Key

Representing repeating decimals as Geometric Series

Using a Geometric Series **In Exercises 35–40, (a) write the repeating decimal as a geometric series, and (b) write its sum as the ratio of two integers.**

35. $0.\overline{4}$

$$=0.4+0.04+0.004+\dots$$

$\times\left(\frac{1}{10}\right)$ $\frac{1}{10}$

$a_1=0.4=\frac{4}{10}$ $r=\frac{1}{10}$

a) $\sum_{n=0}^{\infty}\frac{4}{10}\left(\frac{1}{10}\right)^n$

b) $S=\frac{a_1}{1-r}=\frac{\frac{4}{10}}{1-\frac{1}{10}}=\frac{\frac{4}{10}}{\frac{9}{10}}$

$=\boxed{\frac{4}{9}}$

36. $0.\overline{36}$

$$=0.36+0.0036+0.000036+\dots$$

$\frac{1}{100}$ $\frac{1}{100}$

$a_1=0.36=\frac{36}{100}$

$r=\frac{1}{100}$

a) $\sum_{n=0}^{\infty}\frac{36}{100}\left(\frac{1}{100}\right)^n$

b) $S=\frac{a_1}{1-r}=\frac{\frac{36}{100}}{1-\frac{1}{100}}=\frac{\frac{36}{100}}{\frac{99}{100}}=\frac{36}{99}$

$=\boxed{\frac{4}{11}}$

37. $0.\overline{81}$

$$=0.81+0.0081+0.000081+\dots$$

$a_1=0.81=\frac{81}{100}$

$\frac{1}{100}$ $\frac{1}{100}$

$r=\frac{1}{100}$

a) $\sum_{n=0}^{\infty}\frac{81}{100}\cdot\left(\frac{1}{100}\right)^n$

b) $S=\frac{a_1}{1-r}=\frac{\frac{81}{100}}{1-\frac{1}{100}}=\frac{\frac{81}{100}}{\frac{99}{100}}$

$=\frac{81}{99}=\boxed{\frac{9}{11}}$

38. $0.\overline{01}$

$$0.01+0.0001+0.000001+\dots$$

$a_1=0.01=\frac{1}{100}$

$\frac{1}{100}$ $\frac{1}{100}$

$r=\frac{1}{100}$

a) $\sum_{n=0}^{\infty}\frac{1}{100}\left(\frac{1}{100}\right)^n$

b) $S=\frac{a_1}{1-r}=\frac{\frac{1}{100}}{1-\frac{1}{100}}=\frac{\frac{1}{100}}{\frac{99}{100}}=\boxed{\frac{1}{99}}$

39. $0.0\overline{75}$

$$0.075 \underbrace{+\;0.0}_{\frac{1}{100}}0075 \underbrace{+\;0.0}_{\frac{1}{100}}000075+\cdots$$

$$a_1 = 0.075 = \frac{75}{1000} \qquad r = \frac{1}{100}$$

a) $$\sum_{n=0}^{\infty} \frac{75}{1000}\left(\frac{1}{100}\right)^n$$

b) $$S = \frac{\frac{75}{1000}}{1-\frac{1}{100}} = \frac{\frac{75}{1000}}{\frac{99}{100}} =$$

$$= \frac{75}{1000}\cdot\frac{100}{99} = \frac{75}{990} \;\; 15 \;\; \boxed{\frac{5}{66}}$$

40. $0.2\overline{15}$

$$0.2+\underbrace{0.015 \underbrace{+\;0.0}_{\frac{1}{100}}0015 \underbrace{+\;0.0}_{\frac{1}{100}}000015}$$

$$a_1 = 0.015 = \frac{15}{1000} \qquad r = \frac{1}{100}$$

a) $$0.2 + \sum_{n=0}^{\infty} \frac{15}{1000}\left(\frac{1}{100}\right)^n$$

b) $$\frac{1}{5} + \frac{\frac{15}{1000}}{1-\frac{1}{100}} = \frac{\frac{15}{1000}}{\frac{99}{100}} = \frac{15}{1000}\cdot\frac{100}{99}$$

$$= \frac{15}{990} = \frac{1}{66}$$

$$\frac{1}{5}+\frac{1}{66} = \boxed{\frac{71}{330}}$$

41. $2.535\overline{35}$

$$2.5+\underbrace{0.035 \underbrace{+\;0.0}_{\frac{1}{100}}0035 \underbrace{+\;0.0}_{\frac{1}{100}}000035+\ldots}$$

$$a_1 = 0.035 = \frac{35}{1000} \qquad r = \frac{1}{100}$$

a) $$\frac{5}{2} + \sum_{n=0}^{\infty} \frac{35}{1000}\left(\frac{1}{100}\right)^n$$

$$S = \frac{\frac{35}{1000}}{1-\frac{1}{100}} = \frac{\frac{35}{1000}}{\frac{99}{100}} = \frac{35}{1000}\cdot\frac{100}{99}$$

$$= \frac{35}{990} \;5\!= \frac{7}{198}$$

b) $$\frac{5}{2}+\frac{7}{198} = \boxed{\frac{251}{99}}$$

42. $12.542\overline{42}$

$$12.5+\underbrace{0.042 \underbrace{+\;0.0}_{\frac{1}{100}}0042 \underbrace{+\;0.0}_{\frac{1}{100}}000042+\ldots.}$$

$$a_1 = 0.042 = \frac{42}{1000} \qquad r = \frac{1}{100}$$

a) $$12.5 + \sum_{n=0}^{\infty} \frac{42}{1000}\left(\frac{1}{100}\right)^n$$

$$\frac{25}{2} + \frac{\frac{42}{1000}}{1-\frac{1}{100}} = \frac{\frac{42}{1000}}{\frac{99}{100}} = \frac{42}{1000}\cdot\frac{100}{99}$$

$$= \frac{42}{990} \;6\!= \frac{7}{165}$$

b) $$\frac{25}{2}+\frac{7}{165} = \boxed{\frac{4139}{330}}$$