80. $\sum_{n=2}^{\infty} \ln(n)$

$\lim_{n\to\infty} \ln(n) = \infty$

Diverges by Theorem 9.9

81. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$

Let $f(x) = \frac{1}{x(\ln x)^3}$.

f is positive, continuous, and decreasing for $x \geq 2$.

$$\int_2^{\infty} \frac{1}{x(\ln x)^3}\,dx = \int_2^{\infty} (\ln x)^{-3}\frac{1}{x}\,dx$$

$$= \left[\frac{(\ln x)^{-2}}{-2}\right]_2^{\infty}$$

$$= \left[-\frac{1}{2(\ln x)^2}\right]_2^{\infty} = \frac{1}{2(\ln 2)^2}$$

Converges by Theorem 9.10. See Exercise 47.

82. $\sum_{n=2}^{\infty} \frac{\ln n}{n^3}$

Let $f(x) = \frac{\ln x}{x^3}$.

f is positive, continuous, and decreasing for $x \geq 2$ since

$f'(x) = \frac{1 - 3\ln x}{x^4} < 0$ for $x \geq 2$.

$$\int_2^{\infty} \frac{\ln x}{x^3}\,dx = \left[-\frac{\ln x}{2x^2}\right]_2^{\infty} + \frac{1}{2}\int_2^{\infty} \frac{1}{x^3}\,dx$$

$$= \frac{\ln 2}{8} + \left[-\frac{1}{4x^2}\right]_2^{\infty}$$

$$= \frac{\ln 2}{8} + \frac{1}{16} \text{ (Use integration by parts.)}$$

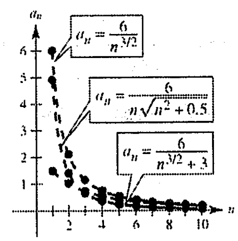
Converges by Theorem 9.10. See Exercise 34.

Section 9.4 Comparisons of Series

1. (a) $\sum_{n=1}^{\infty} \frac{6}{n^{3/2}} = \frac{6}{1} + \frac{6}{2^{3/2}} + \cdots;\ S_1 = 6$

$\sum_{n=1}^{\infty} \frac{6}{n^{3/2} + 3} = \frac{6}{4} + \frac{6}{2^{3/2} + 3} + \cdots;\ S_1 = \frac{3}{2}$

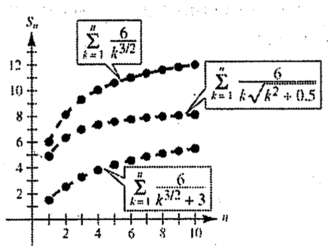
$\sum_{n=1}^{\infty} \frac{6}{n\sqrt{n^2 + 0.5}} = \frac{6}{1\sqrt{1.5}} + \frac{6}{2\sqrt{4.5}} + \cdots;\ S_1 = \frac{6}{\sqrt{1.5}} \approx 4.9$



(b) The first series is a p-series. It converges $\left(p = \frac{3}{2} > 1\right)$.

(c) The magnitude of the terms of the other two series are less than the corresponding terms at the convergent p-series. So, the other two series converge.

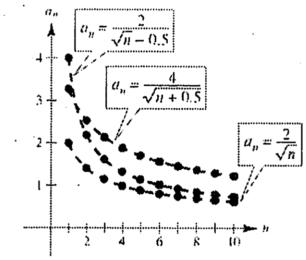
(d) The smaller the magnitude of the terms, the smaller the magnitude of the terms of the sequence of partial sums.



