

33) $\frac{(n+1)!}{n!} = \frac{(n+1)\cdot n!}{n!} = n+1$

34) $\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)\cdot n!}{n!} = (n+2)(n+1)$

<u>Find the limit:</u>

41) $a_n = \sin\left(\frac{1}{n}\right)$ *As $\frac{1}{n}$ approaches 0, $\sin 0 = 0$

$\lim_{n\to\infty} \sin\left(\frac{1}{n}\right) = 0$

42) $a_n = \cos\left(\frac{2}{n}\right)$

$\lim_{n\to\infty} \cos\left(\frac{2}{n}\right) = 1$

*As $\frac{2}{n}$ approaches 0, $\cos 0 = 1$

Recall: 1) Sequence is <u>convergent</u> if $\lim_{n\to\infty} a_n$ exists and approaches a single finite value (as n increases)

2) Sequence is <u>divergent</u> if $\lim_{n\to\infty} a_n$ does not exist and is infinite.

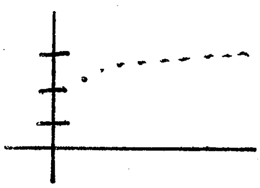
Make inference about convergence/divergence of sequence after graphing first 10 terms of sequence.

45) $a_n = \cos\left(\frac{n\pi}{2}\right)$ $\quad \{a_n\} = \{0, -1, 0, 1, 0, -1, \ldots\}$

<u>calculator steps:</u>
Mode: <u>Seq</u>
$Y_1 \to u(n) = \cos(n\pi/2)$

$\lim_{n\to\infty} a_n$ Does not exist: sequence is divergent

46) $a_n = 3 - \frac{1}{2^n}$ $\quad \lim_{n\to\infty} a_n = \lim_{n\to\infty} 3 - \frac{1}{2^n} = 3 - 0 = 3$



*sequence converges to 3.

49) Determine convergence/divergence of sequence: If convergent, find the limit:

$a_n = \frac{3n^2 - n + 4}{2n^2 + 1}$ $\quad \lim_{n\to\infty} \frac{3n^2 - n + 4}{2n^2 + 1} = \boxed{\frac{3}{2}}$ * sequence converges

50) $a_n = \frac{\sqrt[3]{n}}{\sqrt[3]{n} + 1}$ $\quad \lim_{n\to\infty} \frac{\sqrt[3]{n}}{\sqrt[3]{n} + 1} = \boxed{1}$ * sequence converges

53) $a_n = \frac{1 + (-1)^n}{n}$ $\quad \{a_n\} = \left\{\frac{0}{1}, \frac{2}{2}, \frac{0}{3}, \frac{2}{4}, \frac{0}{5}, \frac{2}{6}, \frac{0}{7}, \frac{2}{8}, \ldots\right\}$

$\lim_{n\to\infty} a_n = 0$ converges

54) $a_n = \frac{1 + (-1)^n}{n^2}$ $\quad \lim_{n\to\infty} a_n = 0$, converges

61) $a_n = \frac{n-1}{n} - \frac{n}{n-1}, n \geq 2$ $\quad a_n = \frac{(n-1)(n-1) - n(n)}{n(n-1)}$

$a_n = \frac{n^2-2n+1-n^2}{n^2-n} = \frac{-2n+1}{n^2-n}$ $\quad \lim_{n\to\infty} \frac{-2n+1}{n^2-n} = \boxed{0}$, converges

62) $a_n = \frac{n^2}{2n+1} - \frac{n^2}{2n-1} = \frac{n^2(2n-1) - n^2(2n+1)}{(2n+1)(2n-1)} = \frac{2n^3-n^2-2n^3-n^2}{4n^2-1}$

$\lim_{n\to\infty} \frac{-2n^2}{4n^2-1} = \boxed{\frac{-1}{2}}$, converges

65) $a_n = \left(1+\frac{k}{n}\right)^n$ $\quad \lim_{n\to\infty}\left(1+\frac{k}{n}\right)^n = e^k$, converges.

66) $a_n = 2^{\frac{1}{n}}$ $\quad \lim_{n\to\infty} 2^{\frac{1}{n}} = 2^0 = 1$, converges

Write expression for n^{th} term of sequence:

69) $\underbrace{1, 4}_{+3}, 7, 10$ $\quad a_n = a_1 + (n-1)d$ $\quad a_n = 1+(n-1)(3) = 1+3n-3$

$\boxed{a_n = 3n-2}$

70) $\underbrace{3, 7}_{+4}, 11, 15$ $\quad a_n = a_1 + (n-1)d$ $\quad a_n = 3+(n-1)4 = 3+4n-4$

$\boxed{a_n = 4n-1}$

73) $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$ $\quad \boxed{a_n = \frac{n+1}{n+2}}$

74) $2, -1, \frac{1}{2}, \frac{-1}{4}, \frac{1}{8} \rightarrow \frac{1}{1/2}, \frac{1}{-1}, \frac{1}{2}, \frac{1}{-4}, \frac{1}{8}$

* multiply denominator by -2

$a_n = \frac{(-1)^{n-1}}{(2)^{n-2}}$

77) $\frac{1}{2\cdot 3}, \frac{2}{3\cdot 4}, \frac{3}{4\cdot 5}, \frac{4}{5\cdot 6}$ $\quad a_n = \frac{n}{(n+1)(n+2)}$

78) $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}$ $\quad a_n = \frac{1}{n!}$

81) $\underset{2!}{2}, \underset{4!}{24}, \underset{6!}{720}, \underset{8!}{40,320}, \underset{10!}{3,628,800}$ $\quad \boxed{a_n = (2n)!}$

82) $\underset{1!}{1}, \underset{3!}{6}, \underset{5!}{120}, \underset{7!}{5040}, \underset{9!}{362,880}$ $\quad \boxed{a_n = (2n-1)!}$