9.2 : Series and Convergence   p. 612-613 #2-72

Find first 5 terms of sequence of partial sums:

2) $\frac{1}{2\cdot 3}+\frac{2}{3\cdot 4}+\frac{3}{4\cdot 5}+\frac{4}{5\cdot 6}+\frac{5}{6\cdot 7}$

$S_1=\frac{1}{6}\approx \underline{0.167}$ $\quad S_4=\frac{1}{6}+\frac{1}{6}+\frac{3}{20}+\frac{4}{30}\simeq \underline{0.6167}$

$S_2=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}\approx \underline{0.33}$ $\quad S_5=\frac{1}{6}+\frac{1}{6}+\frac{3}{20}+\frac{4}{30}+\frac{5}{42}\approx \underline{0.7357}$

$S_3=\frac{1}{6}+\frac{1}{6}+\frac{3}{20}=\underline{0.4833}$

4) $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\dots$

$S_1=1$ $\quad S_4=1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}\approx 1.676$

$S_2=1+\frac{1}{3}\simeq 1.333$

$S_3=1+\frac{1}{3}+\frac{1}{5}\approx 1.533$ $\quad S_5=1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}\approx 1.7873$

6) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n!}=\frac{1}{1}+\frac{-1}{2}+\frac{1}{3!}-\frac{1}{4!}+\frac{1}{5!}$

$S_1=1$ $\quad S_4=1-\frac{1}{2}+\frac{1}{6}-\frac{1}{24}=0.625$

$S_2=1-\frac{1}{2}=0.5$

$S_3=1-\frac{1}{2}+\frac{1}{6}=0.667$ $\quad S_5=1-\frac{1}{2}+\frac{1}{6}-\frac{1}{24}+\frac{1}{120}=0.633$

Recall Geometric Series: a) geometric series with ratio r diverges if $|r|\geq 1$

b) geometric series with ratio r converges if $0<|r|<1$

c) ↳ If series converge, the sum is given by $\sum_{n=0}^{\infty}ar^n=\boxed{\frac{a}{1-r}}$

8) $\sum_{n=0}^{\infty}\left(\frac{4}{3}\right)^n$ This is a geometric series.

Since $r=\frac{4}{3}>1$, series diverge.

10) $\sum_{n=0}^{\infty}2(-1.03)^n$ This geometric series has $r=-1.03$.

Since $|-1.03|>1$, series diverges.

Recall n^{th}-Term Test for Divergence:

* If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges

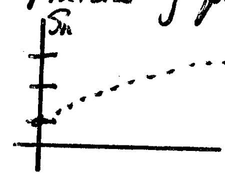
*Caution: The converse of this is <u>not true</u>.
If $\lim_{n\to\infty} a_n = 0$, this n^{th}-term test is inconclusive.

12) $\sum_{n=1}^{\infty} \frac{n}{2n+3}$ $\lim_{n\to\infty} \frac{n}{2n+3} = \frac{1}{2} \neq 0$ Series diverges by n^{th}-term test.

14) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$ $\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1 \neq 0$ series diverge by n^{th}-term test.

16) $\sum_{n=1}^{\infty} \frac{n!}{2^n}$ $\lim_{n\to\infty} \frac{n!}{2^n} = \infty$ series diverge by n^{th}-term test.

18) $\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n$ *geometric series $S = \frac{1}{1-2/3} = \frac{1}{1/3} = 3$ Matches graph (b)



20) $\sum_{n=0}^{\infty} \frac{17}{3}\left(\frac{-8}{9}\right)^n$ *geometric series $S = \frac{17/3}{1-(-8/9)} = 3$ Matches graph (d)

$S_0 = 17/3$ $S_1 = 0.63$, $S_2 = 5.1$ ...

22) $\sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n$ *geometric series $S = \frac{1}{1-2/5} = \frac{5}{3}$ Matches graph (e)

24) Verify infinite series converge. $\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2} = \frac{A(n+2)+Bn}{n(n+2)}$

let $n=0$, $1 = A(0+2)+B(0)$
$1 = 2A$ $A = 1/2$
let $n=-2$ $1 = A(-2+2)+B(-2)$
$1 = -2B$ $B = -1/2$

$\sum \frac{1/2}{n} + \frac{-1/2}{n+2} = \sum_{n=1}^{\infty}\left[\frac{1}{2n} - \frac{1}{2(n+2)}\right]$

$= \left(\boxed{\frac{1}{2}} - \cancel{\frac{1}{6}}\right) + \left(\boxed{\frac{1}{4}} - \cancel{\frac{1}{8}}\right) + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{10}}\right) + \left(\cancel{\frac{1}{8}} - \boxed{\frac{1}{12}}\right) + \left(\cancel{\frac{1}{10}} - \boxed{\frac{1}{14}}\right) + \ldots$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[\frac{1}{2} + \frac{1}{4} - \frac{1}{2(n+1)} - \frac{1}{2(n+2)}\right] = \frac{1}{2} + \frac{1}{4} - 0 - 0 = \boxed{\frac{3}{4}}$

23) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} = \frac{A(n+1)+Bn}{n(n+1)}$

let $n=0$ $1 = A(0+1)+B(0)$ $1 = A$ $A=1$.
let $n=-1$ $1 = A(-1+1)+B(-1)$ $1 = -B$ $B=-1$

$\sum_{n=1}^{\infty}\left[\frac{1}{n} - \frac{1}{n+1}\right] = \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \frac{1}{4}\right) + \ldots$

$\lim_{n\to\infty}\left[1 - \frac{1}{n+1}\right] = 1 - 0 = \boxed{1}$

26) $\sum_{n=1}^{\infty} 2\left(\frac{-1}{2}\right)^n$ *geometric series

Since $r=\frac{-1}{2}$ and $\left|\frac{-1}{2}\right|<1$, series converges.

30) a) $\sum_{n=1}^{\infty} \frac{4}{n(n+4)} = \frac{A}{n}+\frac{B}{n+4}$

$n=-4 \rightarrow 4=A(n+4)+Bn$

$4=A(-4+4)+B(-4) \quad -4B=4 \quad B=-1$

$n=0 \rightarrow 4=A(n+4)+Bn$

$4=A(4)+0 \quad A=1$

$\sum_{n=1}^{\infty} \frac{1}{n}-\frac{1}{n+4}$

$=\left(1-\cancel{\frac{1}{5}}\right)+\left(\frac{1}{2}-\cancel{\frac{1}{6}}\right)+\left(\frac{1}{3}-\cancel{\frac{1}{7}}\right)+\left(\frac{1}{4}-\cancel{\frac{1}{8}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{9}}\right)+\left(\cancel{\frac{1}{6}}-\cancel{\frac{1}{10}}\right)+\dots$

$=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4} = \frac{25}{12} \approx \boxed{2.0833}$

b) Find partial sums: * Math → summation Σ

n	5	10	20	50	100
S_n	2.79	3.164	3.394	3.551	3.6678

c) To graph partial sums →

1) Mode: seq, Dot

2) $Y_1 \rightarrow u(n)=\text{sum}(\text{seq}(a_n, n, 1, n))$

↳ 2nd → Stat → Math → sum

2nd → stat → OPS → seq

3) window → Adjust n, x, y min/max values

36) Find sum of convergent series: $\sum_{n=1}^{\infty} \frac{4}{n(n+2)} = 4\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \frac{A}{n}+\frac{B}{n+2}$

$n=0 \rightarrow 1=A(n+2)+Bn \rightarrow 1=A(2) \quad A=\frac{1}{2}$

$n=-2 \rightarrow 1=A(-2+2)+B(-2) \rightarrow 1=-2B \quad B=-\frac{1}{2}$

$4\sum \frac{1}{2n}-\frac{1}{2(n+2)} = 2\sum_{n=1}^{\infty}\left[\frac{1}{n}-\frac{1}{n+2}\right]$

$=2\left[\left(1-\cancel{\frac{1}{3}}\right)+\left(\frac{1}{2}-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{6}}\right)\right] = \lim_{n\to\infty} 2\left[1+\frac{1}{2}+\frac{1}{n}-\frac{1}{n+2}\right] = 2\left(\frac{3}{2}\right) = \boxed{3}$

38) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n+3)} = \frac{A}{2n+1}+\frac{B}{2n+3}$

$n=-\frac{1}{2} \rightarrow 1=A(2n+3)+B(2n+1) \rightarrow A(2)+B(0)=1 \quad A=\frac{1}{2}$

$n=-\frac{3}{2} \rightarrow 1=A(0)+B(-2) \rightarrow -2B=1 \quad B=-\frac{1}{2}$

$=\frac{1}{2(2n+1)}-\frac{1}{2(2n+3)}$

$\sum_{n=1}^{\infty} \frac{1}{2}\left[\frac{1}{2n+1}-\frac{1}{2n+3}\right] = \frac{1}{2}\left[\left(\frac{1}{3}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right)+\left(\cancel{\frac{1}{7}}-\cancel{\frac{1}{9}}\right)+\dots\right] = \lim_{n\to\infty}\frac{1}{2}\left[\frac{1}{3}-\frac{1}{2n+3}\right]$

$=\frac{1}{2}\left(\frac{1}{3}+0\right) = \boxed{\frac{1}{6}}$

44) $8+6+\frac{9}{2}+\frac{27}{8}$ $r=\frac{3}{4}$ $S=\frac{8}{1-3/4}=\frac{8}{1/4}=\boxed{32}$

46) $4-2+1-\frac{1}{2}$ $r=\frac{-1}{2}$ $S=\frac{4}{1-(\frac{-1}{2})}=\frac{4}{3/2}=\boxed{\frac{8}{3}}$