2. (a) $\sum_{n=1}^{\infty} \frac{2}{\sqrt{n}} = 2 + \frac{2}{\sqrt{2}} + \cdots S_1 = 2$

$\sum_{n=1}^{\infty} \frac{2}{\sqrt{n} - 0.5} = \frac{2}{0.5} + \frac{2}{\sqrt{2} - 0.5} + \cdots S_1 = 4$

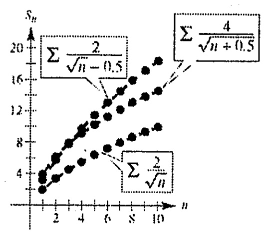
$\sum_{n=1}^{\infty} \frac{4}{\sqrt{n} + 0.5} = \frac{4}{\sqrt{1.5}} + \frac{4}{\sqrt{2.5}} + \cdots S_1 \approx 3.3$



(b) The first series is a p-series. It diverges $\left(p = \frac{1}{2} < 1\right)$.

(c) The magnitude of the terms of the other two series are greater than the corresponding terms of the divergent p-series. So, the other two series diverge.

(d) The larger the magnitude of the terms, the larger the magnitude of the terms of the sequence of partial sums.



3. $\frac{1}{2n - 1} > \frac{1}{2n} > 0$ for $n \geq 1$

Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{2n - 1}$$

diverges by comparison with the divergent p-series

$$\frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n}.$$

4. $\frac{1}{3n^2 + 2} < \frac{1}{3n^2}$

Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{3n^2 + 2}$$

converges by comparison with the convergent p-series

$$\frac{1}{3}\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

5. $\frac{1}{\sqrt{n} - 1} > \frac{1}{\sqrt{n}}$ for $n \geq 2$

Therefore,

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} - 1}$$

diverges by comparison with the divergent p-series

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}.$$

6. $\frac{4^n}{5^n + 3} < \left(\frac{4}{5}\right)^n$

Therefore,

$$\sum_{n=0}^{\infty} \frac{4^n}{5^n + 3}$$

converges by comparison with the convergent geometric series

$$\sum_{n=0}^{\infty} \left(\frac{4}{5}\right)^n.$$

7. For $n \geq 3$, $\frac{\ln n}{n + 1} > \frac{1}{n + 1} > 0$.

Therefore,

$$\sum_{n=1}^{\infty} \frac{\ln n}{n + 1}$$

diverges by comparison with the divergent series

$$\sum_{n=1}^{\infty} \frac{1}{n + 1}.$$

Note: $\sum_{n=1}^{\infty} \frac{1}{n + 1}$ diverges by the Integral Test.

8. $\dfrac{1}{\sqrt{n^3+1}} < \dfrac{1}{n^{3/2}}$

Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$$

converges by comparison with the convergent p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}.$$

9. For $n > 3$, $\dfrac{1}{n^2} > \dfrac{1}{n!} > 0$.

Therefore,

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

converges by comparison with the convergent p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

10. $\dfrac{1}{4\sqrt[3]{n}-1} > \dfrac{1}{4\sqrt[4]{n}}$

Therefore,

$$\sum_{n=1}^{\infty} \frac{1}{4\sqrt[3]{n}-1}$$

diverges by comparison with the divergent p-series

$$\frac{1}{4}\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n}}.$$

11. $0 < \dfrac{1}{e^{n^2}} \le \dfrac{1}{e^n}$

Therefore,

$$\sum_{n=0}^{\infty} \frac{1}{e^{n^2}}$$

converges by comparison with the convergent geometric series

$$\sum_{n=0}^{\infty} \left(\frac{1}{e}\right)^n.$$

12. $\dfrac{3^n}{2^n-1} > \left(\dfrac{3}{2}\right)^n$ for $n \ge 1$

Therefore,

$$\sum_{n=1}^{\infty} \frac{3^n}{2^n-1}$$

diverges by comparison with the divergent geometric series

$$\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n.$$

13. $\displaystyle\lim_{n\to\infty} \frac{n/(n^2+1)}{1/n} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$

Therefore,

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1}$$

diverges by a limit comparison with the divergent p-series

$$\sum_{n=1}^{\infty} \frac{1}{n}.$$

14. $\displaystyle\lim_{n\to\infty} \frac{5/(4^n+1)}{1/4^n} = \lim_{n\to\infty} \frac{5 \cdot 4^n}{4^n+1} = 5$

