9.3: Integral Test and p-Series   p.621 #1-42 D≥51

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* Integral Test: If f is positive, continuous, decreasing for $x \geq 1$ then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)dx$ either both converge or diverge

* P-Series Test: p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges if $0 < p \leq 1$

2) $\sum_{n=1}^{\infty} \frac{2}{3n+5}$ $f(x) = \frac{2}{3x+5}$ $f(x)$ is positive, decreasing, continuous for $x \geq 1$

$\int_1^{\infty} \frac{2}{3x+5}dx$ $u = 3x+5$ $dx = \frac{du}{3}$ $\frac{du}{dx} = 3$ $\int \frac{2}{u} \cdot \frac{du}{3} = \frac{2}{3}\int \frac{1}{u}du = \frac{2}{3}\ln|3x+5| \Big]_1^{\infty}$

$= \frac{2}{3}\ln|\infty| - \frac{2}{3}\ln 8 = \infty$

Since $\int_1^{\infty} f(x)dx$ diverges, $\sum_{n=1}^{\infty} a_n$ diverges

4) $\sum_{n=1}^{\infty} ne^{-n/2}$ $f(x) = xe^{-x/2}$ $f(x)$ is positive, decreasing, continuous for $x \geq 3$

$f'(x) = \frac{1e^{x/2} - xe^{x/2}(\frac{1}{2})}{e^{x}} = \frac{e^{x/2}(1 - \frac{x}{2})}{e^{x}}$

LIPET $\int xe^{-x/2}dx$

u	dv
$+x$	$e^{-x/2}$
-1	$-2e^{-x/2}$
$+0$	$4e^{-x/2}$

$-x2e^{-x/2} - 4e^{-x/2}\Big]_3^{\infty} = \frac{-2x}{e^{x/2}} - \frac{4}{e^{x/2}}\Big]_3^{\infty} = (0-0) - \left(\frac{-6}{e^{3/2}} - \frac{4}{e^{3/2}}\right)$

$= \frac{10}{e^{3/2}} = \boxed{\frac{10}{e^{3/2}}}$

Series converges.

8) $\frac{\ln 2}{\sqrt{2}} + \frac{\ln 3}{\sqrt{3}} + \ldots$ $f(x) = \frac{\ln x}{\sqrt{x}}$ for $x \geq 2$

$f(x)$ is positive, decreasing, continuous for $x \geq 2$

$\int_2^{\infty} \frac{\ln x}{\sqrt{x}}dx = \int (\ln x)(x^{-1/2})dx$

$u = \ln x$ $dv = x^{-1/2}$

$\frac{du}{dx} = \frac{1}{x}$ $v = 2x^{1/2}$

$uv - \int v\,du$

$2\sqrt{x}\ln x - \int \frac{2x^{1/2}}{x}dx = 2\sqrt{x}\ln x - 4\sqrt{x}\Big]_2^{\infty} = \infty$, series diverges

$\int 2x^{-1/2} = \frac{2x^{1/2}}{1/2}$

10) $\sum_{n=1}^{\infty} \frac{n}{n^2+3}$ $f(x) = \frac{x}{x^2+3}$ $f'(x) = \frac{1(x^2+3) - x(2x)}{(x^2+3)^2} = \frac{x^2+3-2x^2}{(x^2+3)^2}$ $\frac{-x^2+3}{(x^2+3)^2}$ $\sqrt{3}$

f(x) is positive, decreasing, continuous for $x \geq 2$

$\int_2^{\infty} \frac{x}{x^2+3}dx$ $u = x^2+3$, $\frac{du}{dx} = 2x$, $dx = \frac{du}{2x}$ $\int \frac{2}{u} du = 2\ln|u| \Big]_2^{\infty} = 2\ln|\infty| - 2\ln 2 = \infty$

<u>Series diverges.</u>

14) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ $f(x) = \frac{1}{x\sqrt{\ln x}}$ $f'(x) = -\frac{2\ln x + 1}{2x^2(\ln x)^{3/2}}$ f(x) continuous, positive, decreasing for $x \geq 2$

$\int \frac{1}{x\sqrt{\ln x}}$ $u = \ln x$, $\frac{du}{dx} = \frac{1}{x}$, $dx = x\,du$ $\int \frac{1}{\sqrt{u}} du = \int u^{-1/2} du = 2u^{1/2}$
$= 2\sqrt{\ln x}\Big]_2^{\infty} = \infty$ Series diverges.

