Key

9.3 AP Practice Problems (p.668) – Polar Equations

1. What is the slope of the tangent line to the cardioid $r = 1 + \cos\theta$ at $\theta = \frac{\pi}{6}$?

$\boxed{\text{(A)}}$ -1 (B) $-\frac{\sqrt{3}+1}{2}$

(C) $\frac{\sqrt{3}+1}{2}$ (D) 1

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\Bigg|_{\theta=\pi/6} = \frac{\cos\theta - \sin^2\theta + \cos^2\theta}{-\sin\theta - 2\sin\theta\cos\theta}$$

$x = r\cos\theta$

$x = (1+\cos\theta)\cos\theta$

$\frac{dx}{d\theta} = -\sin\theta\cos\theta + (1+\cos\theta)(-\sin\theta)$

$= -\sin\theta - 2\sin\theta\cos\theta$

$y = r\sin\theta$

$y = (1+\cos\theta)\sin\theta$

$\frac{dy}{d\theta} = -\sin\theta(\sin\theta) + (1+\cos\theta)(\cos\theta)$

$= \cos\theta - \sin^2\theta + \cos^2\theta$

$$= \frac{\cos\pi/6 - \sin^2\pi/6 + \cos^2\pi/6}{-\sin\pi/6 - 2\sin\pi/6\cos\pi/6}$$

$$= \frac{\sqrt{3}/2 - \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}{-\left(\frac{1}{2}\right) - 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}$$

$$= \frac{\frac{\sqrt{3}}{2} + \frac{1}{2}}{-\frac{1}{2} - \frac{\sqrt{3}}{2}} \rightarrow \boxed{-1}$$

2. Which integral gives the perimeter of one petal of the rose $r = 2\sin(3\theta)$?

(A) $\int_0^{\pi/2} \sqrt{\sin^2(3\theta) + 9\cos^2(3\theta)}\, d\theta$

$\boxed{\text{(B)}}$ $2\int_0^{\pi/3} \sqrt{\sin^2(3\theta) + 9\cos^2(3\theta)}\, d\theta$

(C) $\int_0^{\pi/3} \sqrt{1 + 8\cos^2(3\theta)}\, d\theta$

(D) $\int_0^{\pi/3} [2\sin(3\theta) + 3\cos(3\theta)]\, d\theta$

$$L = \int_0^{\pi/3} \sqrt{(2\sin3\theta)^2 + (6\cos3\theta)^2}\, d\theta$$

$$L = \int_0^{\pi/3} \sqrt{4\sin^2 3\theta + 36\cos^2 3\theta}\, d\theta$$

$$\int \sqrt{4(\sin^2 3\theta + 9\cos^2 3\theta)}\, d\theta$$

$$\boxed{L = 2\int_0^{\pi/3} \sqrt{\sin^2 3\theta + 9\cos^2 3\theta}\, d\theta}$$

*set $r=0$ to find a polar zero (boundary for a petal)

$r = 2\sin(3\theta)$

$2\sin(3\theta) = 0$

$\sin(3\theta) = 0$

$3\theta = \sin^{-1}(0)$

$3\theta = 0, \pi, 2\pi, \ldots$

$\theta = 0, \pi/3, 2\pi/3$

$$L = \int_0^{\pi/3} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$\frac{dr}{d\theta} = 2\cos(3\theta)\cdot 3$

$\frac{dr}{d\theta} = 6\cos(3\theta)$

3. Parametric equations for the polar equation $r = 5\theta$ are

(A) $x = 5\theta\cos\theta;\ y = 5\theta\sin\theta$

(B) $x = 5\cos\theta;\ y = 5\sin\theta$

(C) $x = 5r\cos\theta;\ y = 5r\sin\theta$

(D) $x = \cos(5\theta);\ y = \sin(5\theta)$

$x = r\cos\theta$ | $y = r\sin\theta$

$\boxed{x = 5\theta\cos\theta}$ | $\boxed{y = 5\theta\sin\theta}$

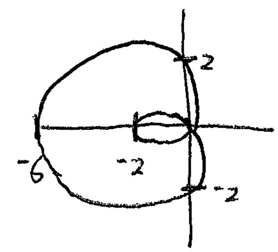
4. The graph of the polar equation $r = 2 - 4\cos\theta$ is a

(A) limaçon without an inner loop.

(B) limaçon with an inner loop.

(C) a cardioid that has symmetry with respect to the x-axis.

(D) a cardioid that has symmetry with respect to the y-axis.



$r = 2 - 4\cos\theta$ | limaçon with inner loop if $\frac{a}{b} < 1$

$* \ r = a \pm b\cos\theta$

5. The arc length of the logarithmic spiral represented by $r = 4e^{3\theta/2}$ from $\theta = 0$ to $\theta = \ln 9$ is

(A) $\frac{26}{3}\sqrt{13}$ (B) $\frac{52}{3}\sqrt{13}$

(C) $\frac{104}{3}\sqrt{13}$ (D) $36\sqrt{13}$

$$L = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$\frac{dr}{d\theta} = 4e^{3\theta/2} \cdot \frac{3\theta}{2} = 6e^{3\theta/2}$$

$$L = \int_0^{\ln 9} \sqrt{(4e^{3\theta/2})^2 + (6e^{3\theta/2})^2}\, d\theta$$

$$\int \sqrt{16e^{3\theta} + 36e^{3\theta}}$$

$$\int \sqrt{52e^{3\theta}}\, d\theta$$

$$\int \sqrt{52}\sqrt{e^{3\theta}}$$

$$\sqrt{52}\int e^{3\theta/2}\, d\theta \qquad u = \frac{3\theta}{2} \qquad \frac{du}{d\theta} = \frac{3}{2} \qquad d\theta = \frac{2}{3}du$$

$$\sqrt{52}\cdot\frac{2}{3}\int e^u\, du$$

$$\frac{2\cdot 2\sqrt{13}}{3}\cdot e^{\frac{3\theta}{2}}\Big]_0^{\ln 9}$$

$$\frac{4\sqrt{13}}{3}\left[e^{\frac{3\ln 9}{2}} - e^0\right] \rightarrow e^{\ln 9\cdot\frac{3}{2}} - e^0$$

$$\frac{4\sqrt{13}}{3}\left[9^{3/2} - 1\right] = \boxed{\frac{104}{3}\sqrt{13}}$$