Therefore,

$$\sum_{n=1}^{\infty} \frac{5}{4^n+1}$$

converges by a limit comparison with the convergent geometric series

$$\sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n.$$

15. $\displaystyle\lim_{n\to\infty} \frac{1/\sqrt{n^2+1}}{1/n} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1$

Therefore,

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^2+1}}$$

diverges by a limit comparison with the divergent p-series

$$\sum_{n=1}^{\infty} \frac{1}{n}.$$

16. $\displaystyle\lim_{n\to\infty} \frac{(2^n+1)/(5^n+1)}{(2/5)^n} = \lim_{n\to\infty} \frac{2^n+1}{5^n+1} \cdot \frac{5^n}{2^n} = 1$

Therefore,

$$\sum_{n=1}^{\infty} \frac{2^n+1}{5^n+1}$$

converges by a limit comparison with the convergent geometric series

$$\sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n.$$

17. $\lim_{n\to\infty} \dfrac{\dfrac{2n^2-1}{3n^5+2n+1}}{1/n^3} = \lim_{n\to\infty} \dfrac{2n^5-n^3}{3n^5+2n+1} = \dfrac{2}{3}$

Therefore,

$\sum_{n=1}^{\infty} \dfrac{2n^2-1}{3n^5+2n+1}$

converges by a limit comparison with the convergent p-series

$\sum_{n=1}^{\infty} \dfrac{1}{n^3}.$

18. $\lim_{n\to\infty} \dfrac{1/n^2(n+3)}{1/n^3} = \lim_{n\to\infty} \dfrac{n^3}{n^2(n+3)} = 1$

Therefore,

$\sum_{n=1}^{\infty} \dfrac{1}{n^2(n+3)}$

converges by a limit comparison with the convergent p-series

$\sum_{n=1}^{\infty} \dfrac{1}{n^3}.$

19. $\lim_{n\to\infty} \dfrac{1/\left(n\sqrt{n^2+1}\right)}{1/n^2} = \lim_{n\to\infty} \dfrac{n^2}{n\sqrt{n^2+1}} = 1$

Therefore,

$\sum_{n=1}^{\infty} \dfrac{1}{n\sqrt{n^2+1}}$

converges by a limit comparison with the convergent p-series

$\sum_{n=1}^{\infty} \dfrac{1}{n^2}.$

20. $\lim_{n\to\infty} \dfrac{n/\left[(n+1)2^{n-1}\right]}{1/\left(2^{n-1}\right)} = \lim_{n\to\infty} \dfrac{n}{n+1} = 1$

Therefore,

$\sum_{n=1}^{\infty} \dfrac{n}{(n+1)2^{n-1}}$

converges by a limit comparison with the convergent geometric series

$\sum_{n=1}^{\infty} \left(\dfrac{1}{2}\right)^{n-1}.$

21. $\lim_{n\to\infty} \dfrac{\left(n^{k-1}\right)/\left(n^k+1\right)}{1/n} = \lim_{n\to\infty} \dfrac{n^k}{n^k+1} = 1$

Therefore,

$\sum_{n=1}^{\infty} \dfrac{n^{k-1}}{n^k+1}$

diverges by a limit comparison with the divergent p-series

$\sum_{n=1}^{\infty} \dfrac{1}{n}.$

22. $\lim_{n\to\infty} \dfrac{\sin(1/n)}{1/n} = \lim_{n\to\infty} \dfrac{\left(-1/n^2\right)\cos(1/n)}{-1/n^2} = \lim_{n\to\infty} \cos\left(\dfrac{1}{n}\right) = 1$

Therefore,

$\sum_{n=1}^{\infty} \sin\left(\dfrac{1}{n}\right)$

diverges by a limit comparison with the divergent p-series

$\sum_{n=1}^{\infty} \dfrac{1}{n}.$

23. $\sum_{n=1}^{\infty} \dfrac{\sqrt[3]{n}}{n} = \sum_{n=1}^{\infty} \dfrac{1}{n^{2/3}}$