16) $\sum_{n=3}^{\infty} \frac{1}{n\ln n\,(\ln(\ln n))}$ f is positive, continuous, decreasing for $x \geq 3$.

$f(x) = \frac{1}{x\ln x \cdot \ln(\ln x)}$

$u = \ln(\ln x)$, $\frac{du}{dx} = \frac{1}{\ln x} \cdot \frac{1}{x}$ $\int \frac{1}{x\ln x \cdot \ln(\ln x)} dx = \int \frac{1}{u} du = \ln(\ln(\ln x))\Big]_3^{\infty} = \infty$ <u>Series diverges.</u>

20) $\sum_{n=1}^{\infty} n^k e^{-n}$ $f(x) = x^k e^{-x}$ $f'(x) = x^{k-1} \cdot k \cdot e^{-x} + x^k \cdot e^{-x}(-1) = \frac{x^{k-1}(k-x)}{e^x}$

f(x) continuous, ~~decreasing~~, decreasing for $x > k$.

$\int x^k e^{-x}$ $u = x^k$, $dv = e^{-x}dx$, $du = kx^{k-1}$, $v = -e^{-x}$

$-x^k e^{-x} - \int -e^{-x} kx^{k-1} \Big]_1^{\infty} = -x^k e^{-x}\Big]_1^{\infty} = 0 + k\int_1^{\infty} x^{k-1} e^{-x} dx$

$\frac{1}{e} + \frac{k}{e} + \frac{k(k-1)}{e} + \ldots + \frac{k!}{e}$

Series Converges.

22) $\sum_{n=1}^{\infty} e^{-n}\cos n \rightarrow f(x) = \frac{\cos x}{e^x}$ Integral Test does not apply b/c $f(x)$ is not positive for $x \geq 1$.

26) $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ $f(x) = \frac{1}{x^{1/3}}$ $\int \frac{1}{x^{1/3}}dx$ f is positive, decreasing, continuous for $x \geq 1$

$$\int x^{-1/3}dx = \frac{3}{2}x^{2/3}\Big]_1^{\infty} = \infty \quad \boxed{\text{series diverges}}$$

28) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ $f(x) = \frac{1}{x^2}$ $f(x)$ is positive, decreasing, continuous for $x \geq 1$

$$\int \frac{1}{x^2}dx = \int x^{-2}dx = \frac{x^{-1}}{-1} = \frac{-1}{x}\Big]_1^{\infty} = 0 - (-1) = \boxed{1}$$ series converges.

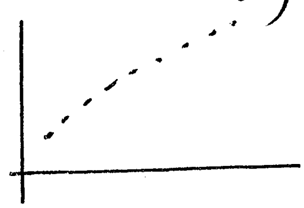
32) $1 + \frac{1}{4} + \frac{1}{9}$ $\sum_{n=1}^{\infty} \frac{1}{n^2}$ convergent p-series, $p = 2 > 1$.

34) $1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \ldots$ $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ divergent p-series, $p = \frac{2}{3} < 1$.

$\hookrightarrow \frac{1}{n^{2/3}}$

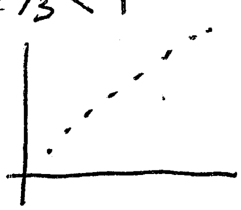
38) $\sum_{n=1}^{\infty} \frac{2}{n}$ divergent p-series, $p = 1$ (harmonic series)

Matches graph (f)



40) $\sum_{n=1}^{\infty} \frac{2}{\sqrt[5]{n^2}} = \frac{2}{n^{2/5}}$ divergent p-series, $p = 2/5 < 1$

matches graph (a)



41) $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}} = \frac{2}{n \cdot n^{1/2}} = \frac{2}{n^{3/2}}$ convergent p-series, $p = 3/2 > 1$

Matches graph (d)