50) $\sum_{n=1}^{\infty}\frac{1}{9n^2+3n-2}=\sum_{n=1}^{\infty}\frac{1}{(3n-1)(3n+2)}$

$n=1/3 \to 1=A(3n+2)+B(3n-1)$

$1=3A \quad A=1/3$

$n=-2/3 \to 1=A(0)+B(-3) \quad B=-1/3$

$\sum\frac{1}{3(3n-1)}-\frac{1}{3(3n+2)}=\frac{1}{3}\sum_{n=1}^{\infty}\left[\frac{1}{3n-1}-\frac{1}{3n+2}\right]=\lim_{n\to\infty}\frac{1}{3}\left[\left(\frac{1}{2}-\cancel{\frac{1}{5}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{8}}\right)+\left(\cancel{\frac{1}{8}}-\cancel{\frac{1}{11}}\right)+\ldots\frac{1}{3n-1}-\frac{1}{3n+2}\right]$

$=\frac{1}{3}\left(\frac{1}{2}+0+0\right)=\boxed{\frac{1}{6}}$

52) $0.\overline{9}=0.999\ldots=\frac{9}{10}+\frac{9}{100}+\frac{9}{1000}\ldots$ $\sum_{n=1}^{\infty}9\left(\frac{1}{10}\right)^n$ $S=\frac{\frac{9}{10}}{1-1/10}=\frac{\frac{9}{10}}{9/10}=\boxed{1}$

54) $0.\overline{01}=0.010101\ldots=\frac{1}{100}+\frac{1}{10000}+\ldots$ $\sum_{n=1}^{\infty}\left(\frac{1}{100}\right)^n$ $S=\frac{\frac{1}{100}}{1-\frac{1}{100}}=\frac{\frac{1}{100}}{\frac{99}{100}}=\boxed{\frac{1}{99}}$

56) $0.2\overline{15}=0.2151515\ldots=0.2+\sum\left(\frac{15}{100}\right)^n=0.2+\cancel{\sum_{n=1}^{\infty}\left(\frac{3}{20}\right)^n}$

$0.2+\sum\frac{15}{1000}\left(\frac{1}{100}\right)^n$

$0.2+\frac{15}{1000}\cdot\left(\frac{1}{1-1/100}\right)=\boxed{\frac{71}{330}}$

$\cancel{0.2+\frac{\frac{3}{20}}{1-3/20}\quad\frac{\frac{3}{20}}{17/20}}$

$\cancel{0.2+\frac{3}{17}}=$

58) $\sum_{n=1}^{\infty}\frac{n+1}{2n-1}$ $\lim_{n\to\infty}\frac{n+1}{2n-1}=\frac{1}{2}\neq 0$, series divergent by n^{th} term test

60) $\sum_{n=1}^{\infty}\frac{1}{n(n+3)}$

$n=0\to A(n+3)+B(n)$

$1=A(3)+0 \quad A=1/3$

$n=-3\to 1=A(0)+-3B$

$B=-1/3$

$\frac{1}{3}\sum_{n=1}^{\infty}\left[\frac{1}{n}-\frac{1}{n+3}\right]=\frac{1}{3}\left[\left(1-\cancel{\frac{1}{4}}\right)+\left(\frac{1}{2}-\cancel{\frac{1}{5}}\right)+\left(\frac{1}{3}-\cancel{\frac{1}{6}}\right)+\ldots\right.$

$\lim_{n\to\infty}\frac{1}{3}\left[1+\frac{1}{2}+\frac{1}{3}\right]=\boxed{\frac{11}{18}}$ converges.

62) $\sum_{n=1}^{\infty}\frac{3^n}{n^3}$ $\lim_{n\to\infty}\frac{3^n}{n^3}=\infty\neq 0$, diverges by n^{th} term test.

(Comparative growth rate)

64) $\sum_{n=0}^{\infty}\frac{1}{4^n}=\left(\frac{1}{4}\right)^n$ geometric series, $\left|\frac{1}{4}\right|<1$, series converges

66) $\sum_{n=1}^{\infty}\frac{2^n}{100}=\frac{1}{100}(2)^n$ geometric series $r=2$ $|2|>1$, series diverges

68) $\sum_{n=1}^{\infty}\ln\left(\frac{1}{n}\right)$

$\lim_{n\to\infty}\ln\left(\frac{1}{n}\right)\neq 0$ <u>diverges</u>

70) $\sum_{n=1}^{\infty}e^{-n}$

$\left(\frac{1}{e}\right)^n$ geometric series

$r=\left|\frac{1}{e}\right|<1$

<u>converges</u>

72) $\sum_{n=1}^{\infty}\ln\left(\frac{n+1}{n}\right)=\lim_{n\to\infty}\ln(n+1)-\ln(n)$

$\neq 0$

<u>diverges.</u>