Diverges;

p-series with $p = \dfrac{2}{3}$

24. $\sum_{n=0}^{\infty} 5\left(-\dfrac{4}{3}\right)^n$

Diverges;

Geometric series with $|r| = \left|-\dfrac{4}{3}\right| = \dfrac{4}{3} > 1$

25. $\sum_{n=1}^{\infty} \dfrac{1}{5^n+1}$

Converges;

Direct comparison with convergent geometric series

$\sum_{n=1}^{\infty} \left(\dfrac{1}{5}\right)^n$

26. $\sum_{n=3}^{\infty} \dfrac{1}{n^3-8}$

Converges; limit comparison with $\sum_{n=3}^{\infty} \dfrac{1}{n^3}$

27. $\sum_{n=1}^{\infty} \dfrac{2n}{3n-2}$

Diverges; n^{th}-Term Test

$\lim_{n\to\infty} \dfrac{2n}{3n-2} = \dfrac{2}{3} \neq 0$

28. $\sum_{n=1}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)=\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\cdots=\frac{1}{2}$

Converges; telescoping series

29. $\sum_{n=1}^{\infty}\frac{n}{(n^2+1)^2}$

Converges; Integral Test

30. $\sum_{n=1}^{\infty}\frac{3}{n(n+3)}$

Converges; telescoping series

$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+3}\right)$

31. $\lim_{n\to\infty}\frac{a_n}{1/n}=\lim_{n\to\infty}na_n$. By given conditions $\lim_{n\to\infty}na_n$ is finite and nonzero. Therefore,

$\sum_{n=1}^{\infty}a_n$

diverges by a limit comparison with the p-series

$\sum_{n=1}^{\infty}\frac{1}{n}$.

32. If $j<k-1$, then $k-j>1$. The p-series with $p=k-j$ converges and because

$\lim_{n\to\infty}\frac{P(n)/Q(n)}{1/n^{k-j}}=L>0$, the series $\sum_{n=1}^{\infty}\frac{P(n)}{Q(n)}$

converges by the Limit Comparison Test. Similarly, if $j\ge k-1$, then $k-j\le 1$ which implies that

$\sum_{n=1}^{\infty}\frac{P(n)}{Q(n)}$

diverges by the Limit Comparison Test.

33. $\frac{1}{2}+\frac{2}{5}+\frac{3}{10}+\frac{4}{17}+\frac{5}{26}+\cdots=\sum_{n=1}^{\infty}\frac{n}{n^2+1}$,

which diverges because the degree of the numerator is only one less than the degree of the denominator.

34. $\frac{1}{3}+\frac{1}{8}+\frac{1}{15}+\frac{1}{24}+\frac{1}{35}+\cdots=\sum_{n=2}^{\infty}\frac{1}{n^2-1}$,

which converges because the degree of the numerator is two less than the degree of the denominator.

35. $\sum_{n=1}^{\infty}\frac{1}{n^3+1}$

converges because the degree of the numerator is three less than the degree of the denominator.

36. $\sum_{n=1}^{\infty}\frac{n^2}{n^3+1}$

diverges because the degree of the numerator is only one less than the degree of the denominator.

37. $\lim_{n\to\infty}n\left(\frac{n^3}{5n^4+3}\right)=\lim_{n\to\infty}\frac{n^4}{5n^4+3}=\frac{1}{5}\neq 0$

Therefore, $\sum_{n=1}^{\infty}\frac{n^3}{5n^4+3}$ diverges.

38. $\lim_{n\to\infty}\frac{n}{\ln n}=\lim_{n\to\infty}\frac{1}{1/n}=\lim_{n\to\infty}n=\infty\neq 0$

Therefore, $\sum_{n=2}^{\infty}\frac{1}{\ln n}$ diverges.

39. $\frac{1}{200}+\frac{1}{400}+\frac{1}{600}+\cdots=\sum_{n=1}^{\infty}\frac{1}{200n}$

diverges, (harmonic)

40. $\frac{1}{200}+\frac{1}{210}+\frac{1}{220}+\cdots=\sum_{n=0}^{\infty}\frac{1}{200+10n}$

diverges

41. $\frac{1}{201}+\frac{1}{204}+\frac{1}{209}+\frac{1}{216}=\sum_{n=1}^{\infty}\frac{1}{200+n^2}$

converges

42. $\frac{1}{201}+\frac{1}{208}+\frac{1}{227}+\frac{1}{264}+\cdots=\sum_{n=1}^{\infty}\frac{1}{200+n^3}$

converges

43. Some series diverge or converge very slowly. You cannot decide convergence or divergence of a series by comparing the first few terms.

44. See Theorem 9.12, page 612. One example is

$\sum_{n=1}^{\infty}\frac{1}{n^2+1}$ converges because $\frac{1}{n^2+1}<\frac{1}{n^2}$ and

$\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges (p-series).

45. See Theorem 9.13, page 614. One example is

$\sum_{n=2}^{\infty}\frac{1}{\sqrt{n-1}}$ diverges because $\lim_{n\to\infty}\frac{1/\sqrt{n-1}}{1/\sqrt{n}}=1$ and

$\sum_{n=2}^{\infty}\frac{1}{\sqrt{n}}$ diverges (p-series).

46. This is not correct. The beginning terms do not affect the convergence or divergence of a series. In fact,

$$\frac{1}{1000} + \frac{1}{1001} + \cdots = \sum_{n=1000}^{\infty} \frac{1}{n} \text{ diverges (harmonic)}$$

and $1 + \frac{1}{4} + \frac{1}{9} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series).

47. (a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \sum_{n=1}^{\infty} \frac{1}{4n^2 - 4n + 1}$

converges because the degree of the numerator is two less than the degree of the denominator. (See Exercise 32.)

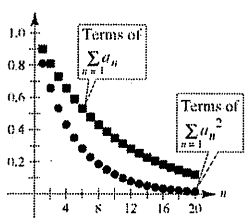
(b)

n	5	10	20	50	100
S_n	1.1839	1.2087	1.2212	1.2287	1.2312

(c) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8} - S_2 \approx 0.1226$

(d) $\displaystyle\sum_{n=10}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8} - S_9 \approx 0.0277$

48.



For $0 < a_n < 1, 0 < a_n^2 < a_n < 1$.

So, the lower terms are those of Σa_n^2.

49. False. Let $a_n = \frac{1}{n^3}$ and $b_n = \frac{1}{n^2}$. $0 < a_n \le b_n$ and both

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converge.

50. True

51. True

52. False. Let $a_n = 1/n, b_n = 1/n, c_n = 1/n^2$. Then,

$a_n \le b_n + c_n$, but $\displaystyle\sum_{n=1}^{\infty} c_n$ converges.

53. True

54. False. $\displaystyle\sum_{n=1}^{\infty} a_n$ could converge or diverge.

For example, let $\displaystyle\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$, which diverges.

$0 < \frac{1}{n} < \frac{1}{\sqrt{n}}$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, but

$0 < \frac{1}{n^2} < \frac{1}{\sqrt{n}}$ and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

55. Because $\displaystyle\sum_{n=1}^{\infty} b_n$ converges, $\lim_{n\to\infty} b_n = 0$. There exists N such that $b_n < 1$ for $n > N$. So, $a_n b_n < a_n$ for $n > N$ and $\displaystyle\sum_{n=1}^{\infty} a_n b_n$ converges by comparison to the convergent series $\displaystyle\sum_{i=1}^{\infty} a_n$.

56. Because $\displaystyle\sum_{n=1}^{\infty} a_n$ converges, then

$\displaystyle\sum_{n=1}^{\infty} a_n a_n = \sum_{n=1}^{\infty} a_n^2$ converges by Exercise 55.

57. $\sum \frac{1}{n^2}$ and $\sum \frac{1}{n^3}$ both converge, and therefore, so does

$$\sum\left(\frac{1}{n^2}\right)\left(\frac{1}{n^3}\right) = \sum \frac{1}{n^5}.$$

58. $\sum \frac{1}{n^2}$ converge, and therefore, so does

$$\sum\left(\frac{1}{n^2}\right)^2 = \sum \frac{1}{n^4}.$$

59. Suppose $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and Σb_n converges.

From the definition of limit of a sequence, there exists $M > 0$ such that

$$\left|\frac{a_n}{b_n} - 0\right| < 1$$

whenever $n > M$. So, $a_n < b_n$ for $n > M$. From the Comparison Test, Σa_n converges.

60. Suppose $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and Σb_n diverges. From the definition of limit of a sequence, there exists $M > 0$ such that

$$\frac{a_n}{b_n} > 1$$

for $n > M$. So, $a_n > b_n$ for $n > M$. By the Comparison Test, Σa_n diverges.

61. (a) Let $\sum a_n = \sum \frac{1}{(n+1)^3}$, and $\sum b_n = \sum \frac{1}{n^2}$, converges.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1/\left[(n+1)^3\right]}{1/(n^2)} = \lim_{n\to\infty} \frac{n^2}{(n+1)^3} = 0$$

By Exercise 59, $\sum_{n=1}^{\infty} \frac{1}{(n+1)^3}$ converges.

(b) Let $\sum a_n = \sum \frac{1}{\sqrt{n}\pi^n}$, and $\sum b_n = \sum \frac{1}{\pi^n}$, converges.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1/\left(\sqrt{n}\pi^n\right)}{1/(\pi^n)} = \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$$

By Exercise 59, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\pi^n}$ converges.

62. (a) Let $\sum a_n = \sum \frac{\ln n}{n}$, and $\sum b_n = \sum \frac{1}{n}$, diverges.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{(\ln n)/n}{1/n} = \lim_{n\to\infty} \ln n = \infty$$

By Exercise 60, $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ diverges.

(b) Let $\sum a_n = \sum \frac{1}{\ln n}$, and $\sum b_n = \sum \frac{1}{n}$, diverges.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{\ln n} = \infty$$

By Exercise 60, $\sum \frac{1}{\ln n}$ diverges.

63. Because $\lim_{n\to\infty} a_n = 0$, the terms of $\Sigma \sin(a_n)$ are positive for sufficiently large n. Because

$$\lim_{n\to\infty} \frac{\sin(a_n)}{a_n} = 1 \text{ and } \sum a_n$$

converges, so does $\Sigma \sin(a_n)$.

64. $$\sum_{n=1}^{\infty} \frac{1}{1+2+\cdots+n} = \sum_{n=1}^{\infty} \frac{1}{\left[n(n+1)\right]/2} = \sum_{n=1}^{\infty} \frac{2}{n(n+1)}$$

Because $\Sigma 1/n^2$ converges, and

$$\lim_{n\to\infty} \frac{2/\left[n(n+1)\right]}{1/(n^2)} = \lim_{n\to\infty} \frac{2n^2}{n(n+1)} = 2,$$

$\sum \frac{1}{1+2+\cdots+n}$ converges.

65. First note that $f(x) = \ln x - x^{1/4} = 0$ when $x \approx 5503.66$. That is,

$\ln n < n^{1/4}$ for $n > 5504$

which implies that

$\frac{\ln n}{n^{3/2}} < \frac{1}{n^{5/4}}$ for $n > 5504$.

Because $\sum_{n=1}^{\infty} \frac{1}{n^{5/4}}$ is a convergent p-series,

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$$

converges by direct comparison.